Uncertainty and Bargaining: A structural econometric approach

Gordon Klein*& Mayra Rebolledo†

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Abstract

This paper proposes a structural econometric model to analyze the influence of uncertain disagreement outcomes on the settlement of new agreements between parties with uneven bargaining power. By including the the findings of theoretical literature on the requirements to reach agreements under uncertain disagreement conditions, and using an asymmetric Nash bargaining framework, our proposal is able to estimate not only the bargaining solutions but also the bargainers' uncertainties and determine whether a new compromise could be reach under such uncertainty or not.

1 Introduction

The vertical interaction along a value chain (e.g., between supplier and retailer, between workers and employers) has been the subject of various threads of literature focusing mainly on the bargaining process at the different links of the chain. One of the workhorse models to analyze those negotiations are Nash-Bargaining models, which are widely used in the applied literature since the work of Horn and Wolinsky (1988). In the last decade, since the work of Draganska, Klapper, and Villas-Boas (2010) or also Grennan (2013), arose empirical literature analyzing surplus splitting through structural econometric frameworks to uncover the bargaining outcomes of the so-called Nash-in-Nash bargaining situations.[1] Since then this topic has proliferated in the literature and the implementa-

*Westfälische Wilhelms-Universität Münster, Germany; Research Affiliate DICE Düsseldorf, Germany

†Westfälische Wilhelms-Universität Münster, Germany; Research Affiliate DICE Düsseldorf, Germany. This research was supported by the Deutsche Forschungsgemeinschaft (DFG) - Project number: 315503248.

[1]Hereafter, we refer the common Nash-bargaining model as Nash-in-Nash solution following Crawford et al. (2018)

tion of Nash bargaining solutions through structural econometric models has allowed us to have a better understanding on topics such as the effects of bundling (Crawford and Yurukoglu, 2012), price discrimination (Grennan, 2013), health insurance competition (Ho and Lee, 2017), or vertical integrations (Crawford et al., 2018) on welfare; or the effects of horizontal integration (Gowrisankaran, Nevo, and Town, 2015) or product charateristics (Bonnet and Bouamra-Mechemache, 2015) on bargaining leverage, to cite a few. However, the bargaining environment considered until now has implicitly assumed smooth and transparent interactions among bargainers, overlooking the potential frictions of some commercial relationships and the possible informational asymmetries within them, being the latter a already known weakness of this bargaining setting[2].

Collard-Wexler, Gowrisankaran, and Lee (2019) has provided a micro-theoretical foundation for the suitability of Nash-in-Nash bargaining to analyze the surplus division under a general mild set of assumptions. They consider a multiple-firm upstream and downstream framework, in which each pair of agents bargain over a single contract (product), and addressing the potential *"inter-relationship uncertainty"* (the uncertainty regarding the competitors contracts) by assuming *passive beliefs*[3]. This assumption may suit well in markets in which there is a single-product exchange within pair of negotiators, but may not thoroughly address the dynamics of a multiproduct relationship, in which the negotiations within the relationship might not always be completely independent from each other, leaving room to analyze the potential interdependence of the negotiations within the same pair of bargainers, opening the discussion regarding the effects of a potential *"intra-relationship uncertainty"*. This leads to the problem, that without explicit consideration of potential sources of uncertainty in *multi-product* relationships, applying the model assuming *inter-* and *intra-relationship passive beliefs* may be too restrictive, which may lead to potential misestimations depending on how prevalent are these type of relationships in the market. This is what we are addressing through our proposal, we include *intra-relationship uncertainty* in the structural econometric approach, to not only uncover the role of this element but also to expand the applications of this kind of analysis to more diverse scenarios, by paying more attention to the disagreement payoff and the part it plays in negotiations. Given that this element would gather the fear/threats that the bargainers develop

[2]See, Crawford and Yurukoglu (2012, pp. 659ff).

[3]According to McAfee and Schwartz (1994), *"[u]nder passive beliefs, when a firm receives an offer different from what it expects in the candidate equilibrium, it does not revise its beliefs about the offers made to others"*, [See, McAfee and Schwartz (1994) pp. 219.]

due to their asymmetries of information within the bilateral relationship.

Examples for such a kind of uncertainty seem to exist in particular in the food supply chain, where the apparent presence of uncertain bargaining conditions have been brought to the public attention through the last decades, in which the discussion regarding bargaining disparities and unfair trade practices UTPs (European Commission, 2016)[4] have been linked to this industry's negotiations. The competition authorities have reacted differently toward this topic. The European Commission, for instance, has been discussing a legislative proposal aiming to prohibit some of the witnessed UTPs between retailers and suppliers, in order *"to grant"* small and middle sized firms *"greater certainty"* and *"to eliminate the "fear factor" in the supply chain"*[5]. Given the multi-product and multi-bargaining intra-commercial relationship setting of this kind of industries, assuming *passive beliefs* among negotiations of the same commercial relationship could be an strong assumption, in particular for markets with witnessed complicated relationships.

Theoretical literature on bargaining has analyzed different sources of uncertainty in negotiations; in particular, situations in which there is a temporary uncertainty regarding the disagreement outcome [e.g. Peters and Van Damme (1991), Chun and Thomson (1990a,b,c), Livne (1988)], describing different incentive conditions that should be fulfilled to prefer reaching an agreement under uncertain conditions instead to delay the decision, given the transitority of the uncertainty source; literature summarized by Thomson (1994, p. 1262ff). In such a situation the questions arises whether a solution of a negotiation should be found either today or tomorrow (when uncertainty disappears). Two of the proposed conditions to create the incentives to reach an agreement in a uncertain situation were the either *Disagreement point concavity* or *Weak disagreement point concavity*, introduced by Chun and Thomson (1990a,b,c), the idea behind these conditions is that the solution reached under uncertainty conditions is at least as good as the evaluated today expected solution under certainty conditions, i.e. that the solution coming from considering an expected disagreement payoff as the outside option of the negotiation is at least as high as the today-evaluated expected solution from the possible outcomes with known disagreement payoffs. Their findings also

[4]According to the European Commission, UTPs *"are practices that deviate grossly from good commercial conduct, are contrary to good faith and fair dealing and are unilaterally imposed by one trading partner on another"*

[5]European Commission acts to ban unfair trade practices in the food supply chain (2018, April 12), European Commission, [Retrieved (2018, September 28): `http://europa.eu/rapid/press-release_IP-18-2702_en.htm`]. Also, in other jurisdictions such as Australia, United Kingdom and Ecuador, this kind of potential issues have been addressed similarly [Junta de Regulación de la Ley Orgánica de Regulación y Control del Poder de Mercado (2017), Mills (2003)]

suggest, that this incentive condition may not be fulfilled in relationships with uneven bargaining power, in such case, the bargainers would prefer not to reach an agreement today (i.e. keeping their current situation) until the uncertainty disappears.

Notice that whereas a new agreement under uncertain conditions could be reached or not, if there is uncertainty in the market, it will be reflected in the observed information, and if it is possible to assess this uncertainty, this would allow us to have an strategic understanding of negotiations. At the same time, and if a new agreement could be reached under uncertain disagreement circumstances, not considering the *intra-relationship uncertainty* in the analysis could lead to misestimations of the bargainers' disagreement profits affecting, consequently, the estimation of the surplus distribution. In this way, we are applying Chun and Thomson (1990a,b,c) incentive condition, *Weak disagreement point concavity*, through our proposed empirical approach to analyze a situation of temporary uncertainty on the disagreement payoff.

Therefore, our approach contributes in a twofold manner. First, by proposing an structural econometric model based on a asymmetric Nash bargaining framework, which regardless on whether an agreement may be reached or not, is able to assess the level of uncertainty of the negotiation. Second, we implement the Chun and Thomson (1990a) incentive condition (*Weak disagreement point concavity*) to determine whether a new compromise could be reached or not given the temporary uncertainty in the market. Both being relevant due to the important refinement of the existing approaches to estimate surplus splitting, and because through these contributions could be opened a new scope of analysis for further issues, such as listing and delisting of products (for instance in the retailing industry), the empirical analysis of strategic bargaining (e.g., the anaylsis of reputation and threats), among other applications, which could be of the interest for both industrial organization and strategic management field.

To exemplify an implmentation of our approach, we analyze the negotiation outcomes of dairies (manufacturers) and retailers before the German-dairy-farmers' strike took place at the end of may 2008. We study how the possibility of a farmers' strike in the upstream market influenced the renegotiations between the retailers and manufacturers in the downstream milk market. Finally, we compare the bargaining results from the before- and after-strike periods to evidence the cost of the strike for both actors. This example is valuable for several reasons. First, it allows us to apply the proposed estimation approach, based on a Nash-in-Nash bargaining analysis, in a multi-product and multi-bargaining type of market (grocery retail)((Bonnet and Bouamra-Mechemache, 2015; Draganska, Klapper, and Villas-Boas, 2010). Second, and given this was a temporary uncertain situation, in which we know that there was a break-up in negotiations between the parties, we can then evaluate empirically the incentive condition by Chun and Thomson (1990a), and evidencing the lack of incentives of the bargainers to reach an agreement before the strike took place, i.e. before the negotiations broke-up.

This paper develops as follows in section 2 the theoretical foundation of our approach is explained. In section 3 we introduce the case of the *milk-strike* and explain the sources of uncertainty of the bargainers in this market. In section 4 is explained our identification strategy, as well as the structural econometric model, to estimate the bargaining outcomes and beliefs. Section 5 describes the data used, as well as presents the results. Finally, section 6 concludes.

2 Uncertain disagreement and bargaining

In some markets prices - and consequently margins- are not exogenous to the bargaining abilites of firms, but rather they result from bilateral negotiations among them. Considering this in the empirical analysis has allowed to have a better understanding on the information available and the implications of the surplus division in vertical relationships.

Recent literature has been addressing the estimation of bargaining outcomes through the implementation of Nash-in-Nash solutions [e.g. (Bonnet and Bouamra-Mechemache, 2015; Crawford et al., 2018; Crawford and Yurukoglu, 2012; Draganska, Klapper, and Villas-Boas, 2010; Grennan, 2013; Ho and Lee, 2017); deriving the empirical specification from the solution of a general Nash product:

$$\underset{w_j}{\text{Max}} \left(\pi_j^m - d_j^m\right)^{\lambda_j^m} \left(\pi_j^r - d_j^r\right)^{\lambda_j^r}$$

in which two bargainers, r and m, engage in a negotiation over w_j, the price of product j, having both agents involved in the negotiation a bargaining position -bargaining power- λ_j^i (being $i = r, m$ and $\sum_i \lambda_j^i = 1$), and taking r and m their decision by comparing their profits from reaching an agreement π_j^i against their profits if no agreement is reached (disagreement payoff) d_j^i. The solution derived from this problem has been proved to be a viable method to empirically approach the bilateral negotiations subject to some assumptions (Collard-Wexler, Gowrisankaran, and Lee, 2019).

Among these assumptions is the use of *passive beliefs*, which according to McAfee and Schwartz (1994) *"[u]nder passive beliefs, when a firm receives an offer different from what it expects in the candidate equilibrium, it does not revise its beliefs about the offers made to others"*. The *passive beliefs* assumption addresses the bargainers' imperfect information during negotiations about the results of the other bargainings that are taking place in the market (Collard-Wexler, Gowrisankaran, and Lee, 2019; Draganska, Klapper, and Villas-Boas, 2010).

This assumption has been used in theoretical analysis in which is also (implicitly) considered that each bilateral relationship bargains over a single product/contract (Collard-Wexler, Gowrisankaran, and Lee, 2019); and by establishing an assumption regarding the beliefs on the negotiations with/of the other players aside from the current negotiation, bargainers would expect only one scenario in regards to disagreement, one in which just the current negotiation was not successful; implying in this way that all other negotiations are independent from the current one.

On the other hand, there are bilateral relationships in which a set of different products are bargained separately from each other; for instance, in the retailing industry, both manufacturers and retailers can have different representatives for brands or product lines, and in this way, manufacturers with a wide variety of products could have several negotiations with the same retailer over different products. In this kind of multi-product and multi-bargaining environment, the general assumption of *passive beliefs* could address the uncertainty regarding the negotiations of the other players, the ones outside the current negotiation, the *"inter-relationship uncertainty"*. However, assuming *passive beliefs* among the bargainings within the same bilateral relationship, could impose an independence among these negotiations that in reality could not always exist, in particular in case of conflict within the bilateral relationship, in such cases there would be an *"intra-relationship uncertainty"*, i.e. the lack of certainty regarding how not reaching agreement in one negotiation would affect the others from the same pair of bargainers; having the disagreement outcome, in this way, a key role when this *intra-relationship uncertainty* is included in the analysis.

As it is already known, the disagreement outcome serves as a reference point to evaluate the gains from an offer, the higher the disagreement payoff (outside option) the better the bargaining position of the agent in the negotiation. Therefore, a better assessment of the disagreement outcome translates in a better evaluation of the offer.

Theoretical literature on bargaining has already analyzed bargaining outcomes under disagreement uncertainty [e.g. (Chun and Thomson, 1990a,b,c; Livne, 1988; Peters and Van Damme, 1991). In particular, Chun and Thomson (1990a,b,c) axiomatically analyzed bargainings when both agents, at the moment of the negotiation, are uncertain about the scenario they would face if no agreement is reached, an uncertainty that disappears in some point in the future. In their work, it was established an incentive condition in order to both bargainers prefer to reach an agreement under in a uncertain situation, instead of waiting until there is no uncertainty to then agree onto something.[6]

The idea behind the *(weak) disagreement point concavity* condition is that the solution under disagreement uncertainty should make bargainers at least as good as they expect to be if the negotiation would develop under no uncertainty. In this way, bargainers would not have the incentive to wait until the uncertainty is gone to reach then an agreement (Chun and Thomson, 1990a,b,c). However, their findings suggest that the set of non-symmetric Nash bargaining solutions may not fulfill this condition.

Hence, bargainers with uneven bargaining power may not have the incentive to reach an agreement under uncertain disagreement conditions and when they have the option of breaking negotiations and wait. However, if a new agreement could be reached under such bargaining circumstances, then assuming *intra-relationship passive beliefs* would misestimate the disagreement profits and, consequently, the surplus division between agents.

At the same time, and regardless whether the solution under uncertainty become the new agreement or not, if there is uncertainty in the negotiations, it will be reflected in the results that we are observing. In this way, and by recalling that in recurrent bargaining relationships there is a preestablished bargaining condition at the moment of the negotiation -*status quo*- (the bargaining power among the agents until that point in time), by including in the analysis the uncertainty that players face, the beliefs and potential new solution can be estimated,as well as Chun and Thomson (1990a,b,c) incentive condition can be implemented, all of these allowing us to strategically analyze negotiations.

[6]Notice that this incentive condition is needed in cases in which the uncertainty is temporary and players have the option of breaking negotiations until the uncertainty is lifted; but such incentive condition would not be needed in negotiations under uncertainty in which players are not allowed to wait, and their only option possible is to reach an agreement in such conditions, as suggested by Chun and Thomson (1990a, p. 951).

Uncertain disagreement conditions: The model

Consider an upstream market consisting of M firms, each upstream firm m offers to the R available downstream firms a set of products J^m, being able each downstream firm r to bargain the price of the available products with the corresponding firm in the upstream market. Let us denote a bargained product between firms m and r as j, and its price as w_j.

The firms belonging to each stage (downstream or upstream) are competitors among each other, and it is assumed that all products are individually bargained in vertical-simultaneous negotiations; given this, we are assuming that the firms will send a *delegated agent*[7] to the negotiation of each of its products with each of its counterparties, i.e. there will not only be concurrent bargainings involving the same firm with different counterparties, but also there will be simultaneous negotiations involving the same pair of firms but regarding different goods[8].

Assume that each *delegated agent* have *passive beliefs* regarding the outcome of the other firms, i.e. when an unexpected outcome arises in the current negotiation, bargainers do not revise their beliefs regarding the outcomes of their competitors.

At the same time, it is assumed, that the *delegated agents* know the outcome when an agreement in their negotiations is reached. In this way, let us denote the profits from an agreement between upstream firm m and downstream firm r over product j as $\pi_j^m(w_j)$ and $\pi_j^r(w_j)$ respectively.

On the other hand, it is also assumed that at the moment of the negotiation the *delegated agents* are not certain about the implications of a disagreement in their current negotiations on the other bargainings involving the same two firms. We assume that the agents of firms m and r consider two different scenarios in case of disagreement over product j: 1) this disagreement does not affect the other negotiations between these two firms, and therefore the disagreement payoff for firm r and m are the profits of these firms when only product j is not exchanged between them, denoting these dissagreement profits as d_j^m and d_j^r for firm m and r respectively; 2) the disagreement implies the break of the commercial relationship between these two firms, i.e. this disagreement affects the other negotiations between the same two firms, and therefore no exchange of products between these

[7]The concept of *delegated agent* has been implicitly or explicitly mentioned before in the literature to address concurrent negotiations of a firm with different counterparties, e.g. Chipty and Snyder (1999); Collard-Wexler, Gowrisankaran, and Lee (2019)

[8]It is a common practice among firms with a wide variety of products to have different representatives for each product line or brand.

firms would take place; then if the set of products negotiated between firms m and r is denoted as J^{mr}, the disagreement payoffs for these firms under this scenario is represented by $d^m_{J^{mr}}$ and $d^r_{J^{mr}}$ respectively[9].

Given these two possible scenarios, the *delegated agents* would develop beliefs regarding facing any of them. Denoting the upstream and downstream firm *delegated agents'* beliefs on the first scenario as δ_j and θ_j respectively (being consequently, their beliefs on the second scenario: $1-\delta_j$ and $1-\theta_j$), where $\delta_j, \theta_j \in [0,1]$, we have that the generalized Nash bargaining product, given the uncertain disagreement conditions, is the following:

$$\underset{w_j}{\text{Max}} \left(\pi^m_j - E(d^m_j)\right)^{\lambda^m_j} \left(\pi^r_j - E(d^r_j)\right)^{\lambda^r_j} \tag{1}$$

where λ^m_j (λ^r_j) represents the upstream (downstream) firm's bargaining power in the negotiation of product j, being $\lambda^m_j + \lambda^r_j = 1$, and $E(d^r_j)$ ($E(d^m_j)$) the upstream (downstream) firm's expected disagreement payoff, i.e. $E(d^r_j) = \theta_j d^r_j + (1-\theta_j) d^r_{J^{mr}}$ ($E(d^m_j) = \delta d^m_j + (1-\delta) d^m_{J^{mr}}$).

Assume further that the source of this uncertainty lays on a possible temporary future event, source that constitutes a threat for the current negotiation, because it condition the counterparty's disposition regarding disagreement. According to

$$F(\pi_j, E(d_j)) \geqslant E(F(\pi_j, d_j)) \tag{2}$$

In this way, under uncertainty at the moment of the negotiations the agents would evaluate that the condition in (2) is fulfilled; e.g. in order to any of the bargaining parties has the incentive to reach an agreement under uncertainty, this agreement should be at least as good as the expected agreement with certainty conditions (linear combination from the possible outcomes under certainty): $F(\pi^m_j, E(d^m_j)) \geqslant \delta F(\pi^m_j, d^m_j) + (1-\delta) F(\pi^m_j, d^m_{J^{mr}})$. Thus, if the function of bargaining solutions under uncertainty is concave in the interval of the possible disagreement payoffs, then the solution under uncertainty is preferred. However, when this condition does not hold, according to

[9]Notice that this setting considers the possibility of a retaliation of the counterparty when no agreement is reached over a particular product, which we consider is a plausible uncertainty (threat) in frictional bilateral multi-product relationships.

3 Uncertainty and Bargaining: A motivating case

The case we are using to analyze uncertainty in negotiations was at the time an unusual episode in the German dairy industry, the 2008 dairy-farmers strike, hereinafter *"milk-strike"* or *"strike"*, which took place at the end of may of 2008, in which dairy farmers stopped the delivery of milk to dairy manufacturers as a sign of protest against the raw-milk prices they were receiving[10].

Despite being dairy manufacturers the direct buyers from farmers, these latter claimed that the raw-milk prices were the result of the consumer-price policy used by retailers that pushed down the consumer-milk price, leaving not enough margin to manufacturers, and consequently to farmers. And at a certain point of the *strike*, both dairy farmers and manufacturers were in a common front against retailers to increase milk prices, and with this their margins[11]

The *milk-strike* drew attention to the relationship among retailers and suppliers, from both public opinion as well as political actors, due to the moral connotations of farmers' main demand: a *"fair price"*. The *strike* finished after some well-known retailers promised to increase milk prices[12]. After the *milk-strike*, the German antitrust agency also opened a sector investigation focussed on buyer power and unfair practices[13].

As it is already known, strikes appear from unsatisfactory negotiations, and they have been more witnessed in wage bargainings between unions and firms, in which context has been studied by the literature [e.g. (Card, 1990a,b; Cramton and Tracy, 1992; Gu and Kuhn, 1998; Hart, 1989; Varoufakis, 1996). However, strikes are not a common leverage strategy in the bargaining between actors of the food supply chain, nor are they meant to transcend the negotiations of the other stages of the value chain.

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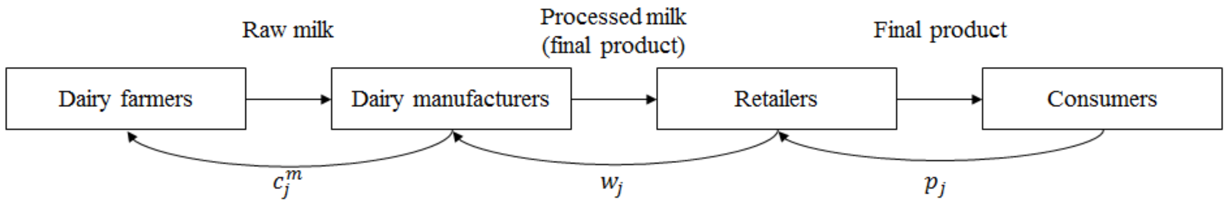
[10]We would like to mention that this *milk-strike* also motivated an unpublished Master Thesis by Anna Popova, in which was compared the suitability of the logit and conditional logit models to derive the demands of milk for the before- and after-strike periods.

[11]Wütende Milchbauern blockieren Molkereien (2008, June 2), Welt, [Retrieved (2018, September 24): `https://www.welt.de/jahresrueckblick-2008/juni/article2737068/Wuetende-Milchbauern-blockieren-Molkereien.html`].

[12]"Ich fordere Sie auf, ab heute Abend wieder Milch zu liefern" (2008, June 5), Frankfurter Allgemeine, [Retrieved (2018, september 24): `http://www.faz.net/aktuell/wirtschaft/unternehmen/milchbauernverband-fuer-boykottende-ich-fordere-sie-auf-ab-heute-abend-wieder-milch-zu-liefern-1544740.html`].

[13]See, Bundeskartellamt (2009).

Figure 1: Milk Value Chain



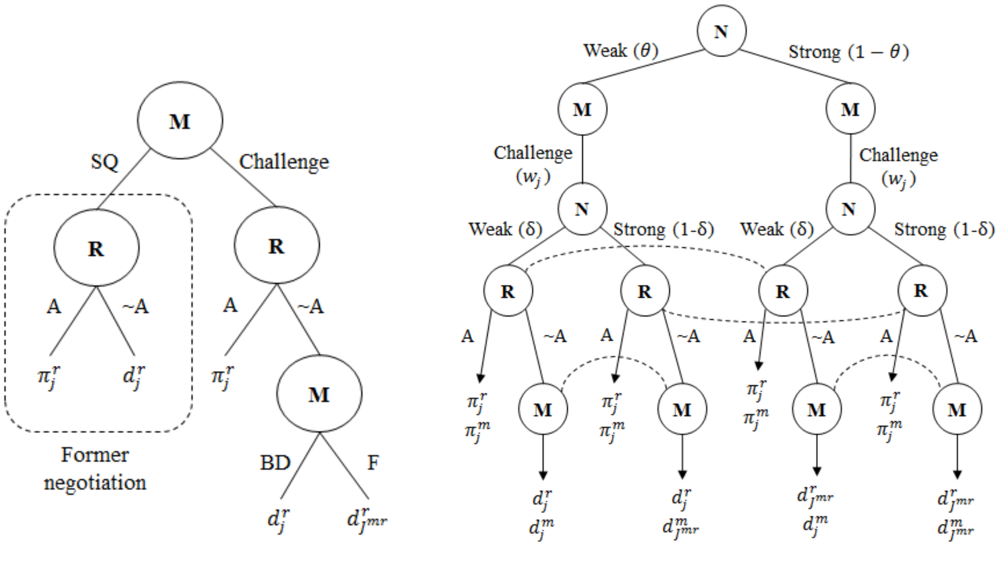
In this way, in order this *strike* had taken place, dairy manufacturers should have not satisfied farmers' demands of a higher raw-milk price (c_j^m) for a while before the *strike* broke. Meanwhile, and given the position of dairy manufacturers in the value chain (as can be observed in figure 1), in order to attend to farmers' demands without compromising their own margins, manufacturers should have also asked retailers for a higher price for their products, which would imply a renegotiation of the wholesale-milk price (w_j) between retailers and manufacturers. And it is in the link of the milk-supply chain, involving manufacturers and retailers, in which we are focusing our analysis, given that the bargaining between these actors would have determined whether farmers' needs could have been satisfied or the strike was inevitable.

Notice that due to the apparent unsatisfaction of the farmers in the upstream market, a wholesale-price renegotiation between retailers and manufacturers would have not developed under the same bargaining environment as the former dealings did, given that the threat of a possible farmers strike could have brought noise to the negotiation table, noise in the shape of uncertainty. And this uncertainty, once the *strike* took place disappeared from future negotiations, given that the threating event already happened, and the commercial relationships between manufacturers and retailers broke.

In this way, if a manufacturer chose renegotiate the wholesale price, challenging then the status quo (sq) of the commercial relationship, from the retailer's perspective this action could lead to two different scenarios when the negotiation was not sucessful: 1) as any other former negotiation, disagreement regarding the wholesale price of product j, would translate in the lost of j from retailer's shelves, but still the retailer could count with the other products of that manufacturer (d_j); 2) given the pressure of the upstream market, if disagreement over the wholesale price of product j took place, the manufacturer would have taken farmers' side, and chose not selling any of the other products to the retailer as well ($d_{J^{mr}}$, where J^{mr} is the set of products that manufacturer m usually sold to retailer r).

Figure 2a, represents the renegotiation from retailer's standpoint above described which, as can be observed, is similar to the classic crisis bargaining game, usually considered when analysing wars in international relations literature [e.g. (Fearon, International Organization; Fey, Meirowitz, and Ramsay, 2013; Fey and Ramsay, 2011; Lewis and Schultz, 2003). In crisis bargaining games one of the bargainers chooses either to stay in the status quo or to challenge it by making an offer to the other party, which in case to be rejected could lead to a war scenario, depending the realization of this scenario on the type of the bargainers. Type of the bargainers that is determined by the source of uncertainty in the game, being one of the possible sources the relative strenght of the opponent (either relative strong or weak); and therefore, the parties have to develop beliefs on the type of their opponent through their interactions (Fey and Ramsay, 2011).

Figure 2: Game representation



(a) (b)

In this way, and for the case under analysis, in the before-strike period, if a manufacturer chooses to challenge the status quo, the retailer could be facing either a "strong" manufacturer which, if disagreement over product j takes place, is ready for a "war" by supporting the farmers and not delivering any of the other products to the retailer (then, being the retailer-disagreement payoff

d^r_{Jmr}); or the retailer could be dealing with a "weak" manufacturer which, even though there is no agreement regarding product j, would still sell the other products to the retailer (d^r_j).

Similarly, under manufacturer's perspective, once they challenge the status quo, and given the noise in the upstream market, if disagreement occurs they could be facing either a retailer that is prepared to resist a "war"(*strike*) scenario: "strong" retailer (then, being the manufacturer-disagreement payoff d^m_{Jrm}); or being in front of a retailer that is not prepared for a *strike* scenario and will pursue to keep the other products of the manufacturer in his shelves: "weak" retailer (disagreement payoff: d^m_j).

Therefore, and as can be seen in figure 2b, in the before-strike period, regardless their type, if the manufacturer chose to challenge the status quo, both manufacturer and retailer were uncertain about their disagreement payoffs in a renegotiation of the wholesale price, and consequently they both would have to develop beliefs on which scenario (type of bargainer) they would face if there was no agreement.

Consequently, if there were renegotiations of the wholesale price in the before-strike period, these would have responded to the kind of dealings presented in section 2. And given the uneven bargaining power among players, the transitority of the uncertainty and the plausibility that the reaching of a new agreement was delayed (actors may not believe the threats, and have no incentives to change their status quo), then the potential agreement under uncertainty would not have fulfilled the incentive condition previously described (*(weak) disagreement point concavity*). Otherwise, farmer's demands would have been met before, and the *strike* would have not broke. In this way, bargainers would have prefer not to continue with the renegotiations, i.e. staying in the status quo, until there is no uncertainty to reach a new agreement, knowing then with certainty the disagreement payoff; resulting in failed wholesale price renegotiations, persisting the unsatisfactory conditions from the status quo, and paving the way to the *strike*, which would disappear the uncertainty to reach new agreements afterwards.

Finally, notice that in the period after the *strike* the noise from the upstream market is already gone and therefore renegotiations over the wholesale price would reach new agreements, due to the lack of uncertainty in the market.

4 Identification strategy

The idea of our paper is to take the case of the *milk-strike* in the German dairy market and to infer the bargaining situation including the distribution of uncertainty. This allows to compare the distribution of the rent among manufacturers and retailers before and after the strike, as well as to verify the lack of incentives to reach a new agreement in the before-strike period due to the uncertainty present in those renegotiations.

For each period we estimate the bargaining outcomes by backward induction. We first estimate the demand of milk, from which we derive the demand elasticities and marginal effects; being able then to assess the retailers margins, by assuming a Nash-Bertrand competition among them. Then, by implementing structural econometric Nash-bargaining models, we analyze the distribution of the rent between retailers and manufacturers including their expectations on their outside option. Additionally, we are including to the analysis a test on whether renegotiation between manufacturers and retailers over the wholesale prices in the before-strike period would have been resolved given the uncertainty produced by a possible *milk-farmers strike*.

We also analyzed the consequences of the strike on the bargaining power and the surplus division between manufacturers and retailers through to the comparison of the before and after bargaining results. Additionally, we are aiming to implement Chun and Thomson (1990a,b,c) theory, and analyzing the bargainers' incentives in the before-strike period to reach a new agreement, given their bargaining power distribution and the uncertainty coming from the upstream market.

4.1 Demand model

For each period (before- and after-strike), we derive the demand by implementing a random-coefficient logit model. We define the utility of a consumer i derived from product j at time t as follows:

$$U_{ijt} = x_j^{'}\boldsymbol{\beta} + \alpha_i p_{jt} + \varepsilon_{ijt} \tag{3}$$

where $\boldsymbol{\beta}$ is a vector of individual-specific coefficients capturing the time invariant effect on the utility of product attributes in x_j, such as brand, the retailer, and the level of fat. p_{jt} represents the price of product j at time t, while the random coefficient is denoted by α_i representing the marginal disutility of the price that varies across consumers, and which distributes as $\alpha_i = \alpha + \sigma_\alpha \vartheta_{\alpha_i}$, where

ϑ_{α_i} is the unobserved consumer characteristics, and σ_α measures the unobserved heterogeneity of consumers. We denote as F_w, the distribution of ϑ_{α_i}, that it is assumed to be independent and normally distributed, such that $\vartheta_{\alpha_i} \sim N(\alpha, \sigma_\alpha)$. Finally, ε_{ijt} denotes a random shock on the utility.

An outside option was included to incorpore a substitute to the J alternatives available in the market, being normalized its utility to zero, such that ε_{ijt} is the consumer indirect utility when choosing this alternative.

Then, and as proposed by Petrin and Train (2010), a control funtion was implemented to account for a possible specification's endogeneity (3) coming from a potential ommitted variable. In this way, the estimation of the market demand consists of two stages. In the first stage a regression of the mean monthly milk prices is perfomed, following the specification below:

$$\overline{p}_{jt} = x_j^{'}\boldsymbol{\tau} + cs_{jt}^{'}\boldsymbol{\psi} + v_{jt} \tag{4}$$

where $\boldsymbol{\tau}$ captures the time invariant effect of variables included in x_j, such as brand and retailer, on the mean monthly price $\overline{p}_{jt}$; while $\boldsymbol{\psi}$ captures the effect of the input prices included in cs_{jt}. Finally, v_{jt} represents the random shock of the regression.

Afterwards, the estimated $\hat{v}_{jt}$ is included in the consumer utility specification (3) as follows:

$$U_{ijt} = x_j^{'}\boldsymbol{\beta} + \alpha_i p_{jt} + \rho\hat{v}_{jt} + \mu_{ijt} \tag{5}$$

where $\mu_{ijt} = \varepsilon_{ijt} - \rho\hat{v}_{jt}$, which is assumed to be independent from p_{jt} and IID extreme value type 1 distributed. In this way, the probability of consumer i buying alternative j on time t conditional to α_i is represented by:

$$L_{ijt}(\alpha_i) = \frac{exp(x_j^{'}\boldsymbol{\beta} + \alpha_i p_{jt} + \rho\hat{v}_{jt})}{1 + \sum_{k=1}^{J} exp(x_k^{'}\boldsymbol{\beta} + \alpha_i p_{kt} + \rho\hat{v}_{kt})}$$

and consequently, the market share of product j in time t is given by integrating the consumer-level choice:

$$s_{jt} = \int \frac{exp(x_j^{'}\boldsymbol{\beta} + \alpha_i p_{jt} + \rho\hat{v}_{jt})}{1 + \sum_{k=1}^{J} exp(x_k^{'}\boldsymbol{\beta} + \alpha_i p_{kt} + \rho\hat{v}_{kt})} dF_w$$

Finally, the own- and cross- price elasticities ($\epsilon_{kjt} = \frac{\partial s_{kt}}{\partial p_{jt}} \frac{p_{jt}}{s_{kt}}$) are computed, after obtaining the marginal effects ($\frac{\partial s_{kt}}{\partial p_{jt}}$) through the simulation process suggested by (Cameron and Trivedi, 2010, p. 353).

4.2 Supply model: Retailers margin

Once the demands for each period have been estimated, then we can continue with the backward induction by estimating the retailers' margins for each period.

Notice that at this stage, the manufacturers' reactions towards disagreement is already known, therefore retailers have no uncertainty at this point; similarly, given the negotiations with manufacturers have already taken place at this point, the wholesale price is taken as given.

Assume Bertrand-Nash competition among retailers to set the price of product j, which is defined as a retailer-brand combination Bonnet and Bouamra-Mechemache (2015); Draganska, Klapper, and Villas-Boas (2010).

In this context, the retailers maximization problem is given by:

$$\underset{p_j}{\text{Max}}\, \pi^r = \sum_{j \in J^r} \left[p_j - w_j - c_j^r\right] s_j(p) M \tag{6}$$

where J^r denotes the set of products sold by retailer r (being $\sum_r J^r = J$), p_j and c_j^r are the retailers price and marginal cost of product j respectively, while the wholesale price of this product is represented by w_j. The total market size is denoted by M, and $s_j(p)$ represents the market share of product j.

Thence, and defining the retailers margin of product j ($p_j - w_j - c_j^r$) as γ_j, the subgame Nash equilibrium prices for the J^r products of retailer r are derived from the first-order condition coming from (6):

$$s_j(p) + \sum_{k \in J^r} \gamma_k \frac{\partial s_k(p)}{\partial p_j} = 0, \tag{7}$$

Which matrix notation would be $(T^r * \Delta)\,\boldsymbol{\gamma} + \boldsymbol{s(p)} = 0$, in which $*$ represents the Hadamard product operator, T^r is the retailers ownership matrix, which element $T^r(k, j) = 1$ if both products k and j are sold by the same retailer and $T^r[j, k] = 0$ otherwise, Δ is a matrix of the marginal effects of the price on the market shares, which general element $\Delta[j, k] = \frac{\partial s_k(p)}{\partial p_j}$, $\boldsymbol{s(p)}$ is the vector of market shares, and $\boldsymbol{\gamma}$ is the vector of retailers margins, (Draganska, Klapper, and Villas-Boas, 2010).

In this way, we have that the retailers margins can be expressed as follows:

$$\boldsymbol{\gamma} = -\left(T^r * \Delta\right)^{\dagger} \boldsymbol{s(p)} \tag{8}$$

where, $(T^r * \Delta)^{\dagger}$ is the Moore-Penrose inverse matrix of $(T^r * \Delta)$.

4.3 Supply model: Manufacturers margin

Once the retailer's margins were computed in each period following (8), we proceed to analyze the bargaining between retailers and manufacturers over the whosale margin (w_j), and derive an estimation of the manufacturers' margins.

As mention in section 3, in the before-strike period, before the farmers' demands appeared, manufacturers and retailers were already in a commercial relationship, meaning they had already an agreement regarding the wholesale price (*status quo*), agreement that should be challenged in a renegotiation. Given that there has been negotiations before challenging the *status quo*, manufacturers and retailers have already a pre-set bargaining power (the one resulted from the *status quo* negotiations). In this way, it is first needed to assess their preestablished bargaining power distribution, which is the one that would rule their renegotiations. And given that the negotiations that derived the *status quo* were held in an environment without the noise coming from the upstream market, then bargainers would not anticipate a possible retaliation in case of disagreement in the negotiation of one product in the *status quo*. In this way, we consider plausible to assume *intra-relationship passive beliefs* to derive the *status quo*'s results.

However, and as explained in section 3, due to a possible farmers' strike the renegotiations would not develop in an *intra-relationship uncertainty* free environment, having both bargainers to assess their bargaining situation, by developing beliefs on the possible scenarios that they expect to face in case of disagreement with the information they have until that time, the information coming from the *status quo*. Therefore, having they to renegotiate considering an expected disagreement outcome, but with a presstablished bargaining power.

As mention before, the potential new solution under uncertain disagreement outcomes should fulfilled the incentive condition in (2). Since we assume that firms act as risk-neutral utility maximizer and that the utility is only dependent on the compensation paid, the new wholesale price under uncertainty (w_j^u) requires to be:

$$w_j^u \geqslant \zeta w_j^{NT} + (1-\zeta) w_j^T \tag{9}$$

in which w_j^{NT} is the wholesale price when the bargainers believe that a disagreement on j would not have an effect on the other negotiations of the bilateral relationship (*no threat*), w_j^T is the wholesale price when they believe that a disagreement on j would translate into the breaking off the commercial relationship, and ζ is the bargainer' belief on facing a *no threat* type of negotiation, i.e. similar to the one faced in the *status quo*.

Notice that while bargaining, the manufacturers would also be uncertain regarding their marginal costs, given that it depends on the actions that the farmers take, and on manufacturers' negotiations. In this way, and defining the manufacturers' margin under uncertainty as: $\Gamma_j^{m,u} = w_j^u - (\delta c_j^{m,NT} + (1-\delta) c_j^{m,T})$, being $c_j^{m,NT}$ and $c_j^{m,T}$ the manufacturers' marginal costs when they faced a *no threat* scenario (similar to the *status quo*) and under the scenario in which commercial relationships are broken due to the milk-farmers strike respectively, *threat* scenario. And denoting as δ the manufacturer's belief on facing a *no threat* scenario. In this way, expression (9) is equivalent to the following:

$$\Gamma^u(w_j^u, \delta c_j^{m,NT} + (1-\delta) c_j^{m,T}) \geq \delta \Gamma^{NT}(w_j^{NT}, c_j^{m,NT}) + (1-\delta) \Gamma^T(w_j^T, c_j^{m,T}) \tag{10}$$

Therefore, through expression (10) is possible to test the fulfillment of condition (2). Notice that given that retailers prices are set in a stage after the bargaining with manufacturers, then they set their prices with full information on the result of the bargainings, for this reason, we analyze the fulfillment of the incentive condition on manufacturers, which margins depend on the negotiation with retailers.

In this way, in before-strike period it is needed to derive the bargaining power parameter in the *status quo* (which would also be the *no threat* scenario), in the scenario in which the commercial relationships break off (*threat* scenario) and the potential manufacturers' margin under uncertainty. Given that the strike took place, we expect to find that in the months before the strike the condition (10) was not fulfilled, which would implied that the renegotiation were not successful and bargainers kept the surplus division already agreed (*status quo*), which were not satisfying farmers' needs in the upstream market.

On the other hand, and as mentioned before, in the after-strike period the uncertainty regarding the strike disappears from the negotiations, in this way we can derive manufacturers' margins by considering a *intra-relationship uncertainty* free scenario (Draganska, Klapper, and Villas-Boas, 2010).

In this way, in the following subsections we present the expression derived to compute the manufacturers' margins before and after the strike.

4.3.1 Manufacturers margin under certainty (Status Quo/*No Threat* Scenario)

As we have mentioned, in the before-strike period manufacturers should challenge the agreement regarding the wholesale price they already have with retailers - an agreement that was reached in an environment without uncertainty (*status quo*) - in order to satisfy farmers' demands without compromising their own margin.

Similarly, and given that the strike would not longer be a threat in the after-strike period, in any renegotiations that took place after the strike the bargainers would not be uncertain rearding the intentions of their counterparties if there is no agreement regarding a product.

Therefore, for both cases we are considering an *intra-relationship uncertainty* free sceanrio, which has been already applied in the literature, assuming that manufacturers bargain with retailers each product separately and where they believe that disagreement on product j would not affect the other negotiations (Draganska, Klapper, and Villas-Boas, 2010), then the bargaining over the wholesale price of j can be expressed by the following Nash product:

$$\underset{w_j}{\text{Max}} \left(\pi_j^r - d_j^r\right)^{\lambda_j} \left(\pi_j^m - d_j^m\right)^{(1-\lambda_j)}$$

in which π_j^r and π_j^m represent respectively the retailers and manufacturers profits from selling product j (agreement payoff), while d_j^r and d_j^m represent respectively the retailer and manufacturer profits when product j is delisted from retailer r (disagreement payoffs):

	Agreements	Disagreement
Manufacturer	$\pi_j^m = \Gamma_j s_j(p)M + \sum_{\substack{k\in J^m \\ k\neq j}} \Gamma_k s_k(p)M$	$d_j^m = \sum_{\substack{k\in J^m \\ k\neq j}} \Gamma_k s_k^{-j}(p)M$
Retailer	$\pi_j^r = \gamma_j s_j(p)M + \sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k s_k(p)M$	$d_j^r = \sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k s_k^{-j}(p)M$

where Γ_j denotes the manufacturer margin of product j ($\Gamma_j = w_j - c_j^m$, being c_j^m the manufacturer marginal cost of producing product j), $s_k^{-j}(p)$ represents the market share of product k when there was no agreement on product j. And J^m represents the set of products of manufacturer m, being $J = \sum_m J^m$.

In this way, from the maximization of the Nash product above presented, we get the following expression:

$$(\pi_j^m - d_j^m)\frac{\partial \pi_j^r}{\partial w_j} = -\frac{(1-\lambda_j)}{\lambda_j}(\pi_j^r - d_j^r)\frac{\partial \pi_j^m}{\partial w_j}$$

Given that $\frac{\partial \pi_j^r}{\partial w_j} = -s_j(p)M$ and $\frac{\partial \pi_j^m}{\partial w_j} = s_j(p)M$, and applying the agreements and disagreements payoffs we get the following:

$$\left(\Gamma_j s_j(p) - \sum_{\substack{k\in J^m \\ k\neq j}} \Gamma_k \Delta s_k^{-j}\right) = \frac{(1-\lambda_j)}{\lambda_j}\left(\gamma_j s_j(p) - \sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k \Delta s_k^{-j}\right) \tag{11}$$

where Δs_k^{-j} represents the change in the market share of product k when product j is not longer available in the market, i.e. $\Delta s_k^{-j} = s_k(p)^{-j} - s_k(p)$.

Denoting $\mathbf{\Gamma}$ as the vector of manufacturer margins, and defining T^m as the manufacturers ownership matrix which element $T^m[j,k] = 1$ if both products k and j are produced by the same manufacturer and $T^m[j,k] = 0$ otherwise, and defining D^j as the matrix of shares and share variations, which $D^j[j,j] = s_j$ and $D^j[j,k] = -\Delta s_k^{-j}$, then the matrix notation of the system of equations from expression 11 is:

$$(T^m * D^j)\mathbf{\Gamma} = \tilde{\boldsymbol{\lambda}} * \left[(T^r * D^j)\boldsymbol{\gamma}\right] \tag{12}$$

where $*$, T^r, and $\boldsymbol{\gamma}$ represent the same as before, and being $\boldsymbol{\gamma}$ obtained through (8), and $\tilde{\boldsymbol{\lambda}}$ is the vector of the bargaining parameters ratio[14].

Therefore, the manufacturer's margin can be expressed by: $\mathbf{\Gamma}^{\boldsymbol{sq}} = (T^m * D^j)^\dagger \left(\tilde{\boldsymbol{\lambda}} * \left((T^r * D^j)\boldsymbol{\gamma}\right)\right)$, where $(T^m * D^j)^\dagger$ represents the Moore-Penrose inverse of matrix $(T^m * D^j)$. By denoting the general

[14] $\tilde{\boldsymbol{\lambda}} = \begin{pmatrix} \tilde{\lambda}_1 \\ \tilde{\lambda}_2 \\ \vdots \\ \tilde{\lambda}_J \end{pmatrix} = \begin{pmatrix} \frac{1-\lambda_1}{\lambda_1} \\ \frac{1-\lambda_2}{\lambda_2} \\ \vdots \\ \frac{1-\lambda_J}{\lambda_J} \end{pmatrix}$

element of the vector $(T^r * D^j)\boldsymbol{\gamma}[i,1] = b_i$, then $\boldsymbol{\Gamma^{sq}}$ can be expressed as $\boldsymbol{\Gamma^{sq}} = C\tilde{\boldsymbol{\lambda}}$, where C is an square matrix of dimension J which general element is $C[i,k] = (T^m * D^j)^{\dagger}[i,k]b_k$.[15]

Then, recalling that $\boldsymbol{\Gamma} = \boldsymbol{w} - \boldsymbol{c^m}$ and $\boldsymbol{\gamma} = \boldsymbol{p} - \boldsymbol{w} - \boldsymbol{c^r}$, we have that $\boldsymbol{c^r} + \boldsymbol{c^m} = \boldsymbol{p} - \boldsymbol{\gamma} - \boldsymbol{\Gamma}$; therefore, $\boldsymbol{c^r} + \boldsymbol{c^m} = \boldsymbol{p} - \boldsymbol{\gamma} - C\tilde{\boldsymbol{\lambda}}$, and due to unobservability of the marginal costs we will assume $\boldsymbol{c^r} + \boldsymbol{c^m} = IP\boldsymbol{\kappa} + \boldsymbol{\eta}$, where $\boldsymbol{\kappa}$ is the vector of the coefficients that capture the effect of the Z cost shifters (inputs), considered in matrix IP, on the total marginal cost, while $\boldsymbol{\eta}$ is an error term (Bonnet and Bouamra-Mechemache, 2015; Draganska, Klapper, and Villas-Boas, 2010). Therefore, $\tilde{\boldsymbol{\lambda}}$ can be estimated by the following specification:

$$\boldsymbol{p} - \boldsymbol{\gamma} = C\tilde{\boldsymbol{\lambda}} + IP\boldsymbol{\kappa} + \boldsymbol{\eta} \tag{13}$$

From the above specification we estimate $\tilde{\boldsymbol{\lambda}}$ by applying a non-linear least squares. Once the vector $\tilde{\boldsymbol{\lambda}}$ (and consequently $\boldsymbol{\lambda}$) was estimated, the vector of manufacturers margins can be recover from (12).

Notice that both the manufacturers' margin in *status quo* and in a *no threat* scenario are the result from negotiations without uncertainty; additionally, the margin in the *no threat* scenario are estimated considering the information until that moment (the *status quo* information), which includes the bargaining power until that point (i.e. $\boldsymbol{\lambda}$ in the *status quo*), then we have that $\boldsymbol{\Gamma^{sq}} = \boldsymbol{\Gamma^{NT}}$. A similar process can be used to recover the manufacturers' margin from the contingent scenario of a strike (*threat* scenario), evaluated at the moment of the uncertainty, more details on this process can be found in Appendix 9

4.3.2 Manufacturers margin under uncertainty

As mention in section 3, in the before-strike period a renegotiation of the wholesale price between manufacturers and retailers would develop under a different environment as the former negotiation (*status quo*) did, this new bargaining would suffer from asymmetric information regarding the possible reaction ("type") of the counterparty (and consequently their profits) when there is no agreement, having both retailer and manufacturer to develop beliefs on the possible scenarios they would be facing, and therefore having to bargain considering expected disagreement payoff.

[15] See Appendix 8 to more details on matrix C.

In this way, a renegotiation over the wholesale price of product j (w_j) is the result of the following Nash product:

$$\underset{w_j}{\text{Max}} \left(\pi_j^r - E(d_j^r)\right)^{\lambda_j} \left(\pi_j^m - E(d_j^m)\right)^{(1-\lambda_j)} \tag{14}$$

just as in (14) π_j^r and π_j^m represent respectively the retailers and manufacturers profits from selling product j, which expressions were presented in section 4.3.1.

On the other hand, $E(d_j^r)$ and $E(d_j^m)$ in (14) are the retailer's and manufacturer's expected disagreement payoff respectively, where $E(d_j^r) = \theta d_j^r + (1-\theta)d_{J^{mr}}^r$ and $E(d_j^m) = \delta d_j^m + (1-\delta)d_{J^{mr}}^m$, in which d_j^r and d_j^m represent the retailer's and manufacturer's disagreement payoff of facing a *no threat* scenario, being θ and δ the retailer's and manufacturer's beliefs on this scenario respectively, and where θ and δ are included in the interval $[0,1]$; while $d_{J^{mr}}^r$ and $d_{J^{mr}}^m$ are the retailer's and manufacturer's disagreement payoffs when agents are facing a retaliation from their counterparties if they do not reach a new agreement (*threat* scenario), which retailer's and manufacturer's beliefs on this possibility are $(1-\theta)$ and $(1-\delta)$ respectively. In this way, the disagreement payoffs for retailers and manufacturers are given by:

	No Threat/Status Quo Scenario	Threat/Strike Scenario
Manufacturer	$d_j^m = \sum_{\substack{k\in J^m \\ k\neq j}} \Gamma_k s_k^{-j}(p)M$	$d_{J^{mr}}^m = \sum_{\substack{k\in J^m \\ k\notin J^{mr}}} \Gamma_k s_k^{-J^{mr}}(p)M$
Retailer	$d_j^r = \sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k s_k^{-j}(p)M$	$d_{J^{mr}}^r = \sum_{\substack{k\in J^r \\ k\notin J^{mr}}} \gamma_k s_k^{-J^{mr}}(p)M$

where J^m, J^r and $s_k^{-j}(p)$ represent the same as before, while $s_k^{-J^{mr}}(p)$ is the market share of product k when disagreement over product j occurs and as result no transaction between that manufacturer and retailer are done, meaning that all products that were traded between these two agents (we called this set of products J^{mr}) are not longer available in the market.

In this way, and given that $\frac{\partial \pi_j^r}{\partial w_j} = -s_j(p)M$ and $\frac{\partial \pi_j^m}{\partial w_j} = s_j(p)M$, from the first order condition of (14) we get the following expression:

$$\pi_j^m - E(d_j^m) = \tilde{\lambda}_j(\pi_j^r - E(d_j^r)) \tag{15}$$

where again $\tilde{\lambda}_j = \frac{(1-\lambda_j)}{\lambda_j}$. After including the agreements and disagreements payoffs in the previous expression, we get the following[16]:

[16] See Appendix 10 to more details on this result.

$$\Gamma_j s_j - \sum_{\substack{k\in J^m \\ k\neq j}} \Gamma_k \Delta s_k^{-j} - \tilde{\delta}_j \left(\sum_{\substack{k\in J^m \\ k\notin J^{mr}}} \Gamma_k s_k^{-J^{mr}} - \sum_{\substack{k\in J^m \\ k\neq j}} \Gamma_k s_k^{-j} \right) = \tilde{\lambda}_j \left[\gamma_j s_j - \sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k \Delta s_k^{-j} - \tilde{\theta}_j \left(\sum_{\substack{k\in J^r \\ k\notin J^{mr}}} \gamma_k s_k^{-J^{mr}} - \sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k s_k^{-j} \right) \right] \tag{16}$$

where $\tilde{\theta} = (1-\theta)$ and $\tilde{\delta} = (1-\delta)$. Notice that the expression presented by Draganska, Klapper, and Villas-Boas (2010) would be the case when $\tilde{\theta}_j = \tilde{\delta}_j = 0$ (i.e. $\theta_j = \delta_j = 1$).

By defining S^j is the matrix of shares which element $S^j[j,j] = 0$ and $S^j[j,k] = s_k^{-j}(p)$ otherwise. Similarly $S^{J^{mr}}$ is the matrix of shares which element $S^{J^{mr}}[j,k] = 0$ if j belong to the same retailer-manufacturer (J^{mr}) that the negotiated product k and $S^{J^{mr}}[j,k] = s_k^{-J^{mr}}(p)$ otherwise, where $s_k^{-J^{mr}}(p)$ is the share of product k when product j and the other products belonging to the same J^{mr} are not longer available in the market. In this way, the matrix notation of equation (16) will be the following:

$$(T^m * D^j)\boldsymbol{\Gamma} - \tilde{\boldsymbol{\delta}} * \left[(T^m * (S^{J^{mr}} - S^j))\boldsymbol{\Gamma}\right] = \tilde{\boldsymbol{\lambda}} * \left[(T^r * D^j)\boldsymbol{\gamma} - \tilde{\boldsymbol{\theta}} * ((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma})\right] \tag{17}$$

where $*$, T^r, T^m, $\boldsymbol{\Gamma}$, $\tilde{\boldsymbol{\lambda}}$, and D^j represent the same as before.

Given that this renegotiation will take place under the bargaining conditions present at that moment, which includes bargaining power distribution at that point, then $\boldsymbol{\lambda} = \boldsymbol{\lambda^{sq}}$ and, as seen in section 4.3.1, $\boldsymbol{\Gamma}$ can be expressed as $\boldsymbol{\Gamma} = \boldsymbol{p} - \boldsymbol{\gamma} - (IP\boldsymbol{\kappa} + \boldsymbol{\eta})$, then (17) becomes[17]:

$$\boldsymbol{p} - \boldsymbol{\gamma} - \left(T^m * D^j\right)^{\dagger} \left[\tilde{\boldsymbol{\lambda}}^{\boldsymbol{sq}} * \left[\left(T^r * D^j\right)\boldsymbol{\gamma}\right)\right] = IP\boldsymbol{\kappa} + E\tilde{\boldsymbol{\delta}} + H\tilde{\boldsymbol{\theta}} + \sum_{z=1}^{Z} F_z \tilde{\boldsymbol{\delta}} \kappa_z + [G+I]\boldsymbol{\eta} \tag{18}$$

where $(T^m * D^j)^{\dagger}$, as before is the Moore-Penrose inverse of matrix $(T^m * D^j)$, and E is an square matrix of dimension J, which general element is $E[i,j] = (T^m * D^j)^{\dagger}[i,j] \left(\sum_{k=1}^{J} ts_{jk}^m (p_k - \gamma_k^r)\right)$, in which again $ts_{jk}^m = (T^m * (S^{J^{mr}} - S^j))[j,k]$.

While H is an square matrix of the same dimension as E, which general element is $H[i,j] = -(T^m * D^j)^{\dagger}[i,j]d_j$, where d_j is the element in position j of the vector $\left(\tilde{\boldsymbol{\lambda}} * ((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma})\right)$.

Additionally, matrix F_z is an square matrix of dimension J which general element is $F_z[i,j] = -(T^m * D^j)^{\dagger}[i,j] \left(\sum_{k=1}^{J} ts_{jk}^m IP[k,z]\right)$, where ts_{jk}^m is the same as before.

[17] For more details on the following matrices E, F_z, G and H, see Appendix 11.

Finally, I is an identity matrix, and G is a matrix of dimension J, which general element is $G[i,j] = -\sum_{k=1}^{J}(T^m * D^j)^{\dagger}[j,k]\tilde{\delta}_k ts_{kj}^w$.

Then, from (18) the beliefs of both retailers and manufacturers can be computed, afterwards the manufacturers' margins of a possible agreement under uncertainty can be recover from equation (17), as can be seen in equation (22) in Appendix (11). Finally, these are the manufacturers' margins resulted from the possible new agreement, which should fulfill the *(weak) disagreement point concavity* condition, presented in (10), in order negotiation between retailers and manufacturers do not break.

5 Data and Results

5.1 Data

The data used in this paper came from the Consumer panel (Verbraucherpanel) collected by GFK Panelservice SE, which is a household-scan data set from german representative consumer panel, thus the data we have is a households individual purchasing decision. We take into account a time-frame from May 2007 to April 2008 and from July 2008 to June 2009 respectively capturing before and after strike time. We are excluding the months of May and June 2008 from the analysis, in which the strike took place[18]. The dataset consisted of 543,368 purchased-milk observations and 621,665 for the after-strike period. This dataset provided information on date, number mililiters, number of packages, paid amounts, fat level, brand, retailer and manufacturer per purchase. Regarding the retailers, all outlets in the dataset were kept except wholesalers, which represented 0.51% of the milk observations in the before-strike period and 0.45% in the after-strike.

The dataset allowed us to distinguish between conventional brands and private labels. Given the fact that private labels are specific of a retailer, and because of this the consumer will not find all private labels in each store but it is highly likely to find a private label where the purchasing choice is taking place; therefore, we grouped all private labels as a single brand. We are assuming that an alternative(product) is defined as the retailer-brand combination. Moreover we define the outside option including the manufacturers and retailers that in the whole dataset did not have at least 10000 observations, which were considered as small and not relevant regarding their strategic

[18] According to the reports the strike took place from May 27, 2008 to June 5, 2008 approx.; therefore, we are excluding the months of May and June, in order to avoid any noise from the strike in the demand estimations.

behavior. Similarly, the small brands whithin retailer were also included in the outside option[19]. In this way, this study considered 13 brands[20], 16 retailers, which resulted in 62 differentiated alternatives aside from the outside option.

The final dataset consists of 1,159,514 observations, 46.62% from them corresponds to the before-strike period and 53.38% to the after-strike period. The outside option represents 13.87% of the observations. Conventional brands constitutes 8.44% of the observations while private labels 77.69%. Excluding the outside option, the mean price in cents per liter of milk for the before-strike period was 66.63 with an standard devitation of 12.541, while the mean(standard deviation) for the after-strike period was 58.07(14.896). Further descriptive statistics for the before- and after-strike periods can be found in Table 1, while Figures 3a and 3b present the evolution of purchased milk quantities. quantified by the number of purchases and the amount of liters bought respectively.

Table 1: Descriptive statistics

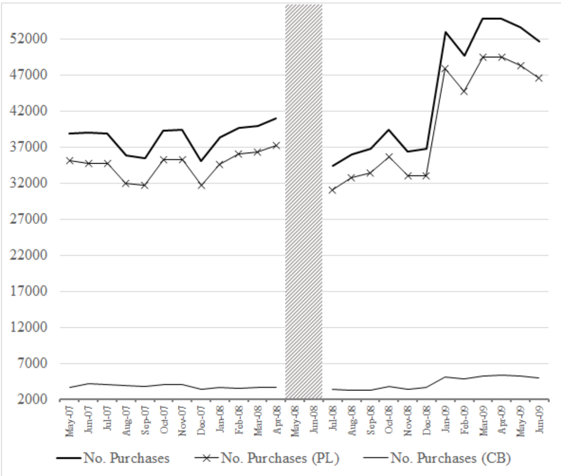
	Before-strike Period			**After-strike Period**		
		Mean			**Mean**	
	Freq.	*Monthly Market Share*[1]	*Price*[2]	Freq.	*Monthly Market Share*[1]	*Price*[2]
Outside Option	14.73%			13.12%		
Private Labels (PL)	76.77%	0.54%(0.00578)	64.77(9.85)	78.49%	0.54%(0.00597)	55.70(11.67)
Conventional Brands (CB)	8.50%	0.02%(0.00026)	83.41(19.72)	8.39%	0.02%(0.00027)	80.20(21.96)

1) Price in cents per liter. 2) Market shares computed from the purchased liters of milk. Standard Deviations are in parenthesis.

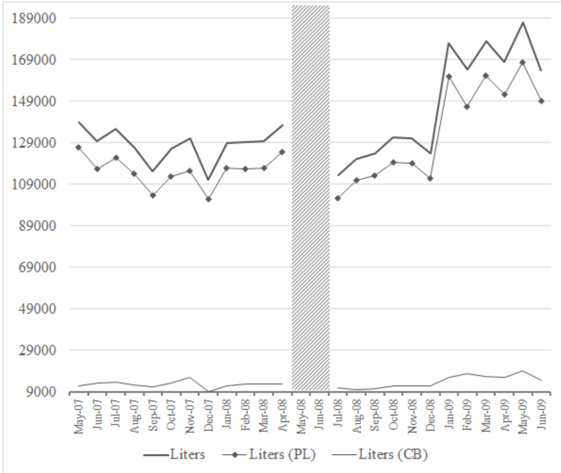
[19]Brands that did not reached at least 624 observations in the whole dataset.

[20]Private labels counted as one brand.

Figure 3: Purchased Milk Quantity



(a)



(b)

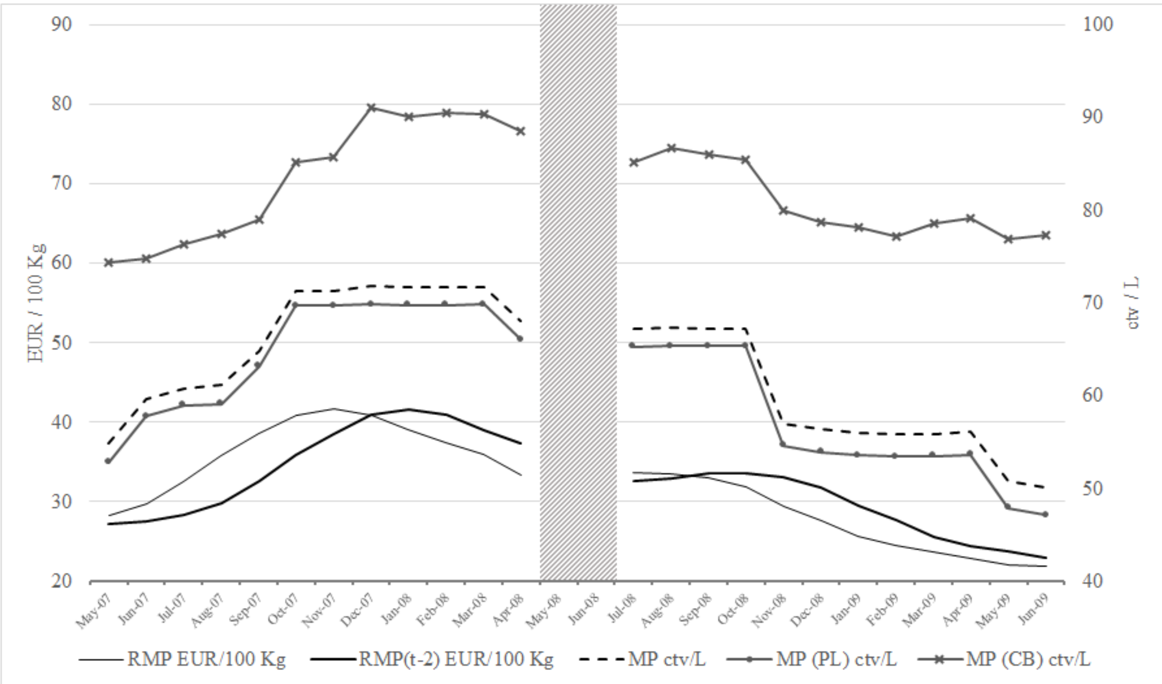
In order to control for the milk fat level on milk prices, a level of fat variable was defined, which considered two levels: a) 0 - 1.5% and, b) 1.6% - 3.9%. Aside from the GFK database, and to control for the input cost, we used as the monthly raw milk prices, the farmers' whole-milk price (3.7% fat and 3.4% protein)[21] coming from the German Federal Ministry of Food and Agriculture(*Bundesministerium für Ernärung, Landwirtschaft und Verbraucherschutz*)[22]. Figure 4 presents the evolution of the monthly raw-milk prices (RMP) and the monthly average milk-prices (MP)[23] coming from the GFK dataset for both periods.

[21]Preise Vollmilch ab Hof bei 3,7% Fettgehalt und 3,4% Eiweissgehalt.

[22]Statistical monthly reports from 03-2008, 03-2009, 03-2010: Table MBT-0301431-0000.

[23]Excluding the prices of the outside option.

Figure 4: Monthly Average Milk Prices



5.2 Demand Estimation

For the estimation of demand, we took into account the potential endogeneity issues coming from the supply side, to control for this, we use a two-step estimation, in which in the first step we implement a control function, as proposed by Petrin and Train (2010), following the specification in (4), in which the farmers milk price was used as a cost shifter using the farmers' monthly whole-milk price (with 3.7% fat and 3.4% protein) lagged two periods and interacted with the fat-level dummies. This estimation reflects a positive strongly significant impact of the two instruments, together with a high R-square for both periods. Moreover, it is reject that the weak instrument hypothesis, given the F-value is well above the value 10, which is known as the threshold suggested by Staiger and Stock(1997), the results can be found in Table 2. Additionally, in Appendix (12) are presented the different control function specifications that have been considered, choosing the presented in table (2) as the one that best suited for both periods.

Table 2: First-Stage: Control Function

Variables	**Before-strike**	**After-strike**
RMP_{t-2} x fat level 1	0.0017***	0.0017***
	(0.00004)	(0.00006)
RMP_{t-2} x fat level 2	0.0019***	0.0020***
	(0.00004)	(0.00006)
Brands	X	X
Retailers	X	X
R^2	0.9931	0.9889
Adj. R^2	0.9928	0.9884
F-Test(Instrs. = 0)	1179.13	520.42
Observations	744	744

Raw-milk Price (RMP): Farmers' whole-milk price (3.7% fat and 3.4% protein).

Dummies: fat level 1 = $\mathbf{1}$(0 - 1.5% fat); fat level 2 = $\mathbf{1}$(1.6 - 3.5% fat).

***, **, * denote 1%, 5% and 10% level of significance respectively.

Standard-Errors are in parenthesis.

In the second step, we follow the demand model introduced in section 4.1, in which the milk demand for each period was estimated using the dataset described in section 5.1, from which a random sample of 200,000 observations was selected, which consisted of 100,000 observations from period May 2007 - April 2008 and 100,000 from July 2008 - June 2009. And based on Revelt and Train (1998) and Train (2003), a random coefficient logit model was estimated by maximizing the simulated log-likelihood for each period. In table 3 are the results from the demand estimation for both period, the pre-treatment (column 1) and the post-treatment period column (2). As can be seen the estimated price coefficients (α) are, as expected, both negative and significant at the 1% level. Also, the standard deviations of the price (σ_α) are significant, indicating in both periods that the impact of the price on the demand is heterogeneous. In addition, it becomes evident that both specifications are rather similar in terms of the coefficient. Still, the impact of the price seems to be lower after the strike took place. This is consistent with the elasticities provided in Table 4.

Table 3: Demand Estimation

	Before	After
Price (α)	$-101.03^{***}(1.36)$	$-137.64^{***}(1.43)$
Price (σ_α)	$50.82^{***}(0.56)$	$63.06^{***}(0.61)$
CF (ρ)	$27.45^{***}(1.95)$	$89.72^{***}(2.01)$
Fat level fixed effects	X	X
Retailers fixed effects	X	X
Brand fixed effects	X	X
Log likelihood	-194758.84	-195424.84

***, **, * denote 1%, 5% and 10% level of significance respectively.

Standard Errors are in parenthesis.

The elasticities derived from the demand estimations have the expected negative sign, as can be observed in Table 4. However, there is an increase (in absolute terms) after the strike, indicating that consumers are more sensitive to price changes.

Table 4: Demand Own-Price Elasticities

Brand	OPE	
	Before-strike	After-strike
B1	−3.11	−3.24
B2	−4.70	−4.31
B3	−5.39	−5.25
B4	−5.94	−7.14
B5	−4.92	−5.69
B6	−5.97	−7.16
B7	−4.49	−4.49
B8	−5.64	−6.58
B9	−5.65	−6.76
B10	−5.78	−6.79
B11	−6.00	−7.18
B12	−4.61	−4.70
B13	−4.01	−4.24
Mean	−4.86	−5.37

The name of the brands cannot be provided due to confidentiality agreement with GFK Panelservice SE.

Those demand side patterns are now used as an input to compute the previously described supply side models. This is, first, we provide the retail margins, before and after the strike in table 5. As can been seen, there is some, but rather slight change in the retail margins. This is not surprising, since there is only minor changes in the elasticities and, thus, also at the main parameters of the model; and since the retail side is solved after the bargaining with the manufacturers takes place, and thus solved before -due to the backward induction-, this is straight forward.

Table 5: Retailer Margins

Brand	Before-strike	After-strike
B1	0.04	0.03
B2	0.02	0.02
B3	0.02	0.02
B4	0.01	0.01
B5	0.02	0.02
B6	0.01	0.01
B7	0.02	0.02
B8	0.01	0.01
B9	0.02	0.01
B10	0.01	0.02
B11	0.01	0.02
B12	0.02	0.02
B13	0.03	0.02
Mean	0.02	0.02

The name of the brands cannot be provided due to confidentiality agreement with GFK Panelservice SE.

After obtaining the retailers margins we compute the manufacturers margins, which are displayed in table 6. We provide information on both: before-strike and after-strike margins. In the before strike scenario, we provide several specifications to uncover describe the steps of the analysis. First, we estimate the *status-quo* scenario, i.e. the manufacturer margins and the corresponding bargaining power parameters without uncertainty. This relies on the assumption that before the strike there has been a pre-established situation with a particular deal. In particular, we interpret the bargaining power parameter according to its meaning as the bargaining power at this particular point in time. We see on average bargaining power parameter that has been estimated using equation (13) of 0.6764 indicating the margins are on average splitted in favor of the retailers.

We use the estimated bargaining power parameter to estimate the players' beliefs on the different scenarios they expect to face during negotiations. This is, firms evaluate their situation in order to settle a further agreement or not, by taking into account their current bargaining power. Thus, we estimate $\boldsymbol{\delta}$ and $\boldsymbol{\theta}$ and the margin under the uncertainty scenario $\boldsymbol{\Gamma^{u}}$.

Clearly, just for the cases in which there was an exchange of more than one product within the bilateral commercial relationship, it would exist an uncertainty regarding the potential disagreement scenario, i.e. just for those cases the market players would have to assess their beliefs on each possible bargaining situation.

We see that the parameter $\boldsymbol{\theta}$ is in general lower than $\boldsymbol{\delta}$. The intuition of those values can be best explained for θ as the belief of the retailer on facing a "no threat" situation in the negotiation with the manufacturer and for δ as the belief of the manufacturer on facing a "not threat" situation in the negotiation with the manufacturer. Thus, the difference in the parameters describes differences in the beliefs of retailer and manufacturer.

Comparing the bargaining power parameter of the Status quo ($\boldsymbol{\lambda_j^{sq}}$) with the bargaining power after the strike ($\boldsymbol{\lambda_j}$), we observe only minor differences on some alternatives, but on average a nearly unchanged bargaining power parameter. This seems to indicate that the original bargaining power, albeit being formally independent across the period, is subject to only minor changes and seems to be rather stable.

Taking into account our uncertainty model, we can simply simulate the missing threat scenario, and use these results with the ones from the *Status Quo* to compute the incentive condition, in order to determine whether it was able to reach a new agreement in the before strike period or bargainers preferred to postpone negotiations. To do so, we follow the models presented in section 4.3.1, for either case under complete certainty (*threat* or *no threat*) and considering the bargaining power distribution until that moment $\boldsymbol{\lambda^{sq}}$. Notice that alternatively, these results can be reached by using the uncertainty model presented in section 4.3.2, using the bargaining power distribution $\boldsymbol{\lambda^{sq}}$ and considering a *no threat* scenario ($\delta = 1$ and $\theta = 1$) or a *threat* scenario ($\delta = 0$ and $\theta = 0$).

Therefore, by comparing Γ^u with $\delta\Gamma_j^{sq} + (1-\delta)\Gamma_j^{\delta=1}$, we can corroborate that the incentive condition (10) is not fulfilled since we have the manufacturer margin under uncertainty is lower than the linear combination of the corner solutions (0.013 < (0.94*0.014) + (0.06*0.012)). Thus, manufacturers would not have the incentive to reach a new agreement under such uncertain conditions and would have preferred to stay with the surplus division that they already agreed with retailers, surplus division that would not satisfied the demands of farmers, being the strike an imminent event and, thus, consistent with our model predictions.

A comparison from the manufacturer margins under uncertainty ($\boldsymbol{\Gamma^u}$) and the manufacturer margin after the strike ($\boldsymbol{\Gamma}$) reveals that the manufacturer margin is on average lower after the strike. This backs the general idea of valuable information. That is, from an ex-post evaluation the uncertainty may have a value. Still, this is the comparison of two estimates.

Table 6: Manufacturers Margins and Bargaining Parameter

	Before-strike Period						After-strike Period	
	*CT-Condition: 0.013 < (0.94*0.014) + (0.06*0.012)*							
Brand	Γ^u	λ_j^{sq}	$\Gamma^{sq}(\Gamma^\delta = 1)$	$\Gamma^T(\Gamma^\delta = 0)$	θ	δ	λ_j	Γ
B1	0.0315	0.7542	0.315	0.0315	0.8628	1	0.6571	0.0158
B2	0.0094	0.5052	0.0094	0.0094	0.5025	1	0.5086	0.0236
B3	0.0037	0.5287	0.0080	0.0010	1	0.9975	0.4843	0.0176
B4	0.0096	0.9137	0.0096	0.0096	1	1	0.8841	0.0012
B5	0.0080	0.4919	0.0080	0.0080	1	1	0.4385	0.0193
B6	0.0103	0.8478	0.0103	0.0103	0.2819	1	0.8960	0.0009
B7	0.0084	0.5267	0.108	0.0051	0.8404	0.4541	0.5089	0.0198
B8	0.0019	0.5461	0.0077	0.0010	1	0.9312	0.5139	0.0109
B9	0.0154	0.9061	0.0154	0.0154	1	1	0.9422	0.0006
B10	0.0077	0.6403	0.0077	0.0077	1	1	0.6484	0.0068
B11	0.0084	0.7524	0.0084	0.0084	1	1	0.9277	0.006
B12	0.0077	0.5350	0.0105	0.0024	1	0.9956	0.6039	0.0138
B13	0.0170	0.6114	0.0170	0.0170	0.8711	0.8860	0.5652	0.0174
Mean	0.0129	0.6764	0.0137	0.0122	0.8974	0.9386	0.6742	0.0111

The name of the brands cannot be provided due to confidentiality agreement with GFK Panelservice SE.

6 Conclusions

Through this work we have shown how current frameworks for structural econometric analysis can be extended to bargaining regimes under uncertainty in recurrent multi-product and multi-bargaining commercial relationships. We show that implementation of these extensions allows to uncover new details of bargaining situations that are in other frameworks only rarely observed or observable at all. Moreover, we apply our framework to a situation of uncertainty and can not only identify all model parameters, but also can show whether an incentive condition to reach a current agreement holds. Given the particularities of the case observed, we can even predict in an ex-ante test that there will be break down of a bargaining situation.

Those developments are important for several reasons. First, the revelation of uncertainty allows for new insights in the analysis of strategic bargaining situations and is expected to enhance the precision of current models. Second, the findings show the value of private information and, thus, may have an important impact on bargaining strategies. Third, the analysis may be applied to further situations where it is ex-ante unclear whether there may be a brake-up of a business relationship. Those cases may allow to predict also listing and delisting of new products. Furthermore, the method may be extended to analyze market players reputation, by assessing how credible are their threats.

7 References

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8 Matrix C in section 4.3.1

In section 4.3.1 was presented expression (12); from which the manufacturer margin vector can be expressed as:

$$\boldsymbol{\Gamma^{sq}} = (T^m * D^j)^{\dagger} \left(\tilde{\boldsymbol{\lambda}} * \left[(T^r * D^j) \boldsymbol{\gamma} \right] \right)$$

where $(T^m * D^j)^{\dagger}$ represents the Moore-Penrose inverse of matrix $(T^m * D^j)$. Finally, being the manufacturer margin rephrased as $\boldsymbol{\Gamma^{sq}} = C\tilde{\boldsymbol{\lambda}}$, where C is an square matrix of dimension J with general element $C[i,k] = (T^m * D^j)^{\dagger}[i,k]b_k$, where b_k is the element at position k of vector $(T^r * D^j)\boldsymbol{\gamma}$.

Proof. Denoting the vector $(T^r * D^j)\boldsymbol{\gamma}$ as $\boldsymbol{b}$, and its general element as $\boldsymbol{b}[i,1] = b_i$; and the general element of matrix $(T^m * D^j)^{\dagger}[i,k] = a_{ik}$, we have:

$$\begin{aligned}
\boldsymbol{\Gamma^{sq}} =& \quad (T^m * D^j)^{\dagger} \left(\tilde{\boldsymbol{\lambda}} * \left[(T^r * D^j) \boldsymbol{\gamma} \right] \right) \\
\boldsymbol{\Gamma^{sq}} =& \quad (T^m * D^j)^{\dagger} (\tilde{\boldsymbol{\lambda}} * \boldsymbol{b}) \\
\boldsymbol{\Gamma^{sq}} =& \quad \begin{pmatrix} a_{11} & \cdots & a_{1J} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{JJ} \end{pmatrix} \begin{pmatrix} \tilde{\lambda}_1 b_1 \\ \vdots \\ \tilde{\lambda}_J b_J \end{pmatrix} \\
\boldsymbol{\Gamma^{sq}} =& \quad \begin{pmatrix} a_{11} b_1 \tilde{\lambda}_1 + \cdots + a_{1J} b_J \tilde{\lambda}_J \\ \vdots \\ a_{J1} b_1 \tilde{\lambda}_1 + \cdots + a_{JJ} b_J \tilde{\lambda}_J \end{pmatrix} \\
\boldsymbol{\Gamma^{sq}} =& \quad \begin{pmatrix} a_{11} b_1 & \cdots & a_{1J} b_J \\ \vdots & \ddots & \vdots \\ a_{J1} b_1 & \cdots & a_{JJ} b_J \end{pmatrix} \begin{pmatrix} \tilde{\lambda}_1 \\ \vdots \\ \tilde{\lambda}_J \end{pmatrix} \\
\boldsymbol{\Gamma^{sq}} =& \quad C \tilde{\boldsymbol{\lambda}}
\end{aligned}$$

Then, C is an square matrix of dimension J, which element in position $[i,k]$ is $C[i,k] = (T^m * D^j)^{\dagger}[i,k]b_k$, where b_k corresponds to $b_k = \left((T^r * D^j)\boldsymbol{\gamma} \right)[k,1]$.

□

9 Manufacturers' margins in the contingent scenario of strike/Threat

The bargaining process of a strike/threat scenario, can be resolved in a similar way as the one presented in section 4.3.1 [Draganska et al. (2010)]. Given, this is a bargaining in which manufacturers and retailers negotiate each product individually, but knowing with certainty about a retaliation from their counterparty if disagreement in the negotiation over the wholesale price of product j occurs, i.e. a disagreement over product j translates in a break of the whole commercial relationship. Therefore, Nash product of this negotiation is:

$$\underset{w_j}{\text{Max}} \left(\pi_j^r - d_{J^{mr}}^r\right)^{\lambda_j} \left(\pi_j^m - d_{J^{mr}}^m\right)^{(1-\lambda_j)} \tag{19}$$

where π_j^r and π_j^m represent respectively the retailer r and manufacturer m profits from selling product j (agreement payoff); being J^{mr} the set of products usually traded between the retailer r and manufacturer m (where $j \in J^{mr}$), then $d_{J^{mr}}^r$ and $d_{J^{mr}}^m$ represent respectively the retailer and manufacturer profits when none of the products belonging to J^{mr} are available in the market (disagreement payoffs). The expressions for the agreements and disagreements payoffs are the following:

	Agreement	Disagreement
Manufacturer	$\pi_j^m = \Gamma_j s_j(p)M + \sum_{\substack{k\in J^m \\ k\neq j}} \Gamma_k s_k(p)M$	$d_{J^{mr}}^m = \sum_{\substack{k\in J^m \\ k\notin J^{mr}}} \Gamma_k s_k^{-J^{mr}}(p)M$
Retailer	$\pi_j^r = \gamma_j s_j(p)M + \sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k s_k(p)M$	$d_{J^{mr}}^r = \sum_{\substack{k\in J^r \\ k\notin J^{mr}}} \gamma_k s_k^{-J^{mr}}(p)M$

as before Γ_j denotes the manufacturer margin for product j, and $s_k^{-J^{mr}}(p)$ represents the market share of product k when there was no agreement on product j and therefore none of the products belonging to J^{mr} are available in the market.

By following the same process presented in section 4.3.1, we get from the maximization of expression (19) the following:

$$\left(\sum_{\substack{j\in J^m \\ j\in J^{mr}}} \Gamma_j s_j(p) - \sum_{\substack{k\in J^m \\ k\notin J^{mr}}} \Gamma_k \Delta s_k^{-J^{mr}} \right) = \frac{(1-\lambda_j)}{\lambda_j} \left(\sum_{\substack{j\in J^m \\ j\in J^{mr}}} \gamma_j s_j(p) - \sum_{\substack{k\in J^r \\ k\notin J^{mr}}} \gamma_k \Delta s_k^{-J^{mr}} \right)$$

where $\Delta s_k^{-J^{mr}}$ represents the change in the market share of product k when the products belonging to J^{mr} are not longer available in the market, i.e. $\Delta s_k^{-J^{mr}} = s_k(p)^{-J^{mr}} - s_k(p)$.

Being as before, $\mathbf{\Gamma}$ the vector of manufacturer margins, T^m the manufacturers ownership matrix which element $T^m[j,k] = 1$ if both products k and j are produced by the same manufacturer and $T^m[j,k] = 0$ otherwise. And defining $D^{J^{mr}}$ as the matrix of shares and share variations, which $D^{J^{mr}}[j,k] = s_k$ if k and j belong to J^{mr}, and $D^{J^{mr}}[j,k] = -\Delta s_k^{-J^{mr}}$ otherwise. Then the matrix notation of the former expression is:

$$(T^m * D^{J^{mr}})\mathbf{\Gamma} = \tilde{\boldsymbol{\lambda}} * \left[(T^r * D^{J^{mr}})\boldsymbol{\gamma}\right]$$

where $*$, T^r and $\tilde{\boldsymbol{\lambda}}$ represent the same as before, $\boldsymbol{\gamma}$ is the retailer margin vector obtained through (8). Notice that this contingent solution is evaluated by agents at the renegotiation; renegotiation that develops under a preestablished bargaining condition, which includes a bargaining power distribution (the bargaining power distribution they have until that moment), ie. $\boldsymbol{\lambda} = \boldsymbol{\lambda^{sq}}$; being then:

$$(T^m * D^{J^{mr}})\mathbf{\Gamma} = \tilde{\boldsymbol{\lambda}}^{\boldsymbol{sq}} * \left[(T^r * D^{J^{mr}})\boldsymbol{\gamma}\right]$$

Therefore, and if $(T^m * D^{J^{mr}})^{\dagger}$ represents the Moore-Penrose inverse of matrix $(T^m * D^{J^{mr}})$, this contingent manufacturer margin can be computed by:

$$\mathbf{\Gamma^T} = (T^m * D^{J^{mr}})^{\dagger} \left(\tilde{\boldsymbol{\lambda}}^{\boldsymbol{sq}} * \left((T^r * D^{J^{mr}})\boldsymbol{\gamma}\right)\right) \tag{20}$$

10 Details on equation (16) in section 4.3.2

Taking into account that $E(d_j^m) = \delta_j d_j^m + (1-\delta_j) d_{J^{mr}}^m$ and $E(d_j^r) = \theta_j d_j^r + (1-\theta_j) d_{J^{mr}}^r$. Then, we have that equation (15) becomes in expression (16).

Proof. Recalling expression (15) introduced in section 4.3.2:

$$\pi_j^w - E(d_j^m) = \tilde{\lambda}_j(\pi_j^r - E(d_j^r))$$

Focusing first on $\pi_j^r - E(d_j^r)$, we have:

$$\begin{aligned}
\pi_j^r - E(d_j^r) &= \pi_j^r - \theta_j d_j^r - (1-\theta_j) d_{J^{mr}}^r \\
&= \gamma_j M s_j + \textstyle\sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k M s_k - \theta_j (\sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k M s_k^{-j}) - (1-\theta_j)(\sum_{\substack{k\in J^r \\ k\notin J^{mr}}} \gamma_k M s_k^{-J^{mr}}) \\
&= \gamma_j M s_j + \textstyle\sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k M (s_k - \theta_j s_k^{-j}) - (1-\theta_j)(\sum_{\substack{k\in J^r \\ k\notin J^{mr}}} \gamma_k M s_k^{-J^{mr}})
\end{aligned}$$

As mentioned before $\Delta s_k^{-j} = s_k^{-j} - s_k$, then $\theta_j s_k^{-j} - s_k = \Delta s_k^{-j} - s_k^{-j} + \theta_j s_k^{-j}$, and defining $\tilde{\theta_j} = (1-\theta_j)$. We have that $\pi_j^r - E(d_j^r)$ becomes in:

$$\pi_j^r - E(d_j^r) \quad = \gamma_j M s_j - \sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k M \Delta s_k^{-j} - \tilde{\theta_j}\left[\sum_{\substack{k\in J^r \\ k\notin J^{mr}}} \gamma_k M s_k^{-J^{mr}} - \sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k M s_k^{-j}\right]$$

Notice that $\pi_j^r - E(d_j^r)$ and $\pi_j^m - E(d_j^m)$ are similar expressions; therefore, the process to derive the solution of one apply to the other. In this way, and defining $\tilde{\delta_j} = (1-\delta_j)$, we get from expression $\pi_j^m - E(d_j^m)$:

$$\pi_j^m - E(d_j^m) \quad = \Gamma_j M s_j - \sum_{\substack{k\in J^m \\ k\neq j}} \Gamma_k M \Delta s_k^{-j} - \tilde{\delta_j}\left[\sum_{\substack{k\in J^m \\ k\notin J^{mr}}} \Gamma_k M s_k^{-J^{mr}} - \sum_{\substack{k\in J^m \\ k\neq j}} \Gamma_k M s_k^{-j}\right]$$

Hence, by replacing $\pi_j^m - E(d_j^m)$ and $\pi_j^r - E(d_j^r)$ into expression (15), we obtain expression (16).

□

11 Matrices on Section 4

Matrices E, F, G and H

Equation (17), introduced in section 4.3.2, can be rearranged as follows:

$$\begin{aligned}\boldsymbol{\Gamma} - (T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\lambda}} * (T^r * D^j)\boldsymbol{\gamma}\right] = {} & (T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\delta}} * \left[(T^m * (S^{J^{mr}} - S^j))\boldsymbol{\Gamma}\right]\right] \\ & - (T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\lambda}} * \tilde{\boldsymbol{\theta}} * ((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma})\right]\end{aligned}$$

Taking into account that $\boldsymbol{\Gamma} = \boldsymbol{p} - \boldsymbol{\gamma} - (IP\boldsymbol{\kappa} + \boldsymbol{\eta})$, and replacing it in the former expression, then we get:

$$\begin{aligned}\boldsymbol{p} - \boldsymbol{\gamma} - (T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\lambda}} * \left[(T^r * D^j)\boldsymbol{\gamma}\right]\right] = {} & IP\boldsymbol{\kappa} + (T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\delta}} * ((T^m * (S^{J^{mr}} - S^j))(\boldsymbol{p} - \boldsymbol{\gamma}))\right] \\ & - (T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\delta}} * ((T^m * (S^{J^{mr}} - S^j))IP\boldsymbol{\kappa})\right] \\ & - (T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\theta}} * \tilde{\boldsymbol{\lambda}} * ((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma})\right] \\ & - (T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\delta}} * ((T^m * (S^{J^{mr}} - S^j))\boldsymbol{\eta})\right] + \boldsymbol{\eta}\end{aligned} \tag{21}$$

Then, by denoting again the general element of matrix $(T^m * D^j)^\dagger[i,j] = a_{ij}$ and the general element of matrix $(T^m * (S^{J^{mr}} - S^j))[i.j] = ts_{ij}^m$, then the second term of the right-hand side of expression (21) can be expressed as $E\tilde{\boldsymbol{\delta}}$, where E is an square matrix of dimension J, which general element is $E[i,j] = (T^m * D^j)^\dagger[i,j]\left(\sum_{k=1}^{J} ts_{jk}^m (p_k - \gamma_k^r)\right)$, in which again $ts_{jk}^m = (T^m * (S^{J^{mr}} - S^j))[j,k]$.

Proof. The second term of the right-hand side of equation (21) is:

$$(T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\delta}} * ((T^m * (S^{J^{mr}} - S^j))(\boldsymbol{p} - \boldsymbol{\gamma}))\right]$$

then by denoting the element in position $[i, j]$ of matrix $(T^m * D^j)^\dagger$ as $a_{i,j}$, and the element in position $[i, j]$ of matrix $(T^m * (S^{J^{mr}} - S^j))$ as ts^m_{ij}, then we have:

$$\begin{aligned}
&(T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\delta}} * ((T^m * (S^{J^{mr}} - S^j))(\boldsymbol{p} - \boldsymbol{\gamma}))\right] = \\
&= \begin{pmatrix} a_{11} & \cdots & a_{1J} \\ \vdots & \ddots & \vdots \\ a_{J1} & \cdots & a_{JJ} \end{pmatrix} \left[\begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} * \begin{pmatrix} ts^m_{11} & \cdots & ts^m_{1J} \\ \vdots & \ddots & \vdots \\ ts^m_{1J} & \cdots & ts^m_{JJ} \end{pmatrix} \begin{pmatrix} p_1 - \gamma_1 \\ \vdots \\ p_J - \gamma_J \end{pmatrix} \right] \\
&= \begin{pmatrix} a_{11}\tilde{\delta}_1(ts^m_{11}(p_1 - \gamma_1) + \cdots + ts^m_{1J}(p_J - \gamma_J)) + \cdots + a_{1J}\tilde{\delta}_J(ts^m_{J1}(p_1 - \gamma_1) + \cdots + ts^m_{JJ}(p_J - \gamma_J)) \\ \vdots \\ a_{J1}\tilde{\delta}_1(ts^m_{11}(p_1 - \gamma_1) + \cdots + ts^m_{1J}(p_J - \gamma_J)) + \cdots + a_{JJ}\tilde{\delta}_J(ts^m_{J1}(p_1 - \gamma_1) + \cdots + ts^m_{JJ}(p_J - \gamma_J)) \end{pmatrix} \\
&= \begin{pmatrix} a_{11}(ts^m_{11}(p_1 - \gamma_1) + \cdots + ts^m_{1J}(p_J - \gamma_J)) & \cdots & a_{1J}(ts^m_{J1}(p_1 - \gamma_1) + \cdots + ts^m_{JJ}(p_J - \gamma_J)) \\ \vdots & \ddots & \vdots \\ a_{J1}(ts^m_{11}(p_1 - \gamma_1) + \cdots + ts^m_{1J}(p_J - \gamma_J)) & \cdots & a_{JJ}(ts^m_{J1}(p_1 - \gamma_1) + \cdots + ts^m_{JJ}(p_J - \gamma_J)) \end{pmatrix} \begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} \\
&= \begin{pmatrix} a_{11}\sum_{k=1}^J ts^m_{1k}(p_k - \gamma_k) & \cdots & a_{1J}\sum_{k=1}^J ts^m_{Jk}(p_k - \gamma_k) \\ \vdots & \ddots & \vdots \\ a_{J1}\sum_{k=1}^J ts^m_{1k}(p_k - \gamma_k) & \cdots & a_{JJ}\sum_{k=1}^J ts^m_{Jk}(p_k - \gamma_k) \end{pmatrix} \begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} \\
&= E\tilde{\boldsymbol{\delta}}
\end{aligned}$$

□

Similarly, and by denoting the general element of matrix IP as Ip_{ij}, the third term of the right-hand side of expression (21) can be expressed as $\sum_{z=1}^{Z} F_z \tilde{\boldsymbol{\delta}} \kappa_z$, where F_z is an square matrix of dimension J, which general element of matrix $F_z[i, j] = -(T^m * D^j)^\dagger[i, j] \left(\sum_{k=1}^{J} ts^m_{jk} IP[k, z]\right)$.

Proof. Having that the third term of the right-hand side of equation (21) is:

$$-(T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\delta}} * ((T^m * (S^{J^{mr}} - S^j))IP\boldsymbol{\kappa})\right]$$

and denoting the general element of matrix IP as $IP[i, j] = Ip_{ij}$, as well as recalling that the general element of matrix $((T^m * (S^{J^{mr}} - S^j))[i, j] = ts^m_{ij}$; then this term can be rearranged as follows:

$$
\begin{aligned}
&-(T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\delta}} * ((T^m * (S^{J^{mr}} - S^j))IP\boldsymbol{\kappa})\right] = \\
&= -\begin{pmatrix} a_{11} & \cdots & a_{1J} \\ \vdots & \ddots & \vdots \\ a_{J1} & \cdots & a_{JJ} \end{pmatrix} \left[\begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} * \begin{pmatrix} ts^m_{11} & \cdots & ts^m_{1J} \\ \vdots & \ddots & \vdots \\ ts^m_{J1} & \cdots & ts^m_{JJ} \end{pmatrix} \begin{pmatrix} Ip_{11} & \cdots & Ip_{1Z} \\ \vdots & \ddots & \vdots \\ Ip_{J1} & \cdots & Ip_{JZ} \end{pmatrix} \begin{pmatrix} \kappa_1 \\ \vdots \\ \kappa_Z \end{pmatrix} \right] \\
&= \begin{pmatrix} -a_{11} & \cdots & -a_{1J} \\ \vdots & \ddots & \vdots \\ -a_{J1} & \cdots & -a_{JJ} \end{pmatrix} \left[\begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} * \begin{pmatrix} \kappa_1 \sum_{k=1}^J ts^m_{1k} Ip_{k1} + \cdots + \kappa_Z \sum_{k=1}^J ts^m_{1k} Ip_{kZ} \\ \vdots \\ \kappa_1 \sum_{k=1}^J ts^m_{Jk} Ip_{k1} + \cdots + \kappa_Z \sum_{k=1}^J ts^m_{Jk} Ip_{kZ} \end{pmatrix} \right] \\
&= \begin{pmatrix} -a_{11} & \cdots & -a_{1J} \\ \vdots & \ddots & \vdots \\ -a_{J1} & \cdots & -a_{JJ} \end{pmatrix} \left[\begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} * \left(\begin{pmatrix} \kappa_1 \sum_{k=1}^J ts^m_{1k} Ip_{k1} \\ \vdots \\ \kappa_1 \sum_{k=1}^J ts^m_{Jk} Ip_{k1} \end{pmatrix} + \cdots + \begin{pmatrix} \kappa_Z \sum_{k=1}^J ts^m_{1k} Ip_{kZ} \\ \vdots \\ \kappa_Z \sum_{k=1}^J ts^m_{Jk} Ip_{kZ} \end{pmatrix} \right) \right] \\
&= \begin{pmatrix} -a_{11} & \cdots & -a_{1J} \\ \vdots & \ddots & \vdots \\ -a_{J1} & \cdots & -a_{JJ} \end{pmatrix} \left[\begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} * \begin{pmatrix} \kappa_1 \sum_{k=1}^J ts^m_{1k} Ip_{k1} \\ \vdots \\ \kappa_1 \sum_{k=1}^J ts^m_{Jk} Ip_{k1} \end{pmatrix} \right] + \cdots \\
&\quad + \begin{pmatrix} -a_{11} & \cdots & -a_{1J} \\ \vdots & \ddots & \vdots \\ -a_{J1} & \cdots & -a_{JJ} \end{pmatrix} \left[\begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} * \begin{pmatrix} \kappa_Z \sum_{k=1}^J ts^m_{1k} Ip_{kZ} \\ \vdots \\ \kappa_Z \sum_{k=1}^J ts^m_{Jk} Ip_{kZ} \end{pmatrix} \right] \\
&= \begin{pmatrix} -a_{11}\tilde{\delta}_1 \kappa_1 \sum_{k=1}^J ts^m_{1k} Ip_{k1} - \cdots - a_{1J}\tilde{\delta}_J \kappa_1 \sum_{k=1}^J ts^m_{Jk} Ip_{k1} \\ \vdots \\ -a_{J1}\tilde{\delta}_1 \kappa_1 \sum_{k=1}^J ts^m_{1k} Ip_{k1} - \cdots - a_{JJ}\tilde{\delta}_J \kappa_1 \sum_{k=1}^J ts^m_{Jk} Ip_{k1} \end{pmatrix} + \cdots \\
&\quad + \begin{pmatrix} -a_{11}\tilde{\delta}_1 \kappa_Z \sum_{k=1}^J ts^m_{1k} Ip_{kZ} - \cdots - a_{1J}\tilde{\delta}_J \kappa_Z \sum_{k=1}^J ts^m_{Jk} Ip_{kZ} \\ \vdots \\ -a_{J1}\tilde{\delta}_1 \kappa_Z \sum_{k=1}^J ts^m_{1k} Ip_{kZ} - \cdots - a_{JJ}\tilde{\delta}_J \kappa_Z \sum_{k=1}^J ts^m_{Jk} Ip_{kZ} \end{pmatrix} \\
&= \begin{pmatrix} -a_{11} \sum_{k=1}^J ts^m_{1k} Ip_{k1} & \ldots & -a_{1J} \sum_{k=1}^J ts^m_{Jk} Ip_{k1} \\ \vdots & \ddots & \vdots \\ -a_{J1} \sum_{k=1}^J ts^m_{1k} Ip_{k1} & \ldots & -a_{JJ} \sum_{k=1}^J ts^m_{Jk} Ip_{k1} \end{pmatrix} \begin{pmatrix} \tilde{\delta}_1 \kappa_1 \\ \vdots \\ \tilde{\delta}_J \kappa_1 \end{pmatrix} + \cdots \\
&\quad + \begin{pmatrix} -a_{11} \sum_{k=1}^J ts^m_{1k} Ip_{kZ} & \ldots & -a_{1J} \sum_{k=1}^J ts^m_{Jk} Ip_{kZ} \\ \vdots & \ddots & \vdots \\ -a_{J1} \sum_{k=1}^J ts^m_{1k} Ip_{kZ} & \ldots & -a_{JJ} \sum_{k=1}^J ts^m_{Jk} Ip_{kZ} \end{pmatrix} \begin{pmatrix} \tilde{\delta}_1 \kappa_Z \\ \vdots \\ \tilde{\delta}_J \kappa_Z \end{pmatrix} \\
&= \sum_{z=1}^Z F_z \tilde{\boldsymbol{\delta}} \kappa_z
\end{aligned}
$$

□

Additionally, and by denoting the general element of vector $\left(\tilde{\boldsymbol{\lambda}} * \left((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma}\right)\right)[i,1] = d_i$, the fourth term of the right-hand side of equation (21) can be expressed as $H\tilde{\boldsymbol{\theta}}$, in which matrix H is an square matrix of dimension J, which general element is $H[i,j] = -(T^m * D^j)^\dagger[i,j]d_j$, where d_j is the element in position j of the vector $\left(\tilde{\boldsymbol{\lambda}} * ((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma})\right)$.

Proof. The fourth term of equation (21) is:

$$-(T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\theta}} * \tilde{\boldsymbol{\lambda}} * ((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma})\right]$$

then by denoting the element in position j of vector $\left(\tilde{\boldsymbol{\lambda}} * \left((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma}\right)\right)$ as d_j, and recalling that the element in position (i,j) of matrix $(T^m * D^j)$ was denoted as a_{ij}, then this term can be reformulated as:

$$\begin{aligned}
-(T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\theta}} * \tilde{\boldsymbol{\lambda}} * ((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma})\right] &= -\begin{pmatrix} a_{11} & \cdots & a_{1J} \\ \vdots & \ddots & \vdots \\ a_{J1} & \cdots & a_{JJ} \end{pmatrix} \left[\begin{pmatrix} \tilde{\theta}_1 \\ \vdots \\ \tilde{\theta}_J \end{pmatrix} * \begin{pmatrix} d_1 \\ \vdots \\ d_J \end{pmatrix}\right] \\
&= \begin{pmatrix} -a_{11}\tilde{\theta}_1 d_1 - \cdots - a_{1J}\tilde{\theta}_J d_J \\ \vdots \\ -a_{J1}\tilde{\theta}_1 d_1 - \cdots - a_{JJ}\tilde{\theta}_J d_J \end{pmatrix} \\
&= \begin{pmatrix} -a_{11}d_1 & \cdots & -a_{1J}d_J \\ \vdots & \ddots & \vdots \\ -a_{J1}d_1 & \cdots & -a_{JJ}d_J \end{pmatrix} \begin{pmatrix} \tilde{\theta}_1 \\ \vdots \\ \tilde{\theta}_J \end{pmatrix} \\
&= H\tilde{\boldsymbol{\theta}}
\end{aligned}$$

□

Finally, the last two terms of the right-hand side of the equation (21) can be expressed as $(G + I)\boldsymbol{\eta}$, where I is an identity matrix and G is an square matrix of dimension J which general element is $G[i,j] = -\sum_{k=1}^{J}(T^m * D^j)^\dagger[j,k]\tilde{\delta}_k ts_{kj}^m$.

Proof. Recalling that the last two terms of the right-hand side of expression (21) are:

$$-(T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\delta}} * ((T^m * (S^{J^{mr}} - S^j))\boldsymbol{\eta})\right] + \boldsymbol{\eta}$$

and considering that the general element of matrix $(T^m * D^j)^\dagger[i,j]$ was denoted as a_{ij}, and the general element of matrix $(T^m * (S^{J^{mr}} - S^j))[i,j] = ts_{ij}^m$, then we have that these terms can be rearranged as:

$$
\begin{aligned}
&-(T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\delta}} * ((T^m * (S^{J^{mr}} - S^j))\boldsymbol{\eta})\right] + \boldsymbol{\eta} = \\
&= -\begin{pmatrix} a_{11} & \cdots & a_{1J} \\ \vdots & \ddots & \vdots \\ a_{J1} & \cdots & a_{JJ} \end{pmatrix} \left[\begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} * \begin{pmatrix} ts_{11}^m & \cdots & ts_{1J}^m \\ \vdots & \ddots & \vdots \\ ts_{1J}^m & \cdots & ts_{JJ}^m \end{pmatrix} \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_J \end{pmatrix}\right] + \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_J \end{pmatrix} \\
&= \begin{pmatrix} -a_{11}\tilde{\delta}_1(ts_{11}^m\eta_1 + \cdots + ts_{1J}^m\eta_J) - \cdots - a_{1J}\tilde{\delta}_J(ts_{J1}^m\eta_1 + \cdots + ts_{JJ}^m\eta_J) \\ \vdots \\ -a_{J1}\tilde{\delta}_1(ts_{11}^m\eta_1 + \cdots + ts_{1J}^m\eta_J) - \cdots - a_{JJ}\tilde{\delta}_J(ts_{J1}^m\eta_1 + \cdots + ts_{JJ}^m\eta_J) \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_J \end{pmatrix} \\
&= \begin{pmatrix} -\eta_1(a_{11}\tilde{\delta}_1 ts_{11}^m + \cdots + a_{1J}\tilde{\delta}_J ts_{J1}^m) - \cdots - \eta_J(a_{11}\tilde{\delta}_1 ts_{1J}^m + \cdots + a_{1J}\tilde{\delta}_J ts_{JJ}^m) \\ \vdots \\ -\eta_1(a_{J1}\tilde{\delta}_1 ts_{11}^m + \cdots + a_{JJ}\tilde{\delta}_J ts_{J1}^m) - \cdots - \eta_J(a_{J1}\tilde{\delta}_1 ts_{1J}^m + \cdots + a_{JJ}\tilde{\delta}_J ts_{JJ}^m) \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_J \end{pmatrix} \\
&= \begin{pmatrix} -(a_{11}\tilde{\delta}_1 ts_{11}^m + \cdots + a_{1J}\tilde{\delta}_J ts_{J1}^m) & \ldots & -(a_{11}\tilde{\delta}_1 ts_{1J}^m + \cdots + a_{1J}\tilde{\delta}_J ts_{JJ}^m) \\ \vdots & \ddots & \vdots \\ -(a_{J1}\tilde{\delta}_1 ts_{11}^m + \cdots + a_{JJ}\tilde{\delta}_J ts_{J1}^m) & \ldots & -(a_{J1}\tilde{\delta}_1 ts_{1J}^m + \cdots + a_{JJ}\tilde{\delta}_J ts_{JJ}^m) \end{pmatrix} \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_J \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_J \end{pmatrix} \\
&= \begin{pmatrix} -\sum_{k=1}^J a_{1k}\tilde{\delta}_k ts_{k1}^m & \ldots & -\sum_{k=1}^J a_{1k}\tilde{\delta}_k ts_{kJ}^m \\ \vdots & \ddots & \vdots \\ -\sum_{k=1}^J a_{Jk}\tilde{\delta}_k ts_{k1}^m & \ldots & -\sum_{k=1}^J a_{Jk}\tilde{\delta}_k ts_{kJ}^m \end{pmatrix} \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_J \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_J \end{pmatrix} \\
&= G\boldsymbol{\eta} + \boldsymbol{\eta} \\
&= (G + I)\boldsymbol{\eta}
\end{aligned}
$$

□

In this way, expression (17) becomes in the following:

$$
\boldsymbol{p} - \boldsymbol{\gamma} - \left(T^m * D^j\right)^\dagger \left[\tilde{\boldsymbol{\lambda}} * \left[\left(T^r * D^j\right)\boldsymbol{\gamma}\right)\right] = IP\boldsymbol{\kappa} + E\tilde{\boldsymbol{\delta}} + H\tilde{\boldsymbol{\theta}} + \sum_{z=1}^{Z} F_z\tilde{\boldsymbol{\delta}}\kappa_z + [G + I]\boldsymbol{\eta}
$$

Manufacturers margins under uncertainty of section 4.3.2

From equation (17) introduced in section 4.3.2, we get that the manufacturer margin under uncertainty can be expressed as $\boldsymbol{\Gamma^u} = A^\dagger(\tilde{\boldsymbol{\lambda}} * B\boldsymbol{\gamma})$; where $A^\dagger$ is the Moore-Penrose inverse matrix of matrix A, an square matrix of dimension J which general element $A[i,j] = (T^m * D^j)[i,j] - \tilde{\delta}_i(T^m * (S^{J^{mr}} - S^j))[i,j]$; while B is an square matrix of dimension J, which general element $B[i,j] = (T^r * D^j)[i,j] - \tilde{\theta}_i(T^r * (S^{J^{mr}} - S^j))[i,j]$.

Proof. First, focusing just in the left-hand side of equation (17), which is:

$$
(T^m * D^j)\boldsymbol{\Gamma} - \tilde{\boldsymbol{\delta}} * \left[(T^m * (S^{J^{mr}} - S^j))\boldsymbol{\Gamma}\right]
$$

and denoting the general element of matrix $(T^m - D^j)[i,j]$ as td_{ij}^m; while recalling the general element of matrix $(T^m * (S^{J^{mr}} - S^j))$ was ts_{ij}^m, then we have that the left-hand side can be reformulated as:

$$\begin{aligned}
&(T^m * D^j)\boldsymbol{\Gamma} - \tilde{\boldsymbol{\delta}} * \left[(T^m * (S^{J^{mr}} - S^j))\boldsymbol{\Gamma}\right] = \\
&= \begin{pmatrix} td_{11}^m & \cdots & td_{1J}^m \\ \vdots & \ddots & \vdots \\ td_{J1}^m & \cdots & td_{JJ}^m \end{pmatrix} \begin{pmatrix} \Gamma_1 \\ \vdots \\ \Gamma_J \end{pmatrix} - \begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} * \begin{pmatrix} ts_{11}^m & \cdots & ts_{1J}^m \\ \vdots & \ddots & \vdots \\ ts_{J1}^m & \cdots & ts_{JJ}^m \end{pmatrix} \begin{pmatrix} \Gamma_1 \\ \vdots \\ \Gamma_J \end{pmatrix} \\
&= \begin{pmatrix} td_{11}^m & \cdots & td_{1J}^m \\ \vdots & \ddots & \vdots \\ td_{J1}^m & \cdots & td_{JJ}^m \end{pmatrix} \begin{pmatrix} \Gamma_1 \\ \vdots \\ \Gamma_J \end{pmatrix} - \begin{pmatrix} \tilde{\delta}_1(ts_{11}^m\Gamma_1 + \cdots + ts_{1J}^m\Gamma_J) \\ \vdots \\ \tilde{\delta}_n(ts_{J1}^m\Gamma_1 + \cdots + ts_{JJ}^m\Gamma_J) \end{pmatrix} \\
&= \begin{pmatrix} td_{11}^m & \cdots & td_{1J}^m \\ \vdots & \ddots & \vdots \\ td_{J1}^m & \cdots & td_{JJ}^m \end{pmatrix} \begin{pmatrix} \Gamma_1 \\ \vdots \\ \Gamma_J \end{pmatrix} - \begin{pmatrix} \tilde{\delta}_1 ts_{11}^m & \cdots & \tilde{\delta}_1 ts_{1J}^m \\ \vdots & \ddots & \vdots \\ \tilde{\delta}_J ts_{J1}^m & \cdots & \tilde{\delta}_J ts_{JJ}^m \end{pmatrix} \begin{pmatrix} \Gamma_1 \\ \vdots \\ \Gamma_J \end{pmatrix} \\
&= \begin{pmatrix} td_{11}^m - \tilde{\delta}_1 ts_{11}^m & \cdots & td_{1J}^m - \tilde{\delta}_1 ts_{1J}^m \\ \vdots & \ddots & \vdots \\ td_{J1}^m - \tilde{\delta}_J ts_{J1}^m & \cdots & td_{JJ}^m - \tilde{\delta}_J ts_{JJ}^m \end{pmatrix} \begin{pmatrix} \Gamma_1 \\ \vdots \\ \Gamma_J \end{pmatrix} \\
&= A\boldsymbol{\Gamma}
\end{aligned}$$

Being then A an square matrix of dimension J which general element $A[i,j] = (T^m * D^j)[i,j] - \tilde{\delta}_i(T^m * (S^{J^{mr}} - S^j))[i,j]$.

Notice that the right-hand side of expression (17) is similar to the left-hand side; therefore, by denoting the general elements of matrices $(T^r * D^j)[i,j]$ and $(T^r * (S^{J^{mr}} - S^j))[i,j]$ as td_{ij}^r and ts_{ij}^r respectively, and following the above process we get that the right-hand side becomes in:

$$\begin{aligned}
\tilde{\boldsymbol{\lambda}} * \left[(T^r * D^j)\boldsymbol{\gamma} - \tilde{\boldsymbol{\theta}} * ((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma})\right] &= \begin{pmatrix} \tilde{\lambda}_1 \\ \vdots \\ \tilde{\lambda}_J \end{pmatrix} * \left[\begin{pmatrix} td_{11}^r - \tilde{\theta}_1 ts_{11}^r & \cdots & td_{1J}^r - \tilde{\theta}_1 ts_{1J}^r \\ \vdots & \ddots & \vdots \\ td_{J1}^r - \tilde{\theta}_J ts_{J1}^r & \cdots & td_{JJ}^r - \tilde{\theta}_J ts_{JJ}^r \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \vdots \\ \gamma_J \end{pmatrix} \right] \\
&= \tilde{\boldsymbol{\lambda}} * B\boldsymbol{\gamma}
\end{aligned}$$

where B is also an square matrix of dimension J which general element $B[i,j] = (T^r * D^j)[i,j] - \tilde{\theta}_i(T^r * (S^{J^{mr}} - S^j))[i,j]$.

In this way, expression (17) becomes in $A\boldsymbol{\Gamma} = \tilde{\boldsymbol{\lambda}} * B\boldsymbol{\gamma}$; and therefore, the manufacturers margins under uncertainty can be expressed as:

$$\boldsymbol{\Gamma}^u = A^{\dagger}(\tilde{\boldsymbol{\lambda}} * B\boldsymbol{\gamma}) \tag{22}$$

□

12 Control function

Table 7: Tested Control Functions

Before-strike Period								
Variables	**Instrument(s) in period**							
	t	**t**	**t - 1**	**t - 1**	**t - 2**	**t - 2**	**t - 3**	**t - 3**
RMP	0.0019***		0.0019***		0.0018***		0.0017***	
	(0.00005)		(0.00004)		(0.00004)		(0.00004)	
RMP x fat level 1		0.0018***		0.0018***		0.0017***		0.0016***
		(0.00005)		(0.00004)		(0.00004)		(0.00004)
RMP x fat level 2		0.0019***		0.0020***		0.0019***		0.0018***
		(0.00005)		(0.00004)		(0.00004)		(0.00004)
Brands	X	X	X	X	X	X	X	X
Retailers	X	X	X	X	X	X	X	X
R^2	0.9893	0.9897	0.9920	0.9926	0.9923	0.9931	0.9911	0.9921
Adj. R^2	0.9889	0.9893	0.9917	0.9923	0.9920	0.9928	0.9908	0.9918
F-Test(Inst(s). = 0)	1276.20	675.48	1965.77	1081.41	2054.64	1179.13	1679.00	986.13
Obs.	744	744	744	744	744	744	744	744

After-strike Period								
Variables	**Instrument(s) in period**							
	t	**t**	**t - 1**	**t - 1**	**t - 2**	**t - 2**	**t - 3**	**t - 3**
RMP	0.0018***		0.0018***		0.0019***		0.0020***	
	(0.00006)		(0.00006)		(0.00006)		(0.00006)	
RMP x fat level 1		0.0016***		0.0016***		0.0017***		0.0018***
		(0.00006)		(0.00006)		(0.00006)		(0.00007)
RMP x fat level 2		0.0019***		0.0019***		0.0020***		0.0020***
		(0.00006)		(0.00006)		(0.00006)		(0.00006)
Brands	X	X	X	X	X	X	X	X
Retailers	X	X	X	X	X	X	X	X
R^2	0.9874	0.9885	0.9877	0.9887	0.9881	0.9889	0.9885	0.9892
Adj. R^2	0.9869	0.9880	0.9872	0.9882	0.9876	0.9884	0.9881	0.9888
F-Test(Inst(s). = 0)	835.57	489.40	873.49	502.45	924.61	520.42	988.12	546.83
Obs.	744	744	744	744	744	744	744	744

Raw-milk Price (RMP): Farmers' whole-milk price (3.7% fat and 3.4% protein); Dummies: fat level 1 = $\mathbf{1}$(0 - 1.5% fat), fat level 2 = $\mathbf{1}$(1.6 - 3.5% fat); ***, **, * denote 1%, 5% and 10% level of significance respectively; Standard-Errors are in parenthesis.