A way out of pay-as-you-go without a double burden

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by

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Abstract

It has repeatedly been proposed to reduce conventional pay-as-you-go-systems to a base level, leaving advanced retirement provision for private funded systems. However, pay-as-you-go systems are, in a sense, one way roads, with no available Pareto efficient way out. The paper discusses a combined public debt and taxing strategy which distributes the transition burden equally between future generations, leaving them with only moderate losses in terms of present value. It is shown within both a two generations model and a multiple-generations model of OLG type, that, with this strategy, there results only a temporary increase in public debt ratio, which even turns into a public surplus in the long run. The paper argues that such a transformation towards a base pension system would be both economically advisable and politically feasible.

JEL-classification: D, H, J

1. Introduction

The main problem with existing pay-as-you-go pension systems is that they are, in a sense, a one way road. On the one hand, when the system is implemented, the current elder generation can immediately be paid a pension. Neither a capital stock is required nor will the pension be at the cost of the younger generation, because the latter will benefit from the system themselves when they are old. Their only sacrifice is that their pension contributions pay only interest amounting to the economy`s growth rate, which is generally less than the capital market interest rate (Feldstein 1995, 4).[1] In a dynamically inefficient economy, where the growth rate exceeds the interest rate, the pay-as-you-go scheme was even Pareto-superior (Samuelson 1958).

On the other hand, once the system is at work, there seems to be no way out without imposing a burden on those who live in the transition period (Aaron 1965; Börsch-Supan 1998). Feldstein (1995) argues that a debt financed transformation towards a funded system can be Pareto efficient if the marginal productivity of capital exceeds both the (positive) growth rate and the discount rate of future consumption (see also Feldstein 1997, 5). An obvious reason for such a divergence is capital income taxation. However, this is a relatively weak second best argument, because this distortion does not directly relate to the pension system and could possibly be removed by other means, thus making the Feldstein argument obsolete (Sinn 2000, 399).

So, within a first best world at least, there actually is a problem of return once a pay-as-you-go system is established (Fenge 1995). Suppose, for example, the extreme case that the system is abandoned at all, by the way of scrapping all contributions from any Transition Period t = T on. Then, in the transition period, there are either no more pensions at all, which would mean a fraud on the elder generation in that period. Alternatively, the pensions of the transition period are borne by the tax payer, which would mean a double burden for the younger generation, who have to provide privately for their old age in addition. A third option is financing the transition costs by public debt. However, this would only mean a change in form but not in substance of the pay-as-you-go scheme, at least with respect to the funding issue. Moreover, this proposal immediately raises the question how the government could finance the interest: "If it uses a credit again, it ends up with a Ponzi scheme….Otherwise we are back to the former system, but with higher contributions. It follows from these brief considerations that public debt does not represent a method that could make a transition to a funded system possible." (Brunner 1996, 143).

Nevertheless, it has repeatedly been proposed to reduce conventional pay-as-you-go-systems to a base level, leaving advanced retirement provision for private funded systems (Yoon and Talmain 2001). Several advantages are expected from such a change in system: (1) A greater share of retirement provision would be funded, thereby rendering retirement provision both more profitable and less vulnerable on demography.(2) The substitution of wage related contributions by general taxes could reduce both labor costs and excess burden and thereby spur wealth and employment. (3)A unique redistribution system of general income taxes is assessed both fairer and more efficient than the parallelism of progressive taxes and proportional pension contributions.(4)Because the entitlement on the basic pension is unconditional and thus does not depend on individual wealth, it

[1] Because it is widely known from both theoretical and empirical research that the interest rate will generally exceed the growth rate, we do not deal here with the case of dynamic inefficiency.

gives a strong incentive for both additional private saving and ongoing participation in the labor market, even as a pensioner. (5)Because the base pension covers basic needs by definition, additional public aid for pensioners is no longer required, so scramble on responsibilities and bureaucracy costs are saved. (6) Both the base rent and private retirement provision are less liable to political manipulation than a general pay-as-you-go system is.

In what follows it is argued that most of these advantages could be realized, if the funding gap arising in the Transition Period is financed partly by additional public debt, and partly by a special pension tax to be imposed on those who are entitled to the base pension. The core idea is to distribute the transition costs thereby on all future generations, instead of burdening only those who happen to live in the transition period. We confirm the result by Sinn (2000) that a Pareto-efficient transition towards a funded system is impossible. However, because there are infinitely many future generations, the sacrifice which must be imposed on each of them is only small. Moreover, with this transition scheme, total retirement provision gets more and more funded instead of being financed by pay-as-you-go in the long run. Hence, public debt can indeed allow for a transition from an unfunded system to a funded one without posing a double burden on the transfer generations.

Two options for adjusting the pension tax are discussed below. The first option ensures that the present value (including the tax) is the same like with the standard retirement system for every present and future generation. With this option, there results a public debt which is constant in relation to GNP in all periods following the transition. This would leave intergenerational distribution ultimately the same, only transforming the hidden promise to pay future pensions into an open public liability. The second option is less ambitious concerning individual welfare, allowing present value in the new system to be lower than in the standard system, with the percentage difference being the same for every generation from the transition period on. This option implies only a temporary rise in public debt, which declines after the transition and even converts into a public surplus in the (very) long run. Anyway, with both options the substantial improvements in terms of costs and incentives of a base pension system referred to above can be realized.

The paper is organized as follows. In Section 2, the core idea is demonstrated with the help of a most simple OLG model. In Section 3, the argument is brought closer to reality by employing a more sophisticated version of the model with five rather than only two generations. Section 4 is devoted to more practical issues concerning the implementation of the transformation scheme and its implications. Formal proves are given in the appendix.

2. A simple transition model

In the simplest case every generation lives for only two periods, first as active N earning wealth w and then as retirees R receiving a pension p. With c being the contribution share of wealth and n being population growth, we have

$$(1)\ \frac{N_t}{N_{t-1}} = (1+n)$$

$$(2)\ \frac{R_t}{N_t} = \frac{1}{1+n}$$

$$(3) p_t = w_t c \frac{N}{R} = w_{t-1} c(1+g_w)(1+n)$$

where p is the pension level, which rises in both population growth n and productivity growth g_w, thereby implying a return on the pension contributions equal to the economy`s growth rate g_y.[2] In the sequel we write q for the ratio $(1+i)/(1+g_y)$. With $i > g_y$ and, hence, q > 1, the present value of this standard pension system for every individual is negative:

$$(4) PV_c \equiv -w_{t-1}c + w_{t-1}\frac{c}{q} < 0$$

Now suppose that from Transition Period t = T on pension contributions are lowered to the base level bc (with $0 \le b \le 1$) and, from Period T+1 on, only a reduced pension bp is paid to retirees, while the original pension p is paid a last time in the transition period. So the elder generation in that period does not suffer a loss, but there occurs a funding gab $(1-b)pR$ which someone must account for. If the gab were financed by additional taxes on wealth, the younger generation in T would bear the total burden. Alternatively, in case of additional general income taxes or indirect taxes (e.g. by increasing the VAT), both the younger and the older generation in T would be charged. Anyway, it seems that the change in system is not in favor of those who live in the transition period.

The picture changes, however, if the funding gap is financed by an appropriate combination of public debt d and additional contributions z, the latter being imposed as a tax on the active`s wealth from Transition Period T on.[3] In addition, the active must prepare for supplement retirement provision in order to close their individual pension gap. Suppose that, for this purpose, they devote aw to private retirement provision. Because the latter earns the market interest rate, the present value of their total retirement provision is

$$(5) PV_a = -w_{t-1}(bc + a + z) + w_{t-1}\left(a + \frac{bc}{q}\right)$$

In order to guarantee the base pensioners the same present value as they enjoyed in the standard system, z must be chosen such that $PV_a = PV_c$. From (4) and (5) it follows that the appropriate tax rate is

$$(6) z^* = c(1-b)(1-\frac{1}{q})$$

For the sake of clearness, we require that pensions are the same in both systems:

$$(7) w_{t-1}a(1+i) + w_{t-1}bc(1+g_y) \overset{!}{=} w_{t-1}c(1+g_y)$$

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[2] The latter is easily verified substituting (3) by $p_t = cw_{t-1}(1+g_w)(1+n)$, so $r \equiv p_t/(cw_{t-1}) - 1 = (1+g_w)(1+n) - 1 = g_y$

[3] In fact, contributions to a pay-as-you-go system do include a hidden tax as well, as Sinn (2000) correctly proposes.

Then the appropriate private contribution rate is

$$(8) a^* = \frac{c(1-b)}{q}$$

Therefore, the divergence between the two pension systems reduces to the contribution side, which with equal present values[4] must satisfy

$$(9) cb + a + z = c \ .$$

It can easily be shown that under these conditions the transformation scheme generates a public debt ratio D_T/Y_T which is constant in all following periods. So no real change to a funded pension system actually occurs. This is equivalent to the result in Sinn (2000, 395) that “the introductory gain of the first generation of retirees equals the present value of the implicit taxes that have to be paid by all future generations” . In this limiting case, the formerly hidden public burden of future pensions would only be transformed into the visible form of public debt (which would thoroughly be desirable for reasons to be discusses below).

However, with $i \geq g_y$ and with any tax z which is just slightly above z^*, there accrues only a temporary rise in total public debt, which decreases in every following period and, due to the compound interest effect, eventually even turns into a public fortune.

This can be demonstrated as follows: The additional public deficit d_T arising in Transition Period T (which equals total pension deficit D_T in that period) is given by

$$(10) D_T = d_T = w_T c(1+n)R_T - w_T cbN_T - w_T zN_T$$

where the first summand represents the costs of pensions and the following summands are the receipts from (reduced) contributions and the pension tax respectively.

In the following Period T+1 only the reduced pension bp must be financed, but on the other hand interest must be paid on total debt of Period T, so total debt in Period T+1 is

$$(11) D_{T+1} = D_T(1+i) + d_{T+1} = d_T(1+i) + w_{T+1}cb(1+n)R_{T+1} - w_{T+1}cbN_{T+1} - w_{T+1}zN_{T+1}$$

Analogously, total public debt in Period T+2 is given by

$$\begin{aligned}(12) D_{T+2} &= D_{T+1}(1+i) + d_{T+2} \\ &= \left[d_T(1+i) + d_{T+1}\right](1+i) + w_{T+2}cb(1+n)R_{T+2} - w_{T+2}cbN_{T+2} - w_{T+2}zN_{T+2}\end{aligned}$$

and so on. This system of financing the transition towards a reduced pay-as-you-go system is sustainable if it implies a non-increasing public debt ratio D_t/Y_t, thus

$$(13) \frac{D_T}{w_t N_t} \geq \frac{D_{T+1}}{w_{t+1}N_{t+1}} \geq \frac{D_{T+2}}{w_{t+2}N_{t+2}}$$

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[4] With $z = z^*$, the present value of the base system is $w_{t-1}c\left(q^{-1} - 1\right) < 0$.

In fact, with $z = z^*$, condition (13) holds with the strict inequality sign, and with $z > z^*$, the inequality sign applies.[5] In other words, it is thoroughly possible to reduce the level of a pay-as-you-go pension system with a slightly diminished present value of retirement provision for every generation, including those who live in the transition period.

Figure i

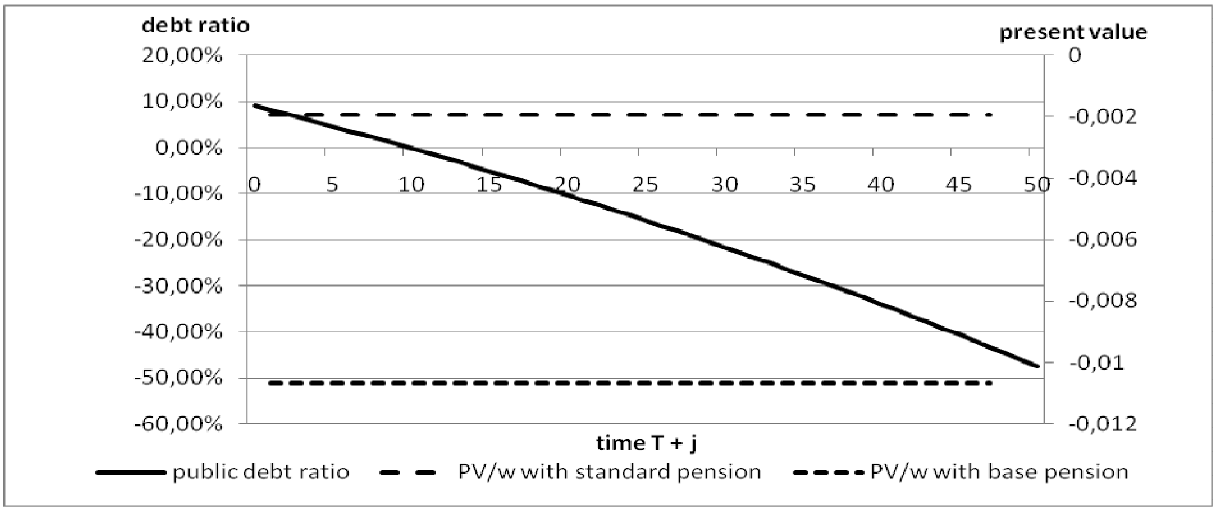
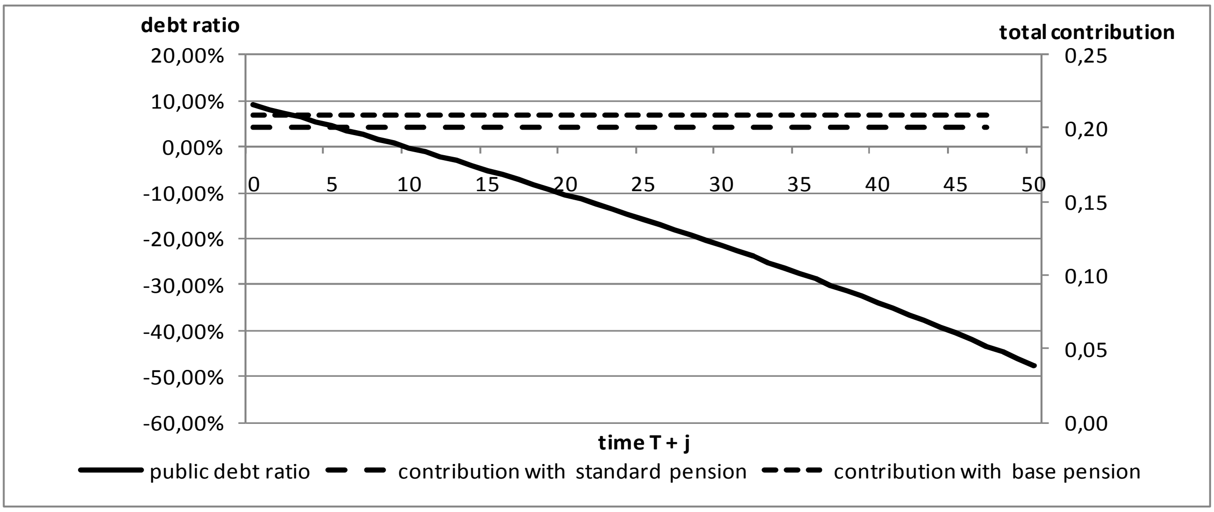


Figure i shows an example for this effect, with the public debt ratio marked on the left hand axis and the present value (in relation to wealth w_{t-1}) on the right hand axis.[6] In this example, it is assumed that a tax z = 0.0971% is raised in order to restore public debt on the zero level within a time span of ten periods (i.e. five generations). After this time span, there accrues even a progressively increasing public surplus. The price for this eventual transformation is a decline in present value from formerly -0.2% to now -1.1% of wealth, which at first glance looks quite substantial. However, in terms of contribution share, it is equivalent to a quite moderate increase from 20.0% to 20.9% (see Figure ii).

Figure ii



[5] For a rigorous proof see the appendix.

[6] The underlying calibration is n = -0.5%, g_w =2.5%, i = 3.0%, c = 20% and b = 50%, i.e. the standard system is reduced to a 50% base level. The resulting variables are z^* = 0.0971% and a^* = 9.9%.

With $PV_a = PV_c$, total profitability of retirement provision in the base system of course equals profitability of the standard system, which is g_y. Note, however, that the latter equality is only a necessary but not a sufficient condition for $PV_a = PV_c$. For profitability g_y in the base system only requires that

$$(14) - w_{t-1}(a+z) + w_{t-1}aq = 0$$

i.e. the growth rate instead of the interest rate is used as discount factor in (14) in order to calculate the particular value of z which generates profitability g_y in the base system. Solving (14) for z yields

$$(15) z^{**} = a(q-1)$$

which is more general than z^* because z^{**} depends on private supplement contributions a. With $a = a^*$ we would have $z^{**} = z^*$ again. In other words: With any $a > a^*$ it would be possible to generate an unchanged profitability g_y of total retirement provision with the base system and a declining public debt ratio as well, because in this case a tax rate $z^{**} < z^*$ would be sufficient to satisfy (14).

In terms of present value and Paretian efficiency, there would of course remain a loss. Moreover, it can also be shown that, with $z = z^*$, only a^* is compatible with utility maximization. Suppose that the individual utility function is logarithmic of the form

$$(16) U = \sum_{t=0}^{1} \frac{\ln(m_t)}{(1+\rho)^t}$$

where m_t is the amount consumed in Period t and ρ is the rate of time preference. When s denotes the voluntarily chosen saving rate in Period 0, with the standard pension system we have the following set of restrictions:

$$(17) w_0(1-c-s) - m_0 = 0$$
$$(18) cw_0(1+g_y) + sw_0(1+i) - m_1 = 0$$

Maximization of (16) then yields for the optimal saving rate with the standard pension system

$$(19) s_c^* = \frac{(1-c)\frac{1+i}{1+\rho} - c(1+g_y)}{\frac{1+i}{1+\rho} + (1+i)}$$

In contrast, with the base system, the respective restrictions are

$$(20) w_0(1-bc-z-s) - m_0 = 0$$
$$(21) bcw_0(1+g_y) + sw_0(1+i) - m_1 = 0$$

Maximization of (16) yields for the optimum saving rate with the base system

$$(22) s_a^* = \frac{(1-bc-z)\frac{1+i}{1+\rho} - bc(1+g_y)}{\frac{1+i}{1+\rho} + (1+i)}$$

Regarding that $(1+\rho) = (1+i)/(1+g_y)$ it is easily verified that, with $z = z^*$,

$$(23) s_a^* - s_c^* = a^*$$

Hence the micro-foundation confirms that (i) the present value criterion is consistent with utility maximizing and that (ii) individuals will chose exactly $a = a^*$ as additional private saving rate when the standard pension system is transformed into a base system with the extra tax z^*. [7]

As was mentioned above, even in the case of a constant public debt and, hence, without transformation to a funded system, some advantages of a base pension system could be picked. In fact, arguments 2 to 5 listed in the introduction would still apply. In particular, the private supplement provision gives the base pensioners a legal entitlement in the form of public bonds, which cannot easily be manipulated or even abandoned by the government. Moreover, concerning the base pension, there is no longer a particular burden on wages resulting from special contributions but a tax burden that can be distributed by the government deliberately in principle. Hence, without violating the principle of *inter*generational distribution, the government can well discriminate between those generations living at the same time and also between the members of each cohort. [8]

3. The model with more than one Transition Period

For empirical applications, a more general model in terms of lifetime division is needed. As a rule of thumb, real life is normally divided in six periods with equal length of 15 years respectively: The first period is childhood, followed by three periods as a worker, and finally two periods in retirement. In the sequel we neglect childhood, hence being left with five periods which are relevant with respect to the pension system. In analogy to the model in Section 2 we then have

$$(2.1) \frac{N_t}{N_{t-1}} = (1+n)$$

$$(2) \frac{R_t}{N_t} = \frac{S_5 - S_3}{S_5} \quad ; \quad \mathrm{S_j} \equiv \frac{\frac{1}{(1+n)^j} - 1}{\frac{1}{1+n} - 1} \quad ; \ \mathrm{j} \neq 0; \mathrm{n} \neq 0$$

[7] Note, however, that total savings (including z) are generally different from the equilibrium saving rate that would result without any compulsory pension system and without any tax.

[8] The latter is intensively discussed in Kotlikoff et. al (1998)

$$(3.2)\, p_t = w_t c \frac{N}{R} = w_t c \frac{S_3}{S_5 - S_3} = w_{t-3}(1+g_w)^3 \frac{S_3}{S_5 - S_3}$$

where S_j is the totals formula of a finite geometric series with j elements. Profitability of the standard pension system is still g_y. Substituting $x \equiv (1+g_w)/(1+i)$, for a generation starting active life in Period t = 0 their present value at discount rate i is

$$(4.2) PV_c \equiv -w_0 c\left(1 + x + x^2\right) + w_t c \frac{N}{R}(x^3 + x^4) < 0$$

Concerning the base pension system, again a fraction a of wealth is assumed to be devoted to private retirement provision. Present value of total retirement in the base system (including the tax burden) is

$$(5.2) PV_a \equiv -w_0(bc + a + z)(1 + x + x^2) + w_0 bc \frac{N}{R}(x^3 + x^4) + \frac{V_3}{(1+i)^3}$$

where V_3 denotes the cumulated amount insured at the beginning of retirement, which is

$$(6.2) V_3 = w_0 a\left((1+i)^3 + (1+g_w)(1+i)^2 + (1+g_w)^2(1+i)\right)$$

Like in the former model, a Pareto-efficient transformation would require $PV_a = PV_c$. In contrast to that model, however, there are now two retirement periods instead of only one. In order to generate the same pension payments as the standard system, private rent payments r_j in the base system must therefore meet the following conditions:

$$(7.2) r_j + bp_j = p_j \qquad \mathrm{j} = 3;4$$

$$(8.2) V_3 + (V_3 - r_3)i = (p_3 + p_4)(1-b)$$

From equations (6.2) to (8.2) the required private contribution rate a can be calculated as

$$(9.2) a^* = \frac{c(1-b)\frac{N}{R}\left[(1+g_w)^3(1+i) + (1+g_w)^4\right]}{1+i}$$

Employing (5.1) and (5.2), the required tax rate to make PV_a equal to PV_c is then

$$(10.2) z^* = c(1-b) - \frac{c(1-b)\frac{N}{R}(x^3 + x^4)(1+i)^3}{(1+i)^3 + (1+g_w)(1+i)^2 + (1+i)(1+g_w)^2}$$

With this tax on wealth of those who are entitled to the base pension, the present value of their total retirement provision is the same as it had been in the standard system, provided they realize $a = a^*$.

Again, public debt ratio $D_t /(w\ N_t)$ is constant in the long run with $z = z^*$. Suppose that the transformation of the pay-as-you-go system into a funded system starts in Period T, i.e. only the contributions of that cohort who start active life in that period are reduced to the base level cbw_T. Consequently, they will receive only the base pension from Period T+3 on, and in addition they have to pay taxes $z^* w_t$ from Period T on up to Period T+2. The same applies to the following generations who start active life in Periods T+1 and T+2 respectively, so after three periods there are no more contributors to the standard pension system.

Concerning public finances, with this transformation scheme the same conclusions apply as in the simple model of Section 2. In particular, with $z = z^*$, public debt is constant in relation to total output in the long run, irrespective of productivity growth, population growth and the interest rate.[9] Thus again, by adjusting the tax rate only slightly above z^*, the government can distribute the costs of the system change equally on all future generations with only limited decrease of their present value respectively, and at the same time generate a public surplus which replaces the initial rise in public debt in the long run.

To see this, we write d_t for the primary deficit in Period t and D_t for total public debt in that period:

$$(11.2) D_t = d_t + D_{t-1}(1+i) = \sum_{j=0}^{t} d_j (1+i)^{t-j}$$

With G_t denoting the size of the particular generation who start active life in Period t, the primary deficit in any period t is

$$(12.2) d_t = [c(1-b) - z]\sum_{j=t-2}^{t}(w_t G_j) - c(1-b)\frac{\mathrm{N}}{\mathrm{R}}\sum_{\mathrm{j}=t-4}^{\mathrm{t-3}}(w_t G_j) \quad ; \mathrm{j} \geq 0$$

The first summand in (9.4) is the loss in contribution receipts and the second summand are the pension expenses saved due to the reduced system. In the first three transition periods there are only public losses but no savings, the latter accruing only from Period T+3 on. Hence the public debt ratio rises in the first three transition periods and decreases thereafter. As is proved in the appendix, like in the model in Section 2, with z = z^* the public debt ratio $D_t /(w_t N_t)$ keeps constant after the transition process has ended. Moreover, with any $z > z*$, public debt constantly decreases after Period T+2 and eventually turns into a public surplus.

[9] A general proof for the six generations model is given in the appendix.

Figure iii

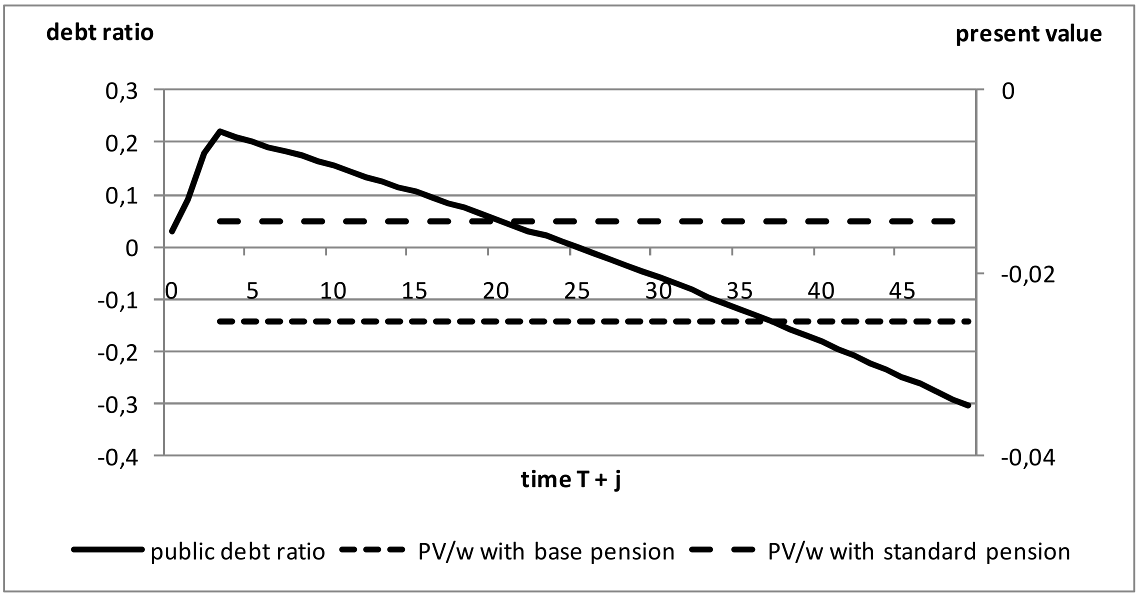
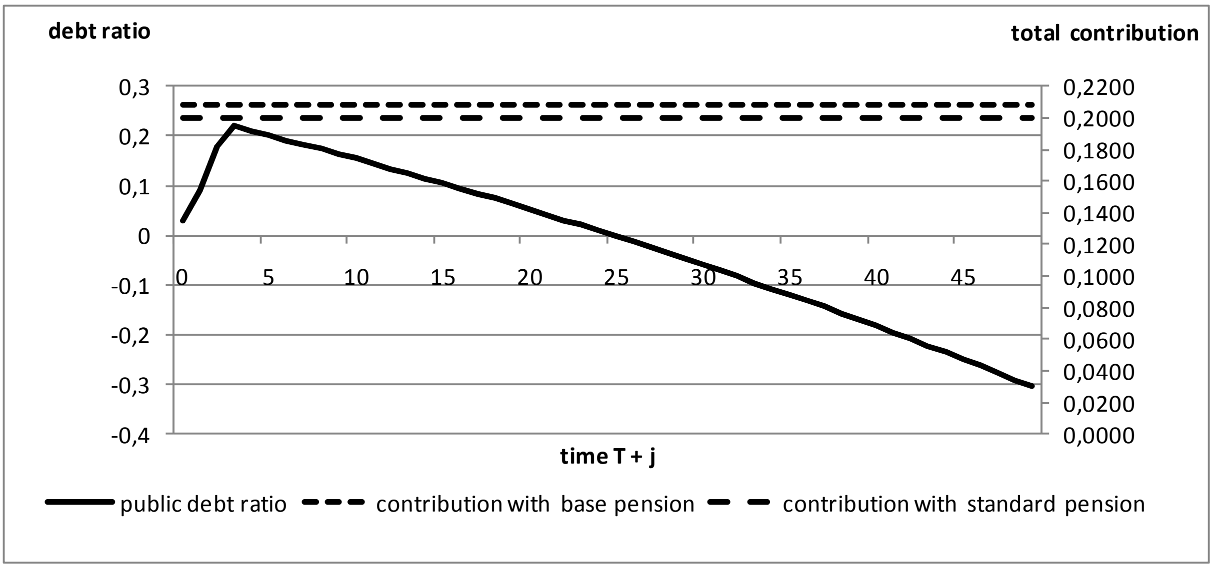


Figure iii shows an example, with the same basic assumptions as in the model of Section 2 above.[10] In order to restore a zero level of public debt after 25 periods (which are again five generations in this model), the tax must be set to 1.12% in this example, thereby reducing the present value for all generations from formerly -1.4% to now -2.5% of wealth w_0 which they earn in their first active period. Again, this sacrifice appears to be less hard in terms of total contributions, which again increase from 20% to 20.9% of wealth like in model of Section 2 (see Figure iv).

Figure iv



Therefore, the multiple generations model does not yield different results from the simple two generations model, neither generally nor in terms of the quantitative relations. In particular, the transformation towards a substantially reduced pay-as-you-go system appears thoroughly possible

[10] See section 2. The resulting z^* is 0.24% with a^* being 9.76% in the five generations model.

without unduly burdening the transition generation, and also within reasonable times given the huge dimension of the problem.

4. Conclusions

It is true that the process described above is not rigorously Pareto-efficient, but it appears to be both economically favorable and politically feasible. Admittedly, in reality, it would hardly be possible to perform the transformation within one period, both because of political restraints against a huge and sudden rise in public debt and because of limited capacity of the capital markets to absorb the respective public bonds. Much more promising appears an incremental procedure like in the multiple generations model above, with only those being released from the standard pension contributions who start their active life in the respective year. That would mean an approximately 45 year`s rise in public debt, until eventually the process is reversed with the first savings to occur when the respective cohorts start retiring. According to our theoretical results, it would then approximately take another five generations (each at 75 years) to restore public debt to zero again.

Although more rapid transformation schemes are also conceivable, we have to ask whether a transformation process with such a long perspective is paying at all. First, we should not forget that, without the transformation, there would not be a reduction of the (implicit) public debt at all. Moreover, an important advantage is the transformation of formerly insecure pension claims into legally protected entitlements in the form of public bonds. In particular, unlike pensions in a pay-as-you-go system, the payout of bonds cannot be made dependent from political opportunity or from individual neediness. The same applies to the base pension, when it is designed as an unconditional social minimum income.[11] So both the base pension and supplement retirement provision can serve as a secure fundament on which further savings can rest. This should spur both private savings and post-retirement incomes in comparison with the standard system where they could easily be counted against the public grants (as it is e.g. the case with the pensions of German civil servants).

Concerning contributions, it is not immediately clear what the effects on the labor market the transformation will be. Homburg (1990) and Homburg and Richter (1990) have argued that private savings should diminish the deadweight loss arising from compulsory payroll taxes (see also Breyer and Straub 1993). This is particularly relevant if the individual pension entitlement is not proportional to the respective individual contribution. Moreover, unwillingness to pay the contributions is likely to be caused by the fact that the profitability of pay-as-you-go systems is lower than the market interest rate.

With the base system discussed above and $z > z^*$, profitability of total retirement provision is lower than with the standard system, where it equals $g_y < i$. The difference is due to the fact that the additional tax z does not pay off at all. On the other hand, while base contributions cbw still pay interest g_y, the profitability of private supplement savings aw is $i > g_y$. So it seems quite reasonable that overall resistance to pay the contributions is lower in the base system than in the standard system, if z does not account for a too large part of contributions in total.

[11] Because the base pension is defined as a constant fraction of wealth in our model, the concept of a dynamic socio-cultural subsistence level is implied. Moreover, as all individuals have the same wealth in our model, we cannot discuss the implications of unequal incomes within one generation. A respective enlargement of the model is left to subsequent research.

Another advantage of a base pension is the redundancy of additional poverty grants for the elder. Thus the system saves bureaucracy and enhances both clearness and fairness of the social system. Because everyone has unconditional access to the base pension, hidden manipulations in favor of particular groups like women, child-raisers, politicians or civil servants become obsolete. In contrast, grants to any of these groups must be made in an open form and immediately paid for, rather than making them dubious promises for the future.

Concerning intergenerational distribution, the model clearly reveals that the real burden on future generations does not stem from the interest payments on public debt but from the decline in real capital formation which possibly results from it. Therefore, even if a pay-as-you-go system is fully preserved, the government can yet relieve future generations by either reducing public debt or raising the investment share of public expenses. On the other hand, the transformation into a formally funded system does not really make a difference if public debt is increased accordingly at the same time. Hence, the pension issue should not be discussed without taking total public debt into consideration.

Appendix

(1) Prove that with $z \geq z^{*}$ in the model of Section 2

$$(A1)\frac{D_{T+1}}{w_{T+1}N_{T+1}} \leq \frac{D_T}{w_T N_T}$$

$$=> \frac{d_T(1+i)+d_{T+1}}{w_{T+1}N_{T+1}} \leq \frac{d_T}{w_T N_T}$$

with

$$(A2) q \equiv \frac{1+i}{1+g_y} = \frac{1+i}{(1+g_w)(1+n)} \geq 1$$

$$(A3)(1+g_w) = \frac{w_{t+1}}{w_t}$$

$$(\mathrm{A4})(1+\mathrm{n}) = \frac{\mathrm{N}_{\mathrm{t+1}}}{\mathrm{N}_{\mathrm{t}}} = \frac{N_t}{R_t}$$

Inserting (A2) to (A3) in (10) and (11) respectively yields

$$(A5)\frac{d_{T+1}}{w_{T+1}N_{T+1}} = -z$$

and

$$(A6)\frac{d_T}{w_{T1}N_T} = c - cb - z$$

Making use of (A2) to (A4) and inserting (A5), (A6) and (10) in (A1) yields

$$(A7) cq - cbq - zq \overset{?}{\leq} c - cb$$

Finally, inserting (9) into (A7) immediately proofs that, first, the strict equality sign in (A7) applies and, second, for any $z > z^*$ and $q \geq 1,$ the inequality sign in (A7) is valid, q.e.d.

(2) Prove that with $z \geq z^*$ in the model of Section 2

$$(A8) \frac{D_{T+2}}{w_{T+2} N_{T+2}} \leq \frac{D_{T+1}}{w_{T+1} N_{T+1}}$$
$$=> \frac{D_{T+1}(1+i) + d_{T+2}}{Y_{T+2}} \leq \frac{d_{T+1}}{Y_{T+1}}$$

By making use of (A5), (A8) can be transformed in

$$(A9) \frac{D_{T+1}}{Y_{T+1}} (q-1) \leq z$$

Insertion of (6) in (A9) yields

$$(A12) \frac{D_{T+1}}{Y_{T+1}} \leq \frac{(1-b)c}{q}$$

From (10) and (11) it follows that

$$(A13) \frac{D_{T+1}}{Y_{T+1}} = c(1-b)q - z(1+q)$$

Finally, inserting (6) in (A13) yields

$$(A14) \frac{D_{T+1}}{Y_{T+1}} = \frac{(1-b)c}{q}$$

Thus, in (A12) the strict equality sign applies with $z = z^*$ and the inequality sign applies with $z > z^*$ q.e.d.

(3) Prove that, with $z \geq z^*$, (A1) applies analogously in the model of Section 3.

With the transition process beginning in Period T, (A1) has to be written

$$(A15) \frac{D_{T+5}}{w_{T+5} N_{T+5}} \leq \frac{D_{T+4}}{w_{T+4} N_{T+4}}$$

Because total output growth is g_y, and $D_{T+5} = D_{T+4}(1+i) + d_{T+5}$, (A15) can be substituted by

$$(A16) D_{T+4}(1+i) + d_{T+5} \leq d_{T+4}(1+g_Y)$$

From (10.2) and (3.2) it follows that

$$(A17) c(1-b) - z^* = \frac{c(1-b)\frac{N}{R}(x^3 + x^4)(1+i)^3}{(1+i)^3 + (1+g_w)(1+i)^2 + (1+i)(1+g_w)^2}$$

For convenience, G_0 is normalized to unity. Then, according to (12.2), for D_4 we have

$$(A18) D_4 = [c(1-b) - z] w_T \begin{bmatrix} (1+i)^4 \\ + (1+i)^3(1+g_w)(1+(1+n)) \\ + (1+i)^2(1+g_w)^2(1+(1+n)+(1+n)^2) \\ + (1+i)(1+g_w)^3((1+n)+(1+n)^2+(1+n)^3) \\ + (1+g_w)^4((1+n)^2+(1+n)^3+(1+n)^4) \end{bmatrix}$$
$$- (1-b) c w_T \frac{N}{R} \left[(1+i)(1+g_w)^3 + (1+g_w)^4(1+(1+n)) \right]$$

By inserting (A17)and (A18) into (A16) and considering that $(1+g_y) = (1+n)(1+g_w)$, all the terms $w_T; c; b; N/R$ cancel out and, after some manipulation of terms, validity of the strict equality sign in (A15) results with $z = z^*$, while the inequality sign in (A15) is valid with $z > z^*$,q.e.d.

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