Inter-generational Distribution of Resources in a Model of Economic Growth
Taking the Land vs. Food Trade-off into Account

Frauke Voosholz*

Corrected Draft, September 2014

CAWM Discussion Paper No. 70

Abstract

This paper considers a model with three overlapping generations of which only the middle one is able to work. There is a trade-off between food and bio-fuel production. We try to study this trade-off, its influence on economic growth and on resource consumption. The paper states that even, if the current generation has the total autonomy of decision about the usage of all resources, they will never use them completely. The members of the current generation will always leave some resources for production because in the phase of retirement they depend on the income of the next generation, the latter paying back the credit obtained when being young. Furthermore, it states that as long as land is an essential input for producing the final product (including food) the amount of land devoted to bio-fuel production will not raise endlessly. The pure consumption loan model by Samuelson (1958) serves as basis for our Overlapping-Generations-Model, which we will integrate into a model of endogenous economic growth with resources, exogenous technical progress and land as a further input. For realization of the optimal consumption pattern, which leads to the maximum of utility, individuals have to make four decisions. They have to commit themselves to the exploitation rate of the non-renewable resource and to the amount of renewable resource used for production. Coincidentally, the allocation of land, necessary for food production and for renewable resource regeneration, and the amount of leisure time devoted for resource production have to be determined. The results are confirmed by numerical examples and will be reviewed by empirical data.

JEL classification: D91, O13, Q24, Q32

Keywords: economic growth, overlapping generations, non-renewable resources, renewable resources, land vs. food trade-off

*Dipl. Volksw. Frauke Voosholz, University of Muenster, Institute of Spacial and Housing Economics, Am Stadtgraben 9, 48143 Muenster / Germany, Tel. +49-251-83-22979, f_voos01@wiwi.uni-muenster.de

1 Introduction

In recent decades the inter-temporal allocation of exhaustible resources has been a frequently discussed topic, especially in the context of sustainable economic growth. Until the 1970s, the Malthusian view that the limited stock of non-renewable resources would restrict the economic growth was dominant (Agnani et al., 2005, p.388). In the neoclassical view, economic growth is possible as long as there is a rise in the technical progress. But is the growth rate sustainable over years, and does it fulfill the idea of an inter-generational distributive justice? To get answers to these questions optimal endogenous growth models, where investments, resource extractions and the used amount of labor are chosen to maximize the discounted lifetime utility, are used (Olson and Knapp, 1997, p.277).

In most models, all individuals live at the same time. They are all the same age, have the same preferences, make the same decisions every time; they are identical and behaving as the representative individual. In 1958, Paul Samuelson introduced a more realistic model of overlapping generations (Weil, 2008, p.115). The individuals are put together to different generations. These generations are taken into account at different points of time. New born generations are not included into their budget by the existing-agents (Weil, 2008, p.117). During one period of time three different generations are living, every generation lives through three periods. In contrast to the Samuelson-Model, most scientists use a two-period version of the OLG-Model (for example Balsko and Shell, 1980; John and Pecchenino, 1994; Olson and Knapp, 1997). Especially Diamond (1965) uses a two-generation model to explain internal and external debt. These two models serve as a basis for most overlapping-generations models.

OLG-Models can be used to illustrate the most varying topics in research. A common field of research with OLG-Models is to investigate the depletion of natural renewable and non-renewable resources and their influence on economic growth, keeping sustainability in mind (Agnani et al., 2005; Mourmouras, 1993; Olson and Knapp, 1997, for example see). If only a non-renewable resource is available and determining the production of an economy, then consumption cannot be uphold positively over an infinite time horizon (Krautkraemer and Batina, 1999, p.169). To maintain sustainable growth and therefore, positive consumption, other factors like technical progress or capital accumulation must act contrarily to the depletion of the resource.

In the OLG-Model of Koskela et al. (2002) a renewable resource is used to serve as a store of value and as input into the production. They reveal that there are two steady-state equilibria when the resource growth is assumed to be concave. Only the one with the higher stock of the renewable resource is stable. Agnani et al. (2005) use an extension of the Diamond-Model adding an exhaustible resource and physical capital. They find the same results as in standard endogenous growth models when technological progress is introduced exogenously. Furthermore, they present as a result that the labor share has to be high enough for positive growth, which is a standard condition in standard OLG-Models without resources.

Frequently, the impact of the use of resource on the environment is considered simultaneously. Gerlagh and Keyzer (2001) use an OLG-Model without any technical progress. They introduce three different policies of how the exhaustible resource can be used, and show that only a trust fund, by which future generations receive claims for the resource, may increase welfare for all generations. But still the introduction and continuation of such a fund depends on the present generation (Gerlagh and Keyzer, 2001).

The model developed here is an Overlapping Generations Model with technological progress. In chapter two the basic assumptions, definitions and notations are presented. Chapter three provides the optimization of consumption with given income. This restriction is abolished in chapter four, and the optimized resource depletion is revealed. This is followed by numerical examples. Chapter six concludes.

2 The Model

Suppose a world existing over an infinite number of periods, with three generations living at the same time. Only the middle one is able to work and generates income. Population growth is zero, and hence, the working population is stable at any level. Apart from their respective age, individuals do not differ from each other; their utility function is the same at all times.

Total income in the economy depends on the total production, and this in turn depends on the work of the middle generation, the land and the exploitation of two resources in the respective period t. On the one hand, there is an exhaustible natural resource, say natural gas, on the other hand, a renewable resource, in our case bio-fuel. We assume that the resources are not storable, so any amount of lifted gas must be immediately used within the period, and the renewable resources have to be either cultivated to regenerate the stock or used for the production. Land used for producing the renewable resource increases the rate of regeneration, and therefore, the production of bio-fuel, which concurrently reduces the amount of input of land for total production (cf. İmrohoroğlu et al. (1999) where the total value of land is considered.). In particular we assume

$$W_t = W\left(TW_t, Z_t, R_t, F_t\right) \tag{1}$$

where A_t and S_t is the stock of the renewable resource R_t and the exhaustible resource Z_t, respectively, at the beginning of period t, and r_t and z_t are the rates of resource depletion in period t. The used amount of the resources in period t is given by $R_t = r_t \cdot A_t$ and likewise by $Z_t = z_t \cdot S_t$. The share of land used for production TW_t is given by $TW_t = (1 - ta_t) \cdot T_t$, where T_t is the total land endowment. F_t denotes an (exogenous) rate of technical progress. The stocks of the resources develop as follows:

$$S_{t+1} = S_t - S_t \cdot z_t \tag{2}$$

$$A_{t+1} = A_t + (A_t - r_t \cdot A_t) \cdot a \cdot ta_t \cdot T_t - r_t \cdot A_t \tag{3}$$

where the stock of the renewable resource regenerates by assumption at the rate $a \cdot ta_t \cdot T_t$. As there is no capital, the whole production W_t is paid in the form of wages to the middle generation.

As the old generation in each period is not able to work, the middle generation makes contracts with the younger generation to provide for their seniority. This entitles them to get benefits from the younger generation during the next period (in analogy to the pure consumption loan model by Samuelson (1958)). The income of the middle generation w_t is allocated between consumption in the current period c_t, savings s_t paid to the young generation, and paying back the credit of the old generation, received at the period before, including interests $q_t \cdot s_{t-1}$. In order to keep the model simple, money is not explicitly included, but it is assumed that savings are done in the form of binding contracts in real terms (cf. Balsko and Shell (1980)). The budget constrains in the three phases of an individual‘s life $(t = j, t = j + 1, t = j + 2)$ are given by

$$c_{1,j} = -s_{1,j} \tag{4}$$

$$c_{2,j+1} = w_{2,j+1} - s_{2,j+1} - q_{j+1} \cdot s_{2,j} \tag{5}$$

$$c_{3,j+2} = q_{j+2} \cdot s_{2,j+1} \tag{6}$$

where $c_{i,t}$ and $s_{i,t}$ are consumption and saving, respectively, of a representative individual of age i, and q_t is the interest factor in period t. Note that $s_{1,j} = -s_{2,j}$. Individual income $w_{i,t}$ is total income W_t in period t divided by the number of individuals which are in their middle age (i.e. $w_{2,t} = W_t/N$). The total income W_t in period t must fulfill the consumption of all three generations in period t with $3 \cdot N$ individuals living at the same time. The three constraints can be added to

$$c_{3,j+2} + q_{j+2} (c_{2,j+1} - w_{2,j+1}) + q_{j+1} \cdot c_{1,j} = 0 \tag{7}$$

This figure gives an overview of the relevant variables of the model:

Note that, while individual consumption optimization is made “diagonal” in this table, total income $W_t = w_{2,t}N$ of the economy is calculated “vertically”, i.e. it must equal total consumption in each period t, keeping in mind that there are N individuals living in each generation.

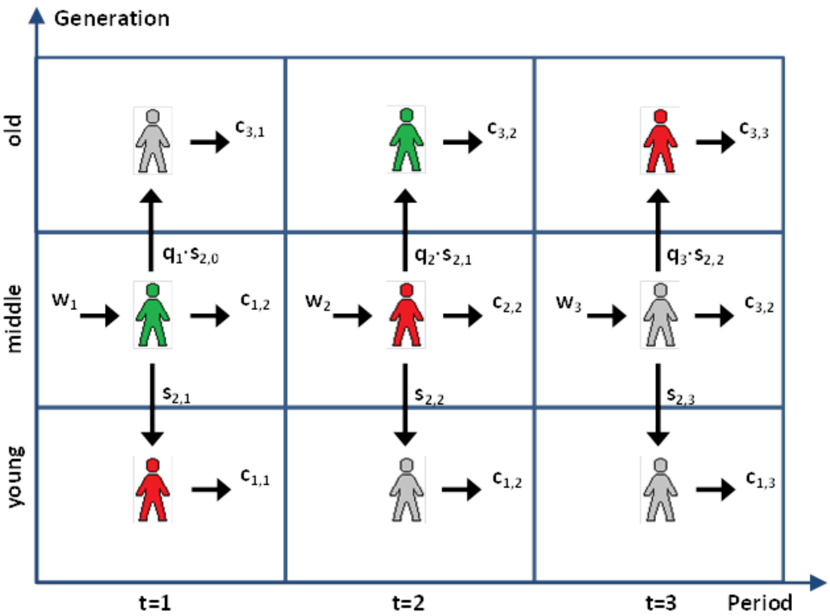

Figure 1: The different amounts of consumptions at the different stages of life

3 Consumption Optimizing

We suppose that, from the current perspective of t=0, the utility function of a representative young individual who lives in period j is given by

$$U = \frac{ln\,(c_{1,j})}{(1+p)^j} + \frac{ln\,(c_{2,j+1})}{(1+p)^{j+1}} + \frac{ln\,(l_{2,j+1})}{(1+p)^{j+1}} + \frac{ln\,(c_{3,j+2})}{(1+p)^{j+2}} \tag{8}$$

where $l_{2,j+1}$ is leisure time (part of which must be sacrificed in the middle period of life in order to earn income) and p is the individual‘s rate of time preference.[1] We assume that the individual seeks to maximize its lifetime utility with currently both income $w_{2,j+1}$ and (thus) the remaining leisure time $l_{2,j+1}$ being exogenously given. Then from maximizing (8) regarding (7) it follows that the optimal consumption plan is given by

$$c_{1,j} = -s_{1,j} = \frac{w_{2,j+1}}{q_{j+1} + \frac{q_{j+1}}{1+p} + \frac{q_{j+1}}{(1+p)^2}} \tag{9}$$

$$c_{2,j+1} = \frac{q_{j+1}}{1+p} \cdot c_{1,j} \tag{10}$$

$$c_{3,j+2} = \frac{q_{j+1} \cdot q_{j+2}}{(1+p)^2} \cdot c_{1,j} \tag{11}$$

It can be shown that the general result in OLG models holds that optimal individual consumption rises by the factor $q_t/(1+p)$ in each period t. Since the individual does not earn any income in the first period of life, it gets into debt in the amount of its consumption in that period according to (9). The amount of consumption depends in each period for $t = j, j+1, j+2$ on the amount of income earned in period $t = j+1$. In

[1] Detailed information on the influence of endogenous labor supply can be found in Nourry and Venditti (2006), and in Duranton (2001).

period j the individual raises a credit, with a prediction of future income, and in period $j+1$ it grants a credit, depending on the current income to receive repayments in period $j+2$ to fulfill the consumption needs.

Now we consider the situation from the view of an individual who also lives in period j, but has already reached the second period of its life. As the amount of consumption when it was young is now an exogenous variable, its maximization problem reduces to

$$maxU_m = \frac{ln\left(c_{2,j}\right)}{\left(1+p\right)^j} + \frac{ln\left(c_{3,j+1}\right)}{\left(1+p\right)^{j+1}} + \frac{ln\left(l_{2,j}\right)}{\left(1+p\right)^j} \tag{12}$$

with the remaining relevant constraints being

$$c_{2,j} = w_{2,j} - \overline{c_{3,j}} - s_{2,j} \tag{13}$$

$$c_{3,j+1} = q_{j+1} \cdot s_{2,j} \tag{14}$$

The term $w_{2,j} - \overline{c_{3,j}}$ denotes disposable income of the representative working individual, which is total income net of the debt which has been taken in the past, including interest. Obviously, $\overline{c_{3,j}}$ is identical to the consumption of the elder generation of period j; because they gave the loan in the previous period. Maximizing (12) with respect to ((13) and (14)) yields the optimal consumption pattern

$$c_{2,j} = \frac{w_{2,j} - \overline{c_{3,j}}}{1+\frac{1}{1+p}} \tag{15}$$

$$c_{3,j+1} = c_{2,j} \cdot \frac{q_{j+1}}{1+p} = q_{j+1} \cdot \frac{w_{2,j} - \overline{c_{3,j}}}{2+p} \tag{16}$$

$$s_{2,j} = \frac{c_{3,j+1}}{q_{j+1}} = \frac{w_{2,j} - \overline{c_{3,j}}}{2+p} \tag{17}$$

With constant population, equilibrium in the capital market requires that $s_{1,j} + s_{2,j} = 0$. Hence, equating (9) and (17) yields the equilibrium interest factor with exogenously given incomes $w_{2,t}$:

$$q^*_{j+1} = \frac{w_{2,j+1} \cdot (2+p)}{\left(w_{2,j} - c_{3,j}\right) \cdot \left(1 + \frac{1}{1+p} + \frac{1}{(1+p)^2}\right)} \tag{18}$$

Total income is given by (1), according to this the individual share of income is represented by $w_{2,t} = \frac{W(ta_t,T_t,z_t,S_t,r_t,A_t,F_t)}{N}$ in period t. Substituting $w_{2,j}$ and $w_{2,j+1}$ yields

$$q^*_{j+1} = \frac{W\left(ta_{j+1}, T_{j+1}, z_{j+1}, S_{j+1}, r_{j+1}, A_{j+1}, F_{j+1}\right) \cdot \left(1+p\right)^2}{W\left(ta_j, T_j, z_j, S_j, r_j, A_j, F_j\right)} \tag{19}$$

Note that $N \cdot c_{1,t} + N \cdot c_{2,t} + N \cdot c_{3,t} = W_t$, i.e. total consumption in period t must equal real income in that period (because the resource cannot be stored). Moreover, from the optimal consumption plans it follows that $c_{2,t}/c_{1,t} = c_{3,t}/c_{2,t} = (1+p)$.[2] Given all that, the right hand expression in (19) results.[3]

---

[2]See appendix A.

[3]See appendix B.

4 Optimizing Resource Depletion

It seems natural that the middle generation decides on the rates of resource depletion because by their work they determine the amount of income available in the current period. At first sight, it seems to be optimal for them to fully exploit the exhaustible resource within their lifespan. However, there are two limiting factors to such a behavior: First, the younger generation would not agree to any saving contract which would leave them without any resources in their own seniority. Secondly, lifting the resource requires input of leisure time L_t by the middle agers, which is also a scarce resource for every individual.

We assume the simple leisure function

$$L_t = L\left(R_t, Z_t\right) = N \cdot H - R_t - \omega \cdot Z_t \tag{20}$$

where H are the maximum working hours of one individual during each period. One unit of renewable resource can be extracted with the input of one working hour, whereas for the production of one unit of non-renewable resource a higher amount of labor is needed, indicated by $\omega > 1$.[4] Because $L_t = l_{2,t} \cdot N$, this can be transformed into

$$l_{2,t} = H - \frac{r_t A_t + z_t S_t}{N} \geq 0 \tag{21}$$

There is an upper limit to the exploiting rates r_t and z_t because leisure cannot be negative.

Now we turn back to the overall maximization problem of the middle generation ($i = 2$) in the initial period j, with the relevant constraints being (13), (14) and additionally,

$$w_{2,j} = \frac{W\left(ta_j, T_j, z_j, S_j, r_j, A_j, F_j\right)}{N} \tag{22}$$

which can be added to

$$\frac{W\left(ta_j, T_j, z_j, S_j, r_j, A_j, F_j\right)}{N} - c_{3,j} - c_{2,j} - \frac{c_{3,j+1}}{q_{j+1}} = 0 \tag{23}$$

The optimal consumption plan given by (15) to (17) is still valid. But now income $w_{2,j}$ and the interest factor q^*_{j+1} are dependent on the exploiting rates r_j and z_j. Keeping in mind the determination of leisure time by deciding on the extent of the extraction rates (20), the utility function (12) translates to

$$\begin{aligned} max\ U_m(r,z) \quad = \quad & \frac{\ln\left(\frac{W(ta_j,T_j,z_j,S_j,r_j,A_j,F_j)\cdot(1+p)}{N(3+3p+p^2)}\right)}{(1+p)^j} + \frac{\ln\left(\frac{L_{R_j},Z_j}{N}\right)}{(1+p)^j} \\ & + \frac{\ln\left(\frac{(1+p)^2\cdot W(ta_{j+1},T_{j+1},z_{j+1},S_{j+1},r_{j+1},A_{j+1},F_{j+1})}{N(3+3p+p^2)}\right)}{(1+p)^{j+1}} \end{aligned} \tag{24}$$

[4] More complex production functions would not alter our principal results.

For further calculations the production function (4) is specified using the Cobb-Douglas type,

$$W\left(ta_t, T_t, z_t, S_t, r_t, A_t, F_t\right) = TY_t^{\beta} \cdot \left(S_t \cdot gZ_t\right)^{\alpha} \cdot \left(A_t \cdot gJ_t\right)^{1-\alpha-\beta} \cdot (1+f)^t \cdot F \tag{25}$$

To reveal the optimal extraction of resources and the optimal allocation of land, the current value Hamiltonian is set up using the further constraints of the stocks of resources ((2) and (3)). The shadow prices of the resources are π_t for the non-renewable resource, and θ_t for the renewable resource. The resulting first order condition (FOC) for the control variables $(r_{t+1}, z_{t+1}, ta_{t+1})$ and state variables $(A_{t+1}, S_{t+1}, T_{t+1})$, using (25) and (21) are

$$z_{t+1}: \quad \frac{2\alpha}{z_{t+1}(1+p)^{t+1}} - \frac{2S_{t+1}}{\left(HN - A_{t+1}r_{t+1} - 2S_{t+1}z_{t+1}\right)(1+p)^{t+1}} + \pi_{t+1}S_{t+1} = 0 \tag{26}$$

$$r_{t+1}: \quad \frac{2-2\alpha-2\beta}{r_{t+1}(1+p)^{t+1}} - \frac{A_{t+1}}{\left(HN - A_{t+1}r_{t+1} - 2S_{t+1}z_{t+1}\right)(1+p)^{t+1}} + \theta_{t+1}A_{t+1}\left(aT_{t+1}ta_{t+1}+1\right) = 0 \tag{27}$$

$$ta_{t+1}: \quad \frac{aA_{t+1}T_{t+1}\theta_{t+1}\left(ta_{t+1}-1\right)\left(r_{t+1}-1\right)(1+p)^{1+t} + 2\beta}{\left(ta_{t+1}-1\right)(1+p)^{1+t}} = 0 \tag{28}$$

$$S_{t+1}: \quad \frac{2\alpha}{S_{t+1}(1+p)^{t+1}} - \frac{2z_{t+1}}{\left(HN - A_{t+1}r_{t+1} - 2S_{t+1}z_{t+1}\right)(1+p)^{t+1}} + \pi_{t+1}\left(z_{t+1}-1\right) + \pi_t = 0 \tag{29}$$

$$A_{t+1}: \quad \frac{2-2\alpha-2\beta}{A_{t+1}(1+p)^{t+1}} - \frac{r_{t+1}}{\left(HN - A_{t+1}r_{t+1} - 2S_{t+1}z_{t+1}\right)(1+p)^{t+1}} + \theta_{t+1}\left(r_{t+1}-1\right)\left(aT_{t+1}ta_{t+1}+1\right) + \theta_t = 0 \tag{30}$$

$$T_{t+1}: \quad \frac{aA_{t+1}T_{t+1}\theta_{t+1}ta_{t+1}\left(r_{t+1}-1\right)(1+p)^{1+t} + 2\beta}{T_{t+1}(1+p)^{1+t}} = 0 \tag{31}$$

From the first order conditions we can reveal the optimal extraction rates and the optimal allocation of land. Because of the complexity of the equations and the dependencies of the variables on each other, this can only be done by using numerical simulations.

Note that the rate of technical progress does neither affect the FOC of the control variables nor the FOC of the state variables. Therefore, the rate of technical progress does not affect the optimal exploiting rates or the optimal allocation of land. However, a higher rate of technical progress increases the interest factor according to (19) as well as the resulting income path according to (1).

5 Numerical Example

We consider first an example without technological progress (i.e. $f = 0$). Individual time preference is $p = 4\%$, the initial renewable resource stock is $A_0 = 1500$, the initial non-renewable resource stock is S_0=3800. The amount of land being available for use is set constant with $T_t = T_{t+1} = 1$ (case 1).[5]

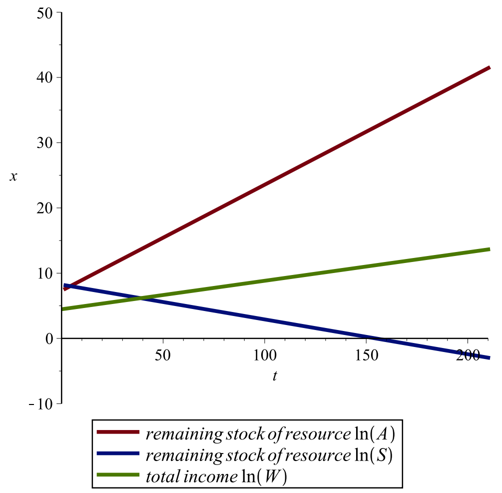

Figure 2: Remaining resource stocks S and A, total income W, all values in logarithmical depiction, case 1

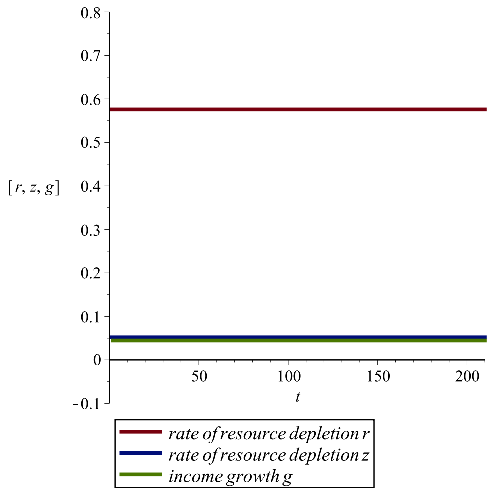

Figure 3: Resource depletion rates z and r, income growth rate g, case 1

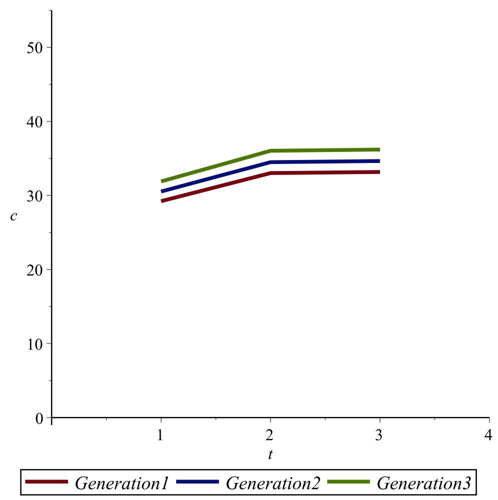

Figure 4: Consumption of the different generations during the first three periods of their life, case 1

As Fig. 2 and Fig. 3 reveal, in this scenario the stock of the non-renewable resource rapidly decreases at the rate of $z_t \approx 5,19\%$ and the stock of the renewable resources

[5]The further variables are set constant during the whole simulations, with $F = 1$, $\alpha = 0.4$, $\beta = 0.2$, $a = 2.15$ and working population size $N = 1$.

increases at the rate of $r_t \approx 57,6\%$. With this depletion rate, steady growth of income at a rate of $g_t \approx 4,46\%$ is possible. Due to this, each new generation is able to generate a higher amount of consumption than the previous generation in each period (see. Fig. 4). Although the non-renewable resource is not totally exhausted until later, the remaining stock is very quickly reduced to a negligible amount. This is compensated by an even higher increase of the renewable resource. Due to the fact that the total land endowment T and the share of land used for resource regeneration $ta_t \approx 82,49\%$ is constant, this is only possible, if the productivity of land would rise extremely. It seems to be possible that there might be a certain increase, but not to infinity. Therefore, there is a natural maximum of the stock of the renewable resource A_t^{max}. If $A_t^{max} = 5000$ is assumed, the results change to the following (case 2).

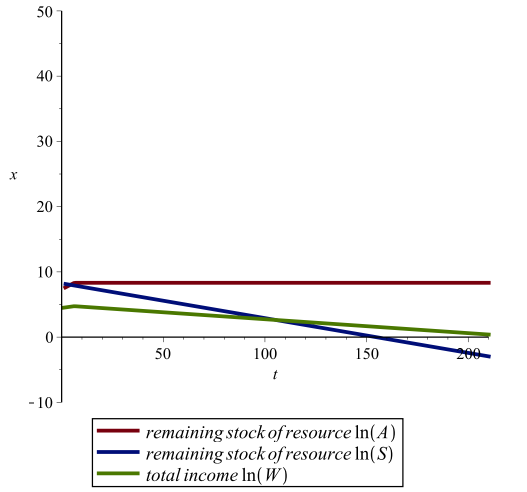

Figure 5: Remaining resource stocks S and A, total income W, all values in

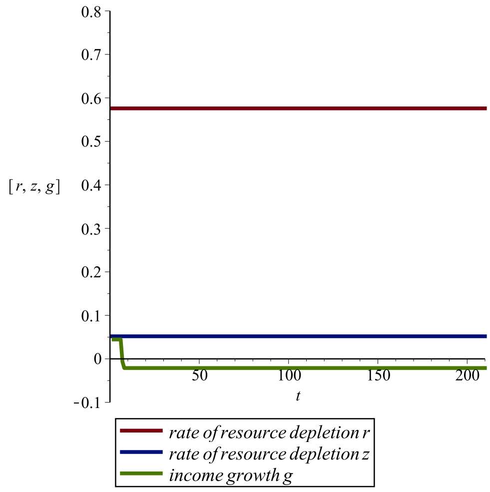

Figure 6: Resource depletion rates z and r, income growth rate g, case 2

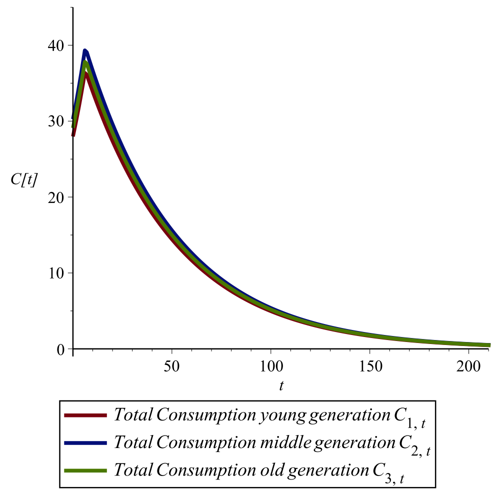

Figure 7: Total Consumption of the different generations, case 2

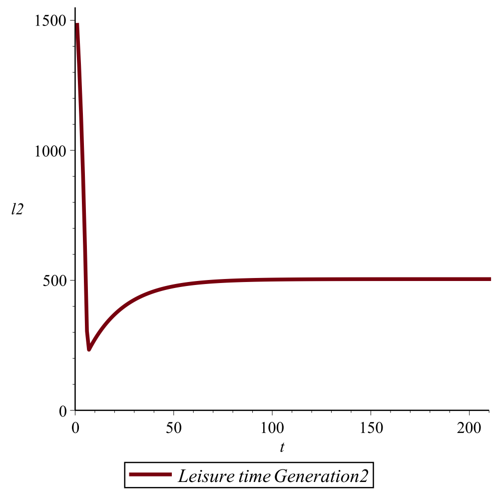

Figure 8: Leisure time, case 2

There is no change in the resource depletion rates, but the stock of the renewable resource is quickly increased to its maximum, and as soon as A_t^{max} is reached, the rate of income growth shrinks and becomes even negative (see Fig. 6). There is not enough renewable resource to substitute for the decreasing amount of non-renewable resource available. Although this reduction-rate is the same for many future generations and - in this sense - they all treat their successors like they have been treated themselves, one would hardly call that a particularly convincing kind of inter-generational justice. On the other hand, without technological progress, a negative rate of income growth is inevitable, irrespective of the way the initial stock is divided among generations (see Fig. 7). Hence, sooner or later the economy will no longer be able to provide even for its basic needs.

From Fig. 8 we can see that in the beginning more labor has to be invested to accumulate the higher stock of renewable resource. If this state is reached, less leisure time has to be devoted to resource extraction, and therefore, the additional leisure time can compensate for the decreasing amount of consumption.

Now we suppose that there is technological progress, with resource productivity rising by the rate $f = 8\%$ per year, all other variables being unvaried, keeping the assumption of A_t^{max} (case 3).

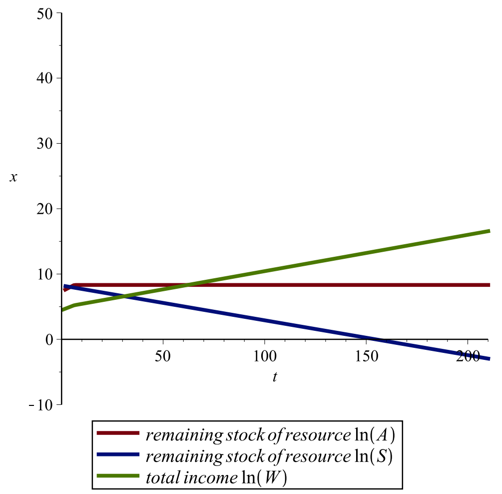

Figure 9: Remaining resource stocks S and A, total income W, all values in logarithmical depiction, case 3

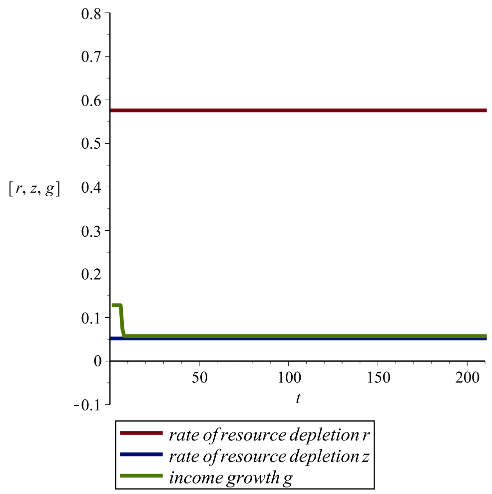

Figure 10: Resource depletion rates z and r, income growth rate g, case 3

As we have already seen, the exhaustion rate is not affected thereby at all and remains at its previous level $r_t \approx 57,6\%$. As it can be learned from Fig. 9 in contrast to Fig. 2, we see the limiting influence of A_t^{max}. But in contrast to Fig. 5 the total amount of income is now rising, due to the influence of the technological progress. A quite similar effect would be achieved if it is possible to extend the total amount of land being available for total production by investments. This would lead, on the one hand, to a higher regeneration capacity, which induces a higher amount of resource usable for production, and on the other hand, directly to a higher production.

6 Summery

It is shown that economic growth is possible even if one generation can decide on the extraction rates of the renewable and non-renewable resources. It is necessary that there is a compensation for the used resources in form of a higher stock of technical progress or a higher amount of land available for production. We reach the same results as it is postulated in the Hartwick's rule to reach intergenerational equity (cf. Hartwick (1977)). The future generations must be able to reach at least the same amount of consumption as the previous ones. This is fulfilled, even without any central planner or any other political intervention.

To fit the model more to a realistic setting, in a first step three generations are regarded instead of two, as mostly done. This underlines that the current deciding generation is always effected by the past and the future. Furthermore, the technological progress should be endogenized, by introducing investments to research. Consequently, labor has to be divided between final production, research and leisure time. The technological progress may first affect final production, but it also may affect the regeneration rate, by inventing e.g. new fertilizer, or enlarge the total amount of usable land.

A Mathematical Appendix

Proposition I: $c_{2,t}/c_{1,t} = c_{3,t}/c_{2,t} = 1 + p$

Proof of proposition I: We have

$$N \cdot c_{1,j} + N \cdot c_{2,j} + N \cdot c_{3,j} = w_{2,j} N \tag{A.1}$$

$$N \cdot c_{1,j+1} + N \cdot c_{2,j+1} + N \cdot c_{3,j+1} = w_{2,j+1} N \tag{A.2}$$

From the conditions for consumption optimization (9) to (11) and (15) to (17) we know that

$$\frac{c_{2,j+1}}{c_{1,j}} = \frac{q_{t+1}}{1+p} \tag{A.3}$$

and

$$\frac{c_{3,j+1}}{c_{2,j}} = \frac{q_{t+2}}{1+p} \tag{A.4}$$

Moreover, we have

$$\frac{w_{2,j+1}}{w_{2,j}} = \frac{c_{1,j+1}}{c_{1,j}} = \frac{c_{2,j+1}}{c_{2,j}} = \frac{c_{3,j+1}}{c_{3,j}} \tag{A.5}$$

and from the capital equilibrium equation (18) it follows that

$$q^*_{j+1} = \frac{w_{2,j+1}(2+p)}{(w_{2,j} - c_{3,j})(1 + \frac{1}{1+p} + \frac{1}{(1+p)^2})} \tag{A.6}$$

By combining (A.1), (A.2) and (A.6) we find

$$\frac{(c_{1,j+1}+c_{2,j+1}+c_{3,j+1})(2+p)}{q_{j+1}(1+\frac{1}{1+p}+\frac{1}{(1+p)^2})}=c_{1,j}-c_{2,j} \tag{A.7}$$

Dividing (A.7) by $c_{2,j}$ yields

$$\frac{c_{1,j}}{c_{2,j}}=\frac{(\frac{c_{1,j+1}+c_{2,j+1}+c_{3,j+1}}{c_{2,j}})(2+p)}{q_{j+1}(1+\frac{1}{1+p}+\frac{1}{(1+p)^2})}-1\overset{?}{=}\frac{1}{1+p} \tag{A.8}$$

According to (A.3) q^*_{j+1} can be substituted by $(1+p)c_{2,j+1}/c_{1,j}$. By rearranging terms in (A.8), this leads to

$$\frac{c_{1,j}}{c_{2,j}}\frac{c_{1,j+1}}{c_{2,j+1}}+\frac{c_{1,j}}{c_{2,j}}+\frac{c_{3,j+1}}{c_{2,j+1}}\frac{c_{1,j}}{c_{2,j}}\overset{?}{=}\frac{(1+p)+2+\frac{2}{1+p}+\frac{1}{(1+p)^2}}{2+p} \tag{A.9}$$

Obviously, the relation $c_{1,j}/c_{2,j}$ must be equal to $c_{1,j+1}/c_{2,j+1}$ and to $c_{2,j+1}/c_{3,j+1}$, because the optimal consumption plan (9) to (11) is identical for each generation. Thus, denoting this relation as x, we have

$$\frac{1}{x^2}+\frac{1}{x}+\frac{x}{x}\overset{?}{=}\frac{(1+p)+2+\frac{2}{1+p}+\frac{1}{(1+p)^2}}{2+p} \tag{A.10}$$

According to our proposition, we should have $x=(1+p)$. By inserting this into (A.10), it is easily shown that both sides of the equation are identical and, hence, the proposition holds true. q.e.d.

Proposition II: $\frac{\frac{r_{j+1}B_j(1-r_j)(1+f)^{(j+1)}}{N}(2+p)}{(\frac{r_jB_j(1+f)^j}{N}-c_{31})(1+\frac{1}{1+p}+\frac{1}{(1+p)^2})}=\frac{r_{j+1}(1-r_j)(1+f)^{(j+1)}(1+p)^2}{r_j(1+f)^j}$

Proof of proposition II:

$$c_{3,j}=\frac{w_{2,j}}{1+\frac{1}{1+p}\frac{1}{(1+p)^2}}=\frac{\frac{r_jB_j(1+f)^j}{N}}{1+\frac{1}{1+p}\frac{1}{(1+p)^2}} \tag{A.11}$$

Inserting (A.11) into proposition II yields

$$\frac{\frac{r_{j+1}B_j(1-r_j)(1+f)^{(j+1)}}{N}(2+p)}{(\frac{r_jB_j(1+f)^j}{N}-\frac{\frac{r_jB_j(1+f)^{(j-1)}}{N}}{1+\frac{1}{1+p}\frac{1}{(1+p)^2}})(1+\frac{1}{1+p}+\frac{1}{(1+p)^2})} \tag{A.12}$$
$$=\frac{r_{j+1}(1-r_j)(1+f)^{(j+1)}(1+p)^2}{r_j(1+f)^j}$$

References

Agnani, B., Gutiérrez, M.-J., and Iza, A. (2005). Growth in Overlapping Generation Economies with Non-renewable Resources. *Journal of Environmental Economics and Management*, *50*(2), 387–407.

Balsko, Y. and Shell, K. (1980). The Overlapping-Generations Model, I: The Case of Pure Exchange without Money. *Journal of Economic Theory*, *23*(3), 281–306.

Diamond, P. A. (1965). National Debt in a Neoclassical Growth Model. *The American Economic Review*, *55*(5), 1126–1150.

Duranton, G. (2001). Endogenous Labour Supply, Growth and Overlapping Generations. *Journal of Economic Behavior & Organization*, *44*, 295–314.

Gerlagh, R. and Keyzer, M. A. (2001). Sustainability and the Intergenerational Distribution of Natural Resource Entitlements. *Journal of Public Economics*, *79*(2), 315–341.

Hartwick, J. M. (1977). Intergenerational Equity and the Investing of Rents from Exhaustible Resources. *The American Economic Review*, *67*(5), 972–974.

İmrohoroğlu, A., İmrohoroğlu, S., and Joines, D. H. (1999). Social Security in an Overlapping Generations Economy with Land. *Review of Economic Dynamics*, *2*(3), 638–665.

John, A. and Pecchenino, R. (1994). An Overlapping Generations Model of Growth and the Environment. *The Economic Journal*, *104*(427), 1393–1410.

Koskela, E., Ollikainen, M., and Puhakka, M. (2002). Renewable Resources in an Overlapping Genee rations Economy without Capital. *Journal of Environmental Economics and Management*, *43*(3), 497–517.

Krautkraemer, J. A. and Batina, R. G. (1999). On Sustainability and Intergenerational Transfers with a Renewable Resource. *Land Economics*, *75*(2), 167–184.

Mourmouras, A. (1993). Conservationist Government Policies and Intergenerational Equity in an Overlapping Generations Model with Renewable Resources. *Journal of Public Economics*, *51*(2), 249–268.

Nourry, C. and Venditti, A. (2006). Overlapping Generations Model with Endogenous Labor Supply: General Formulation. *Journal of Optimization Theory and Applications*, *128*(2), 355–377.

Olson, L. J. and Knapp, K. C. (1997). Exhaustible Resource Allocation in an Overlapping Generations Economy. *Journal of Environmental Economics and Management*, *32*(3), 277–292.

Samuelson, P. (1958). An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money. *Journal of Political Economy*, *66*(6), 467–482.

Weil, P. (2008). Overlapping Generations: The First Jubilee. *Journal of Economic Perspectives*, *22*(4), 115–134.