The Influence of Different Production Functions on Modeling Resource Extraction and Economic Growth

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Abstract

In this paper we discuss the influence of using different production functions on modeling the resource extraction rates and economic growth. The focus is set on the modeling of the production sector, which requires either non-renewable resources, renewable resources or a combination of both resources for production. There are great differences between the possible assumptions when modeling the substitution process between the different input factors. It is shown that the existence of an optimal extraction rate in conjunction with economic growth depends on the specification of the production function even if the same parameterization is used. The target is to provide an overview on the different possibilities of modeling, and to support the decision which kind of production function should be used for modeling different aspects of economic growth.

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1 Introduction

In recent years, the substitution of different energies is among the most prominent issues in environmental economics. This is due to the fact that there has to be an alternative to burning fossil fuels and emitting carbon dioxide into the environment, an alternative which causes less damage. In addition, we have to keep in mind that the stock of non-renewable resources is limited, and the depletion of the total stock of natural resources including the renewable ones has to be prevented.

This paper focuses on the influence of using different production functions on modeling the resource extraction rates and economic growth. A wide field of economic research deals with use of non-renewable and renewable resources and their optimal depletion or extraction rates. In the past most researchers concentrated on non-renewable resources, while in recent years renewable resource became the focus of attention. As both kinds of resources are determined by different properties, most research is done by focusing on only one of the two subject areas. By using both resources in one model, the substitution effects can be shown.

Most models are first developed in a very general way without specifying any functions. Often the Cobb-Douglas specification is used to visualize and clarify the results, because of its simplicity and easy mathematical structure. It can be shown that, if you use different kinds of specifications of production functions, you will receive different results.

In the second section, an overview of the different kinds of specifications of production functions is given. Models using either only non-renewable resources, only renewable resources, or both types of resources are analyzed. Section three presents a basic theoretical model, which is taken as a basis for our comparing study in section four, where the different kinds of production functions are used for finding the optimal solution in these models. For visualization, the model is solved numerically with one set of calibrated variables. Section five shows the main conclusions. To keep the model clear a mathematical appendix is added.

2 Different Kinds of Specification

2.1 General Distinction of Production Function

Different kinds of production functions are used by various economists. There are four main types in the theoretical and empirical literature, the Cobb-Douglas (CD) and the Constant Elasitcity of Substitution (CES) production functions, as well as production functions with variable elasticity of substitution (VES) and Leontief production functions. The last one is quite uninteresting if you analyze economic growth because of the assumption that there exists a fixed input ration and the scarcest factor limits the production (cf. Meyer et al. (1998) pp. 21-22). The special feature of the first two kinds of production functions is that they start from the assumption of constant elasticity of substitution. They are common in economic modeling because they involve quite easy mathematical solving. Production functions with a changing elasticity of substitution are quite complex in modeling and, therefore, hardly ever used.

All production functions incorporate different kinds of inputs as capital, labor, human capital, energy and/or different resources, land and other factors as pollution or environmental quality. Thereby each paper concentrates on the main inputs relevant for the research question to simplify the model. This paper concentrates on the energy and resource aspects of the production functions and on how they are linked.

Resources in total have to be divided into different groups. There are renewable resources, which grow over time, and non-renewable resources, which do not. Often non-renewable is used as synonymous with exhaustible; this is not completely correct because over-exploitation of renewable resources also leads to exhaustion.

To compare the different models more easily, a general notation is used. The variables used by the cited authors are renamed. Equations are used, where Y is the total production output, K is the capital stock, L is the labor force, Z is the non-renewable resource used for production, S is the stock of the non-renewable resource, R is the renewable resource used for production and A is the stock of the renewable resource, P describes the technical progress. The production coefficients of Cobb-Douglas production functions are marked with αi, with $i = 1, 2, .., \infty$ and the constraint $\sum \alpha_i = 1$. Further notations are made when necessary.

2.2 Using Only Non-renewable Resources

In a special issue of the Review of Economic Studies in 1974 the first economic growth models which included the influence of non-renewable resources as it is considered today were published. Stiglitz (1974a,b) uses the following Cobb-Douglas production function

$$Y = K^{\alpha 1} \cdot L^{\alpha 2} \cdot Z^{\alpha 3} \cdot e^{P \cdot t} \tag{1}$$

The stock of the resource develops with

$$\dot{S} = -Z \tag{2}$$

If a non-renewable resource is used in production, the steady-state is the path along which consumption is growing. There is only one state where the capital-output ratio K/Y and the ratio of used resource to resource stock Z/S are constant. It can only be reached due to the rate of resource augmenting technological progress P. In case the economy departs from this path, there exists a finite time horizon after which consumption stops growing and the resource is fully exhausted. Pezzey and Withagen (1998) proved that in a model with static technology and non-renewable resources the optimal consumption path is single-peaked, when maximizing total utility. From some point in time it always declines monotonically.

Dasgupta and Heal (1974) use a quite different approach to solve the same problem. They use a CES production function in the form of

$$Y = \left(\beta \cdot K^{\theta} + (1 - \beta) \cdot Z^{\theta}\right)^{\frac{1}{\theta}} \tag{3}$$

where $0 < \beta < 1$ and $\theta = \frac{\sigma - 1}{\sigma}$ with $0 \leq \sigma \leq \infty$. How the substitution parameter β is determined is further discussed by Klump and Preissler (2000) pp. 51-52. In the case of $\sigma \leq 1$ the non-renewable resources are essential to production. For $\sigma < 1$ the evolution of the stock of the non-renewable resources develops as before. The production and therefore the consumption level rises at first, but tends to zero during time. For $\sigma = 1$, the production function is not defined, because $\theta = 0$. It leads to the special case of a Cobb-Douglas production function, as used in (1). If $\sigma > 1$, there is no problem because the resource is inessential for production. To overcome this results technical change is introduced at a specific date T, when a perfect substitute for the non-renewable resource, in form of a flow of services at a constant rate, is discovered. At point T the economy switches, and production and consumption rise again.

A quite similar approach is used by Scholz and Ziemes (1999). Research activity in a growing amount of different firms N leads to an increasing variety of capital inputs X_i, which can overcome the scarcity of the essential input of the non-renewable resources. The production function reads as follows

$$Y = P \cdot \sum_{i=1}^{N} \left(X_i^{\alpha 1}\right) \cdot L^{\alpha 2} \cdot Z^{\alpha 3} \tag{4}$$

Even though research activities do not directly reduce the non-renewable input, they decrease the energy-intensity of output as they increase the productivity of all factors. A similar approach is used by Antony (2010), in which research activity is not only addressing the resource sector but also the labor sector. Because of the not-resource- or energy-specific forces at work, the results of models where the scarcity of resources is compensated by research in other fields, are not completely satisfying (Pittel and Rübbelke, 2011, p. 8).

In the model of de V. Cavalcanti et al. (2001) the used resource is as well non-renewable, but in contrast to the other models the authors assume that new reserves can be found and old fields can be developed to produce more resources. Both activities require investments I by the owner. Now, in contrast to (2), the development of the stock of the non-renewable resources reads

$$\dot{S} = -Z + I \tag{5}$$

The production function is nearly the same as (3), despite the fact that the technological progress is now labor augmenting. Stürmer and Schwerhoff (2012) use the same idea; they assign that the stock of the non-renewable resources is growing by innovations in the extraction technology. Both models lead to long-run growth because the resource is in fact non-renewable, but at the same time inexhaustible.

So far it is shown that, in theory, long-run growth under special circumstances is possible. There is a wide critical literature about this topic, due to the fact that the previously discussed models do not consider that the use of non-renewable resources is associated to negative environmental externalities. On the one hand, the negative effect of pollution Q by the use of non-renewable resources can be incorporated into the utility function, on the other hand, into the production function. Grimaud and Rougé (2005) use a fairly standard Cobb-Douglas production function $Y = L^{\alpha 1} \cdot Z^{\alpha 2} \cdot P^{\lambda}$. Pollution is generated by the use of the resource within the production process. The pollution effect is incorporated into a separable instantaneous utility function with a negative partial derivation

$$U\left(C, Q\right) = \frac{C^{1-\epsilon 1}}{1-\epsilon 1} - \frac{Q^{1+\epsilon 2}}{1+\epsilon 2} \tag{6}$$

Under optimal conditions, a non-selfish present generation, or a generation with preference for environment quality, decelerates the extraction of the resource. But in a decentralized economy the negative effect of the use of the resource is not incorporated and the same types of households would generate even faster resource depletion. In contrast to Grimaud and Rougé (2005), Schou (2000) introduces the pollution into the production function with constant returns to scale with respect to the variable inputs of a firm.

$$Y = P \cdot K^{\alpha 1} \cdot L^{\alpha 2} \cdot Z^{\alpha 3} \cdot Q^{-\delta} \tag{7}$$

The elasticity of the pollution function depending on the resources is constant, and the positive effect in production outbalances the negative one of pollution by using the resources. In this case long run growth is only possible under special circumstances. The resource use must fall over time along the balanced growth path, and the flow of pollution is bound to get reduced as well. Production does not have to be zero because of the technological parameters, but there has to be a declining consumption in the long run.

2.3 Using Only Renewable Resources

If you only use renewable resources in production, on first sight there seems to be no problem. The simplest case is an economy without production, in which the resource is the only good (Chichilnisky et al., 1998). The dynamics of a renewable resource are described by

$$\dot{A} = \eta_A(A) - R \tag{8}$$

with η_A being the rate of resource generation. In this case, if consumption C is less than the regeneration rate $\eta_A(A)$, the stock of the resource A is rising, if it is more, the stock declines and tends to zero, and at the same time the consumption possibilities are falling. If a simple production function as

$$Y = K^{\alpha 1} \cdot R^{\alpha 2} \tag{9}$$

is used, the same results arise, only with the difference that there are higher consumption possibilities for the same amount of resource due to the capital input (Chichilnisky et al., 1998, p. 66-69).

Valente (2005) includes in his considerations labor as an essential input, which is growing with the constant population growth rate and different specifications of technological progress. They maximize total discounted utility. The production function is the same as in (1), except for the substitution of the non-renewable Z by the renewable resource R.

$$Y = K^{\alpha 1} \cdot L^{\alpha 2} \cdot R^{\alpha 3} \cdot e^{P \cdot t} \tag{10}$$

If the rate of resource regeneration $\eta_A(A)$ is higher than the social discount rate, sustainable per capita consumption is possible. But if the rate is relatively low, the time-path is still single-peaked as in the case of an exhaustible resource.

A different development of the stock of the renewable resources is assumed by Ruiz-Tamarit and Sánchez-Moreno (2006). As only the not extracted amount $(1 - r) \cdot A$ can be regenerated, the extraction rate r is determined endogenously and influences the regeneration rate.

$$\dot{A} = \eta_A (1 - r) \cdot A - r \cdot A \tag{11}$$

They reveal basically the same results; positive long-run growth is possible under an optimal harvest rate. But this rate is restricted by the regeneration rate of the renewable resources. Under special circumstances (an impatient society with a high discount rate) a future collapse could also be the optimal, but not preferable strategy.

2.4 Using Both Kinds of Resources

The more interesting and realistic cases are those when both, non-renewable and renewable, or different kinds of resources are used. There are two possibilities, either the non-renewable resources are first fully physically depleted and then substituted by the renewable resources, or there is a smooth process where substitution develops over time and both resources are used simultaneously. Additionally, Tahvonen and Salo (2001) (p. 1381) introduce the possibility that the substitution process can evolve in both directions (from renewable into fossil and back into renewable) at different stages of the long run development of the economy.

$$Y = K^{\alpha 1} \cdot (Z + R)^{\alpha 2} \tag{12}$$

The substitution process depends on the assumed extraction cost functions. This kind of simple CES production function is also used by Pittel and Bretschger (2010). The resources are used to produce different intermediates in different sectors with different resource intensities.

In almost the same manner Grimaud et al. (2007) use a standard Cobb-Douglas production function to model the production of a homogeneous good. They regard energy as an input factor which is discribed by a function using non-renewable and renewable resources as well as human capital (here simplified), see also Popp (2006)

$$Y = P \cdot K^{\alpha 1} \cdot L^{\alpha 2} \cdot ENERGY^{\alpha 3} \tag{13}$$

$$ENERGY = \left(H^{\theta 1} + \left(Z^{\theta 2} + R^{\theta 2}\right)^{\frac{\theta 1}{\theta 2}}\right)^{\frac{1}{\theta 1}} \tag{14}$$

Using this kind of production function for the energy input allows the resources to be imperfect substitutes. Other users of this kind of production function are Gerlagh and van der Zwaan (2003). They integrate the CES energy function into the final good production function by using again a CES function, where the second part consists of the further inputs.

$$Y = \left(P \cdot \left(K^{\alpha 1} \cdot L^{\alpha 2}\right)^{\theta 1} + \left(H \cdot \left((1-\beta)\right) \cdot Z^{\theta 2} + \beta \cdot R^{\theta 2}\right)^{\frac{1}{\theta 2}}\right)^{\theta 1}\right)^{\frac{1}{\theta 1}} \tag{15}$$

Nearly the same specification for energy production is used by Growiec and Schumacher (2008). They, additionally, introduce the technological progress, either into the substitution elasticity $\theta 2$ or into the distribution parameter β. Thereby they increase the flexibility of the model. With the changing substitution elasticity they can figure out the state where the resources become perfect substitutes.

In contrast to this, in the model of Nguyen and Nguyen-Van (2008) both kinds of resources are essential for production and cannot be totally substituted.

$$Y = K^{\alpha 1} \cdot L^{\alpha 2} \cdot Z^{\alpha 3} \cdot R^{\alpha 4} \tag{16}$$

They have proven the existence of an optimal solution to the social planner's problem. An example of applying such a production function to real data of a special country is given by Lafforgue (2008). André-García and Cerdá-Tena (2001) use this Cobb-Douglas production function without labor and capital to show how the output is changed and if a steady state exists, when different kinds of resources are used. They illustrate three different cases, the use of two non-renewable, the use of two renewable and the use of one renewable and one non-renewable resource.

Another specification is that resources which are used for production are not only extracted from the stock, but also recycled out of waste, see Pittel et al. (2010). The share of consumption C which was made out of the resources and waste is added to the stock of waste, the used amount is deducted.

$$\dot{Waste} = -UsedWaste + \frac{Waste + Z}{Y} \cdot C \tag{17}$$

Recycled waste is then used as an essential input in a Cobb-Douglas production function.

3 Basic Model

Now the general model which is taken as the basis for the following comparing study is introduced. A standard model of endogenous economic growth is used. The production consists of capital K, labor L, human capital H, technological change P and the two resources Z and R with the option that not all input factors have to be used. The general production function reads

$$Y = F\left(K, L, H, P, Z, R\right) \tag{18}$$

The accumulation of capital is fairly standard

$$\dot{K} = Y - C \tag{19}$$

The stocks of the resources evolve as follows (see (2) and (8))

$$\dot{A} = \eta_A\left(A\right) - R \tag{20}$$

$$\dot{S} = -Z \tag{21}$$

Total population equals total labor input. Therefore, the labor force grows exogenously at the population growth rate n.

$$\dot{L} = n \tag{22}$$

Human capital is only an important input factor in some models. As a standard evolution of the stock the following is assumed

$$\dot{H} = h\left(LH, H\right) \tag{23}$$

where $h_L H$ is the marginal labor productivities in this sector when LH is the share of labor used in the human capital sector with $L = LH + LY$.

A representative, infinitely living household, which maximizes the discounted lifetime utility, is considered

$$U = \int_0^\infty u\left(C\right) \cdot e^{-\rho \cdot t} \tag{24}$$

where ρ is the time preference rate. A current value Hamiltonian of the social planner's problem is set up with all accumulation equation as constraints

$$HAM\left(C, L, R, Z, K, H, A, S, P\right) = u\left(C\right) + \lambda 1 \cdot \dot{K} + \lambda 2 \cdot \dot{A} + \lambda 3 \cdot \dot{S} + \lambda 4 \cdot \dot{H} \tag{25}$$

where λi, with $i = 1, 2, .., \infty$ are the co-state variables. After eliminating them from the first order conditions, see Appendix A, the following conditions hold

$$Y_K = -\frac{\dot{U_C}}{U_C} + \rho \tag{26}$$

$$Y_K = \frac{\dot{Y_R}}{Y_R} + \eta_A \tag{27}$$

$$Y_K = \frac{\dot{Y_Z}}{Y_Z} \tag{28}$$

$$Y_K = \frac{\dot{Y_{LY}}}{Y_{LY}} - \frac{\dot{h_{LY}}}{h_{LY}} + h_H + \frac{U_C \cdot Y_H}{U_C \cdot Y_{LY} - h_{LY}} \tag{29}$$

where (26) reflects the Keynes-Ramsey Rule and (27) and (28) the Hotelling Rules for renewable and nonrenewable resources. Using these conditions (26) to (29) and the law of motions for the stock variables, it is possible to reveal the corresponding growth rates and starting values for the control variables.

4 Analyzing the Influence

To analyze the influence of the different production functions on the results of the models, now the different production functions from Chapter 2 are inserted in the basic model (see Chapter 3) to solve for the growth rates. The basic specification of the utility function (24) is[1]

$$max \int_0^\infty \frac{C^{1-\epsilon} - 1}{1 - \epsilon} e^{-\rho t} dt \tag{30}$$

4.1 Using Cobb-Douglas Production Functions

The first calculations show the influence of the different production functions on the growth rates, when only non-renewable resources are used or when only renewable resources are used. It is assumed that the size of the population grows exogenously with n.

In case [1] the derivations of the production function (1), $Y = K^{\alpha 1} \cdot L^{\alpha 2} \cdot Z^{\alpha 3} \cdot e^{P \cdot t}$, are used and substituted into conditions (26) and (28). The following growth rates result when only non-renewable resources are used:

$$gC_t = \frac{1}{\epsilon}\left(\left(K_t^{\alpha 1-1} \alpha 1 L_t^{\alpha 2} Z_t^{\alpha 3} e^{P \cdot t}\right) - \rho\right) \tag{31}$$

$$gZ_t = \frac{K_t^{\alpha 1-1} \alpha 1 L_t^{\alpha 2} Z_t^{\alpha 3} e^{P \cdot t} - gC_t \alpha 1 - n\alpha 2 - P}{\alpha 3 - 1} \tag{32}$$

In case [2] when using production function (10),$Y = K^{\alpha 1} \cdot L^{\alpha 2} \cdot R^{\alpha 3} \cdot e^{P \cdot t}$, with only renewable resources and substituting the derivations into (26) and (27) theses growth

[1]In the following we set $\epsilon = 0.8$ and $\rho = 0.04$.

rates are the result

$$gC_t = \frac{1}{\epsilon}\left(\left(K_t^{\alpha 1-1}\alpha 1 L_t^{\alpha 2} R_t^{\alpha 3} e^{P\cdot t}\right) - \rho\right) \tag{33}$$

$$gR_t = \frac{K_t^{\alpha 1-1}\alpha 1 L_t^{\alpha 2} R_t^{\alpha 3} e^{P\cdot t} - gC_t\alpha 1 - n\alpha 2 - P - \eta_A}{\alpha 3 - 1} \tag{34}$$

The first impression may be that the results do not differ much from each other. The only difference seems to be the regeneration rate η_A in equation (34). The differences are revealed by taking the dynamics of the stocks into account (for the non-renewable resource (2) and for the renewable resource (8)). Using the same calibration, the optimal starting values for C and Z and R, respectively, can be revealed and the growth rates and the extraction rates, respectively, can be compared.[2] In both cases long run economic growth is possible, depending to a varying degree on the rate of exogenous economic progress P. In the case of renewable resources [2], even without economic progress, long-run growth is possible. In the case of non-renewable resources, the economic progress compensates for the declining amount of resource used for production. If the rate is too low, the whole economy collapses. With the same amount of economic progress P, the economy of case [1] starts with a level of supply of about $C_0^1 = 0.46$ units, the economy of case [2] with a higher level of $C_0^2 = 0.76$ units, due to the fact, that there is a higher input of resources in the second case ($Z_0^1 = 0.0006$ units compared to $R_0^2 = 0.14$ units). Additionally, the growth rates have different levels; case [1] grows at 0.22 percent, whereas case [2] grows at 4.15 percent per period of time. The amount of resources used for production in each period is increasing in case [2] at a rate of 2.81 percent, in case [1] it is declining at a rate of -1.96 percent in each period.

If there is no exogenous economic progress in production function (10), there is still growth at a rate of 1.95 percent per period of time, due to the fact that in each period more resources are used. The input of renewable resources is growing at a rate of 2.39 percent, in contrast to 2.81 percent in the case of technological progress. The difference is again based on the technological progress which is also affecting the resources as input to further production possibilities.

With the current calibration of the model, we receive the same results as if we do not factor labor as input in the model (result for the production function in equation (9)). If population growth is considered and if labor is employed as input factor in production, all rates and values are slightly higher.

The next step is to compare these results to a model where both resources are used. In the style of (16) we use a CD production function with both resources. Additionally we introduce exogenous technological progress as before (with $e^{P\cdot t}$), for a better

[2]The production coefficients are set to $\alpha 2 = 1/10, \alpha 1 = \alpha 3 = 45/100$. A constant population is assumed, $n = 0$, and the technological progress is $P = 1/100$. The initial endowment of the stocks are $K_0 = 10$, $S_0 = 50$, $A_0 = 50$ and one unit of labor is employed. The regeneration rate is set to $\eta_A = 0.04$.

comparability.[3] The third case yields to the following growth rates

$$gC_t = \frac{1}{\epsilon}\left(\left(K_t^{\alpha 1-1}\alpha 1 L_t^{\alpha 2} R_t^{\alpha 3} Z_t^{\alpha 4} e^{P\cdot t}\right) - \rho\right) \tag{35}$$

$$gZ_t = \frac{K_t^{\alpha 1-1}\alpha 1 L_t^{\alpha 2} R_t^{\alpha 3} Z_t^{\alpha 4} e^{P\cdot t} - (n\alpha 2 + P + \alpha 3\eta_A + gC_t\alpha 1)}{\alpha 3 + \alpha 4 - 1} \tag{36}$$

$$gR_t = \frac{K_t^{\alpha 1-1}\alpha 1 L_t^{\alpha 2} R_t^{\alpha 3} Z_t^{\alpha 4} e^{P\cdot t} - (n\alpha 2 + P + (1-\alpha 4)\,\eta_A + gC_t\alpha 1)}{\alpha 3 + \alpha 4 - 1} \tag{37}$$

The differences between the growth rates using different types of resources are again explained mainly by the rate of regeneration η_A. Additionally, it can be shown that the growth rates strongly depend on the production coefficient and the used amount of the other resource. Because of using an essential non-renewable resource in the production process, the initial consumption level and in unison, the growth rate of consumption are less than in case [2], but because of the use of the renewable resource higher than in case [1] ($C_0^3 = 0.49$ growing at 3.34 percent). If there is no technological progress, the growth rate of consumption tends to zero, being slightly negative. In contrast to the cases, when only one resource is used, the change of input of the renewable resource is slightly higher at 2.66 percent than before, and the decline of input of the non-renewable resource is slightly lower at -1.34 percent.

The production function (4) does not alter the results in our set up. Capital K is split into different capital goods X_i, leading to a different possibility to generate technological progress. On overview on how technical change is directed can be found in Acemoglu (2002).

4.2 Using CES Production Functions

Until now, the focus was on the use of the Cobb-Douglas production function, where all input factors are always essential for production. We now use the more general CES production function, as introduced in (3), $Y = \left(\beta \cdot K^\theta + (1-\beta)\cdot Z^\theta\right)^{\frac{1}{\theta}}$. The equations for the calculation of the growth rates when using a non-renewable resource read

$$gC_t = \frac{1}{\epsilon}\left(\left(\beta K_t^{\frac{\sigma-1}{\sigma}} + (1-\beta)\, Z_t^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} \beta K_t^{-\frac{1}{\sigma}} - \rho\right) \tag{38}$$

$$gZ_t = -\left(\beta K_t^{\frac{\sigma-1}{\sigma}} + (1-\beta)\, Z_t^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \frac{\sigma}{K_t} + gC_t \tag{39}$$

If instead of the non-renewable resource the renewable resource is used, the growth rate stays the same, except for the replacement of the resource. In this case the extraction rate reads

$$gR_t = -\left(\beta K_t^{\frac{\sigma-1}{\sigma}} + (1-\beta)\, R_t^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \frac{\sigma}{K_t} + gC_t + \left(1 + \frac{R}{K}^{\frac{\sigma-1}{\sigma}} \frac{1-\beta}{\beta}\right)\sigma\eta_A \tag{40}$$

There are four different cases [4a-d], depending on the size of the substitution elasticity σ, which have to be distinguished (see Neumeyer (2000) pp. 321-322). In the case [4a]

[3] The production coefficients are now set to $\alpha 2 = 1/10, \alpha 1 = \alpha 3 = \alpha 4 = 3/10$. The other parameterization stays the same.

when $\sigma > 1$ the non-renewable resource is inessential for the production; even if there is no input of resource into production, constant growth of consumption of 3.33 percent is possible.[4] If, additionally, a proportion of the non-renewable resource is used, the growth rate may be even higher at the beginning, until the stock of the resource is exhausted. In the long-run, the growth rate approaches the same rate as without using the resource. But the level of consumption is higher in each period during the whole time, due to the fact that the initial consumption level is higher. If a renewable resource is used instead of the non-renewable one, these effects are even greater. But for those, who include natural resources in growth theory, this case is quite uninteresting.

More interesting is the case [4b] when $\sigma < 1$.[5] In this case long-run production is theoretically possible, even if only non-renewable resources are used; but the actual rate of consumption is declining relative quickly. The production is declining and after a short time, there are hardly any goods left for consumption, so that the capital goods also have to be consumed. The growth rate and the change of the amount of resource extraction tend to -2.5 percent, which is represented by the term ρ/ϵ in equation (38); the rest of the equations (38) and (39) tend to zero. If technological progress is introduced (in the form of enhancement of the total production), economic growth is possible as long as it compensates for the decline in resource input. In our case, the technological progress must not necessarily be higher than 2.5 percent because the production is enlarged in each period.

If only renewable resources are used, long-run growth is possible again, independent of the size of σ, see equation (40). In the current setting, the final growth rate of consumption equals the growth rate of the resource and is about 2.5 percent. The size of σ has influence on the optimal size of the initial levels of consumption and resource input ($C_0 = 0.89$ units and $R_0 = 0.16$ units); if σ decreases, these levels rise. Depending on the size of σ, the regeneration rate η_A and the given stock of the resource A have to be great enough to compensate for the use of the renewable resource and to maintain a sufficiently high growth of resources.[6]

As a special case [4c], if $\sigma = 0$ is assumed, the production function changes to a production function of the Leontief type. The input factors become perfect complements. Substitution is not possible and production is only possible as long as the stock of the non-renewable resource is not depleted. The time-of-use and production depend on the ratio of resource used to the total stock per period. This case is trivial, and in the economics of long-term growth irrelevant because with research and development this stage can be overcome. If $\sigma = 1$ is set, we receive the special case of a Cobb-Douglas production function and therefore, the same results as in case [1] (see Groth (2007) p.12).

Using CES production functions the results are always quite predicable, except for the specific case [4d] if $\sigma = 1$, and therefore, it becomes a CD production function.

[4]The calibration set is now $\beta = 0.6$ and $\sigma = 1.2$. The further variables are set as before.

[5]Now $\sigma = 0.8$ is used.

[6]For $\sigma = 0.5$ the initial stock of A, when the same regeneration rate is assumed, has to be larger, leading to initial values of $C_0 = 2.33$ and $R_0 = 1.22$.

The growth functions in case of production function (12), $Y = K^{\alpha 1} \cdot (Z + R)^{\alpha 2}$ read:

$$gC_t = \frac{K_t^{\alpha 1 - 1} \alpha 1 \left(R_t + Z_t\right)^{\alpha 2} e^{P \cdot t} - \rho}{\epsilon} \tag{41}$$

$$gZ_t = \frac{\left(R_t + Z_t\right)^{\alpha 2 + 1} \alpha 1 K_t^{\alpha 1 - 1} e^{P \cdot t}}{Z_t \left(\alpha 2 - 1\right)} + \frac{\left(-gK_t \alpha 1 + \left(1 - \alpha 2\right) gR_t - P\right) R_t}{Z_t \left(\alpha 2 - 1\right)} + \frac{\left(-gK_t \alpha 1 - P\right)}{\left(\alpha 2 - 1\right)} \tag{42}$$

$$gR_t = \frac{\left(R_t + Z_t\right)^{\alpha 2 + 1} \alpha 1 K_t^{\alpha 1 - 1} e^{P \cdot t}}{R_t \left(\alpha 2 - 1\right)} + \frac{\left(-gK_t \alpha 1 + \left(1 - \alpha 2\right) gZ_t - P - \eta_A\right) R_t}{R_t \left(\alpha 2 - 1\right)} + \frac{\left(-gK_t \alpha 1 - P - \eta_A\right)}{\left(\alpha 2 - 1\right)} \tag{43}$$

In the production functions (12) as well as in (13) and (15) both kinds of resources are introduced and combined as a CES production function. The output of this function is introduced into a CD production function. The resources are treated as intermediates. Both resource extraction rates are interdependent. Again they mainly differ in the rate of the resource regenerations η_A in equation (43) and the dynamics of the stocks.[7] In this setting, when technological progress is considered (case [5a]), long-run economic growth is possible at a rate of 5.05 percent. In comparison to the other cases this rate is quite high; this is due to missing σ in comparison to cases [4], and due to the additive linkage of the resources in comparison to case [3]. The initial consumption is at $C_0^{5a} = 0.7$, the renewable resource is extracted at a rate of 2.97 percent, the non-renewable resource at a rate of -5.0 percent. If there is no technological progress (case [5b]) the optimal initial consumption reduces to $C_0^{5b} = 0.55$, growing at a rate of 2.5 percent. The extraction rate of the non-renewable resource is slightly higher at -5.5 percent and the one of the renewable resource slightly lower at 2.5 percent. If in the same setting, as case [5a], the non-renewable resource is missing, the initial amount of renewable resource has to be higher; if the renewable resource is missing, the growth rate tends to zero but remains above zero, so that constant consumption is possible due to the rate of technological progress.

To simplify the equations (13) and (15) one can choose a broader definition of capital and subsume under this term the influences of capital, human capital and labor. Then nearly the same equation as in (12) results, supplemented by the substitution factor σ.[8] In the same setting as before, in the long-run the same growth rates are achieved, starting at lower initial levels. There is an adjustment path for the extraction rate of the renewable resource from a higher rate of 5.05 percent to the rate of 2.97 percent. Associated with this is an adjustment of the growth rate of consumption from 1.6 percent to 5.05 percent. The seriously complex production functions are chosen, if you want to model the higher complexity of the reality including further influences on the extraction rates. For the purpose of mainly making statements about extraction rates and growth rates and comparing these rates with regard to the different modeling of resources, the simplified equations will meet the requirements.

[7] Now $\alpha 1 = \alpha 2 = 0.5$ is used.

[8] Assuming that the resource may be substituted one by one, $\sigma = 0.5$ is supposed.

4.3 Further Modifications on the Production Functions

The difference between the dynamics of the stock of the non-renewable resource described in equation (2) and in equation (5) is that in the latter there are additional investments to the stock of the non-renewable resource. These investments are used to develop new sources for resource extraction. Therefore the total amount of S is not fixed, and if new sources are made available, economic growth without technological progress is possible for a longer time (in contrast to case [1], when (1) is used). If equation (11) is used instead of equation (8) to describe the dynamics of the stock of the renewable resource, there is a smaller amount available for regeneration. This is due to the fact, that the extraction takes place at different points of time. This leads to lower initial amounts of the renewable resource and consumption, and as well to a lower growth rate and extraction rate, respectively (in contrast to case [2], when (10) is used).

In the functions (6) and (7) the polluting effect of the use of the non-renewable resource is additionally modeled. In the setting of (6) the pollution effect decreases the utility, which can be drawn from the consumption goods. In this case only in a setting of a social planner, the consideration leads to a smaller extraction of the non-renewable resource because only then the effect on the utility is already beard in mind, when deciding on the production. A further possibility to consider pollution is, if production function (7) is used. The extraction rates are lower as in comparison to (1) because the firms have to pay an additional amount for the pollution. The economics of scale are reduced identifiable by $\alpha 1 + \alpha 2 + \alpha 3 - \delta < 1$. In equation (17) waste is introduced as an additional input factor, which may substitute for the declining amounts of natural resources. These equations are incorporated in this paper to model some further effects that may influence the extraction rates.

5 Conclusion

In total one can say that, when only non-renewable resources are considered, economic growth is possible as long as there is a high enough rate of technological progress or when the non-renewable resource is not essential for the production process. Technological progress may be on the one hand resource saving or on the other hand supporting the total production. At first glance, there seems to be no problem at all if only renewable resources are modeled and no difference depending on which kind of production function is used. If both kinds of resources are used, the modeling approach has significant influence, because of the different characteristics of the substitution process. Especially in the case when CES production functions are used, there may be enormous changes depending on the size of σ. If the resources are combined as intermediates, the amount used from one resource always depends on the amount used from the other resource. By calculating the optimal growth path, this leads to the fact that for the total production a special amount is necessary, which may be provided by different combinations of inputs from the resources.

The contribution of this paper is, to give a first impression about the influence of choosing the one or another production function, on the used models. But there has to be done further research on specific production functions to analyze all influencing factors, especially the direction and size of the impact on the extraction rates and therefore, on the growth rate of the economy.

A Mathematical Appendix

The first order conditions $\frac{\partial HAM}{\partial C}=0$, $\frac{\partial HAM}{\partial R}=0$ $\frac{\partial HAM}{\partial Z}=0$ $\frac{\partial HAM}{\partial LY}=0$ with $LH=1-LY$ yield to

$$\lambda 1 = U_C \tag{A.1}$$
$$\lambda 2 = \lambda 1 \cdot Y_R \tag{A.2}$$
$$\lambda 3 = \lambda 1 \cdot Y_Z \tag{A.3}$$
$$\lambda 4 = \frac{\lambda 1 \cdot Y_{LY}}{h_{LY}} \tag{A.4}$$

From the Euler equations $\frac{\partial HAM}{\partial K}=\rho\lambda 1-\dot{\lambda 1}$, $\frac{\partial HAM}{\partial A}=\rho\lambda 2-\dot{\lambda 2}$, $\frac{\partial HAM}{\partial S}=\rho\lambda 3-\dot{\lambda 3}$, $\frac{\partial HAM}{\partial H}=\rho\lambda 4-\dot{\lambda 4}$ we get

$$\frac{\dot{\lambda 1}}{\lambda 1} = \rho - Y_K \tag{A.5}$$
$$\frac{\dot{\lambda 2}}{\lambda 2} = \rho - \eta_A \tag{A.6}$$
$$\frac{\dot{\lambda 3}}{\lambda 3} = \rho \tag{A.7}$$
$$\frac{\dot{\lambda 4}}{\lambda 4} = \rho - h_H - \frac{\lambda 1 \cdot Y_H}{\lambda 4} \tag{A.8}$$

The transversality conditions are

$$\lim_{0\to\infty} \lambda 1 \cdot K \cdot e^{-\rho\cdot t} = \lim_{0\to\infty} \lambda 2 \cdot R \cdot e^{-\rho\cdot t} = \lim_{0\to\infty} \lambda 3 \cdot Z \cdot e^{-\rho\cdot t} = \lim_{0\to\infty} \lambda 4 \cdot H \cdot e^{-\rho\cdot t} = 0 \tag{A.9}$$

Differentiating (A.1) to (A.4) with respect to time divided by (A.1) to (A.4) reveals

$$\frac{\dot{\lambda 1}}{\lambda 1} = \frac{\dot{U_C}}{U_C} \tag{A.10}$$
$$\frac{\dot{\lambda 2}}{\lambda 2} = \frac{\dot{\lambda 1}}{\lambda 1} + \frac{\dot{Y_R}}{Y_R} \tag{A.11}$$
$$\frac{\dot{\lambda 3}}{\lambda 3} = \frac{\dot{\lambda 1}}{\lambda 1} + \frac{\dot{Y_Z}}{Y_Z} \tag{A.12}$$
$$\frac{\dot{\lambda 4}}{\lambda 4} = \frac{\dot{\lambda 1}}{\lambda 1} + \frac{\dot{Y_{LY}}}{Y_{LY}} - \frac{\dot{h_{LY}}}{h_{LY}} \tag{A.13}$$

and combining with (A.5) to (A.8) and (A.4) and (A.1) yields

$$\rho - Y_K = \frac{\dot{U_C}}{U_C} \tag{A.14}$$
$$\rho - \eta_A = (\rho - Y_K) + \frac{\dot{Y_R}}{Y_R} \tag{A.15}$$
$$\rho = (\rho - Y_K) + \frac{\dot{Y_Z}}{Y_Z} \tag{A.16}$$
$$\rho - h_H - \frac{U_C \cdot Y_H}{U_C \cdot Y_{LY} - h_{LY}} = (\rho - Y_K) + \frac{\dot{Y_{LY}}}{Y_{LY}} - \frac{\dot{h_{LY}}}{h_{LY}} \tag{A.17}$$

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