

What Drives Total Real Unit Energy Costs Globally? A Novel LMDI Decomposition Approach

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Abstract

This paper presents a novel logarithmic mean Divisia index (LMDI) decomposition framework that is tailor-made for unit cost indicators. It adds four new models to the existing LMDI model family. The main novelty of the new framework lies in the separation of quantity and price effects captured in unit cost indicators, while retaining the same desirable properties of traditional models. Four case studies apply the novel LMDI framework to the total real unit energy costs (total RUEC) indicator. Total RUEC represents the sum of direct energy costs (for energy products) and indirect energy costs (energy costs embedded in intermediate inputs and passed on along global value chains) as a fraction of value added. This yardstick allows for monitoring shifts in the burden of energy costs on industries. The first three case studies, based on the World Input-Output Database, cover the period between 1995 and 2009. For an up-to-date analysis, a fourth case study collects additional data for 2009-2016 from energy and economic statistics' institutions. Globally, up until 2009, rising costs for crude petroleum/natural gas and the rise of China in the global economy were the largest drivers of total RUEC. In general, increases of indirect energy costs were more substantial than were those of direct energy costs. The total RUEC of Chinese car manufacturers increased much more strongly than did that of American car manufacturers. After 2009 (until 2016), prices for crude petroleum/natural gas and value added generation were major decelerating factors of global direct RUEC, while increases in energy consumption had offsetting effects. This paper provides a suitable tool to scientists who want to build on unit cost indicators in their research in general and to all policy-oriented institutions concerned with monitoring and analysing the energy transition in particular.

JEL Classification

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Keywords

Logarithmic mean Divisia index; Structural decomposition analysis; Total real unit energy costs; Monitoring energy transition; Environmental-economic accounting; Multi-regional input-output analysis

Highlights

- This paper presents a novel logarithmic mean Divisia index decomposition framework.
- It contributes four new models to the existing LMDI model family.
- First LMDI models that separate quantity and price effects.
- Four case studies apply the LMDI framework to the total RUEEC indicator.
- This paper provides a new tool, particularly for monitoring the energy transition.

1 Introduction

Between 1995 and 2011, the ratio of direct energy costs to global GDP almost doubled (from 7.6% in 1995 to 14.8% in 2011). Then, in the five years after 2011, the ratio declined sharply (to 9.4% in 2016). Major accelerating and decelerating factors explaining those shifts are trends in energy consumption, energy intensity, energy prices, and economic activity. Between 1995 and 2016, global total primary energy supply increased from approximately 390 to 580 exajoules [IEA, 2018a,e], adding more than two times the total primary energy supply of the United States today. During the same time, global GDP and gross output rose substantially, leading to a drop in energy intensity (reduction from 5.1 to 3.6 megajoules per constant 2010 USD of gross output, calculations based on World Bank [2018b] and WIOD [2018] data) – while OECD’s real energy end-use industrial price index increased by the order of +40% between 1995-2011 (+25% during 1995-2016) [IEA, 2018c]. Regardless of those major trends at the global level, large heterogeneity exists across countries, sectors, and energy products, potentially affecting the competitiveness of industries. The total real unit energy costs (total RUEC) indicator captures all those compound factors in a single yardstick and allows for the analysis of the shifts in the burden of energy costs on single industries. Paying attention to the characteristics of a globalised world economy, this paper investigates the question: “What drives total RUEC globally?”.

At the outset, a short introduction to the total RUEC indicator is warranted. In 2014, both the European Commission [2014] and the Energy Expert Commission of the German Government [Löschel et al., 2014] recommended using direct RUEC (not total RUEC) for assessing and monitoring the burden of energy costs on firms. RUEC are (direct) energy costs as a percentage of value added, and they are a measure of the amount of money spent on energy (electricity, natural gas, petroleum products, etc.) to obtain one unit of value added. In this sense, the indicator measures the energy requirement of industries in EUR to produce one EUR of value added. During the last four years, this indicator has become widespread in European energy policy (see also European Commission [2015, 2016, 2017, 2018a]), as affordable energy is one of the main objectives of the Energy Union. And, even prior to this recent rise in popularity, international institutions and researchers (e.g., OECD/IEA [2004], FitzGerald et al. [2009], Sato and Dechezleprêtre [2015]) used the indicator for trans-national and trans-sectoral comparisons and to analyse the vulnerability of industrial sectors to changes in energy and carbon prices.

However, (direct) RUEC do not capture fully the energy price and cost effects of multi-stage supply chains. By way of illustration, a car manufacturer will be affected not only by changes in direct energy costs (e.g. domestic electricity prices) but also by changes in energy costs embodied in intermediate inputs, such as steel, aluminium, and plastics, bought from suppliers all around the world. The global economy is a complex web of global value chains that “describe the full range of activities that firms and workers do to bring a product/good or service from its conception to its end use and beyond” [Frederick, 2016]. In a globalised economy, value chains are not contained in single nations, but span countries and continents. To account for corresponding energy cost effects, Kaltenegger et al. [2017a] developed the “total” RUEC indicator. Total RUEC are the sum of “direct” and “indirect” energy costs, again as a percentage of industrial value added. Indirect energy costs are embodied in intermediate inputs and reflect the energy costs incurred at earlier stages of the production chain. Though national policy makers tend to consider only differences in direct energy prices and costs, mostly because those are easier to assess and can be influenced more easily, it is the differences in total energy prices and costs that affect energy-related international competitiveness. The authors conclude that indirect energy costs are very substantial: for some German sectors, like machinery and transport equipment, indirect energy costs amount to more than three times the direct energy costs.

For the analysis in the study at hand, Section 2 presents a novel logarithmic mean Divisia index (LMDI) decomposition framework that is tailor-made for unit cost indicators. In Section 3, case studies then apply the LMDI framework, answering the research question by focusing on the global economy, the global manufacturing industry, and the global automotive industry. The first three studies cover the period between 1995 and 2009 based on the World Input-Output Database; for an up-to-date analysis, a fourth collects additional data for 2009-2016 from energy and economic statistics' institutions. Section 4 and Section 5 discuss the findings and conclude.

This paper contributes to the literature in two respects: First, Section 2 adds four new models to the existing LMDI model family, which consists of eight different LMDI decomposition models for quantity or intensity indicators. The derivation of the new set of models was necessary, because the decomposition identities of interest in this paper differ structurally from the traditional decomposition identities of quantity and intensity indicators. Nevertheless, for the derivation of the new formulae and for proving their desirable properties, I draw from almost two decades of Ang's and his colleagues' work on the eight existing LMDI decomposition models, starting with Ang and Choi [1997], who introduced the logarithmic mean weight function to index decomposition analysis, and culminating with Ang [2015], who provides guidelines for implementing the currently existing eight models. The new framework for unit cost indicators fills an important research gap, because unit cost indicators capture more information than do either quantity or intensity indicators, as they combine physical quantities and corresponding prices. Quantity indicators (such as energy consumption or CO₂ emissions) and intensity indicators (such as energy or emissions intensities, i.e., indicators with a physical quantity in the numerator divided by a monetary value – e.g., either gross output or value added – in the denominator) focus on physical flows. See Ang et al. [2010], Ang and Xu [2013], Wang et al. [2017], Xu and Ang [2013] for some examples. However, economists, in particular, are not interested in physical flows alone but are also interested in corresponding monetary data. Unit cost indicators capture both physical (quantities) and monetary flows (prices and costs) in the numerator, divided by a monetary value in the denominator (gross output or value added). Structural decomposition analysis is a suitable analytical tool to track economy-wide quantity*and* price trends captured in unit cost indicators. In this way, Section 2 provides a theoretical contribution to the literature.

Second, the application of the LMDI framework to case studies in Section 3 shows how it can be used for international monitoring of global and national energy transitions. Policy-oriented institutions, such as the IEA, OECD, ministries of energy, expert commissions, and other stakeholders, can use the framework and combine it with (their own) up-to-date data, such as shown in case study 4 in Section 3.4 (and Annex E). Thus, Section 3 provides a rather practical contribution to the literature.

In sum, this paper provides a suitable tool for scientists who want to build on unit cost indicators in their research in general and for all policy-oriented institutions concerned with monitoring and analysing the energy transition in particular.

2 LMDI framework for unit (energy) cost indicators

This section explains the math behind unit cost indicators (Subsection 2.1) and the LMDI framework for their decomposition (Subsection 2.2). As it adds four new models to the existing

eight LMDI decomposition models in the literature, this section concludes with a comparison between the new and the traditional models (Subsection 2.3).

2.1 Unit (energy) cost indicators

The LMDI framework presented in this paper is applicable to unit cost indicators in general. Given the research question, it is applied to RUEC:

$$RUEC(t) = \frac{\sum_i^n EC_i(t)}{\sum_j^m VA_j(t)} \quad (1a)$$

$$RUEC(t) = \frac{\sum_i^n EP_i(t)EQ_i(t)}{\sum_j^m VA_j(t)} \quad (1b)$$

RUEC are energy costs (EC) as a percentage of value added (VA). The numerator of unit cost indicators can be split up into quantities (here energy quantity EQ , i.e., energy use) and corresponding prices (here energy price EP). Of course, instead of “net output” (VA) “gross output” (GO) could be used as a measure of production in the denominator.¹ Indices i and j denote the elements in different categories and sub-categories (Table 1 provides a non-exhaustive list). t refers to time, i.e., the reporting period.

For the sake of conciseness, index i (in EC_i , EP_i and EQ_i) is used instead of various indices (e.g., in $EC_{p,f,s,r,p}$, where p refers to energy products, f to energy forms, s to sectors, r to regions, and p to price concepts; see again Table 1 – even more categories are conceivable). Using just index i , instead of various indices, is an efficient way to refer to either one or several categories at the same time. The elements of index i depend on the specific research question. The same holds for index j (in VA_j). This also means that indirect energy costs (vs. direct energy costs) should be regarded as an additional (sub-)category of the total RUEC indicator and, therefore, as elements of i .

Though most of the data in Table 1 can be found directly in National Accounts and quasi-official statistics (see also Section 3.4 and Annex E), total and indirect energy costs have to be computed separately. This can be done using the following Equations 2a and 2b in matrix notation. By way of reminder, indirect energy costs are embodied in intermediate inputs and reflect the energy costs incurred at earlier stages of the production chain.

$$ruec_{tot}^T = \frac{\underbrace{(e_c)^T \hat{x}^{-1}}_{\text{Direct energy costs requirements}}}{\underbrace{\hspace{10em}}_{\text{Total energy costs requirements}}} L \hat{x} \hat{v}^{-1} \quad (2a)$$

$$ruec_{tot}^T = \frac{\underbrace{(e_p \odot e_q)^T \hat{x}^{-1}}_{\text{Direct energy costs requirements}}}{\underbrace{\hspace{10em}}_{\text{Total energy costs requirements}}} L \hat{x} \hat{v}^{-1} \quad (2b)$$

¹However, this paper focuses on the definition used by the [European Commission \[2014\]](#) and the Energy Expert Commission of the German Government [[Löschel et al., 2014](#)], with VA in the denominator.

$ruec_{tot}^T$ is a transposed vector of total RUEC per sector. e_c^T and $(e_p \odot e_q)^T$ are both transposed vectors of direct energy costs per sector. The vector of energy prices e_p is connected to the vector of energy quantities e_q via Hadamard product. The multiplication of the transposed vector of direct energy costs by the inverse of a diagonalised matrix of gross outputs per sector \hat{x}^{-1} and by the Leontief inverse L results in a transposed vector of total energy cost requirements. Their multiplication with a diagonalised matrix of gross outputs per sector gives absolute total energy costs per sector. Finally, multiplication by the inverse of a diagonalised matrix of value added per sector \hat{v}^{-1} yields sectoral total RUEC. Note, indirect energy costs can be calculated as the difference between total and direct energy costs.

Table 1: RUEC categories and elements.

Numerator and denominator	Categories	Possible sub-categories	WIOD elements
Referring to index i in Equations 1a and 1b.			
Total energy costs ¹⁾	Energy product	Primary and secondary	Coal and lignite Crude petroleum and natural gas Coke and refined petroleum products Electrical energy, gas, steam and hot water
	Energy form	Direct and indirect	N/A (values are computed separately ²⁾)
	Sectoral origin	Industry and other; energy-intensive industries and other	Agriculture, hunting, forestry and fishing Mining and quarrying Food, beverages and tobacco Textiles and textile products :
	Regional origin	Domestic and foreign; EU27, NAFTA, ...	Australia Austria Belgium Brazil :
	Price concept	Basic prices and purchasers' prices	Intermediate inputs at basic prices Taxes less subsidies on products International transport margins
Referring to index j in Equations 1a and 1b.			
Value added	Factors of production	Labour and capital	High-skilled labour compensation Medium-skilled labour compensation Low-skilled labour compensation Capital compensation
	Sectoral origin	see above	see above
	Regional origin	see above	see above

Notes: 1) As shown elsewhere in this paper, energy prices and energy quantities can be separated; 2) See Equations 2a and 2b.

Source: Own elaboration.

Because the total RUEC indicator takes both direct and indirect energy costs into account, it bears similarities to the more familiar energy footprint indicator: $ef = e_q^T \hat{x}^{-1} Ly$. Energy footprints (ef^T) are consumption-based indicators that record the energy used to produce a country's final demand (see e.g., Akizu-Gardoki et al. [2018], Arto et al. [2016], Kaltenecker et al. [2017b], Kucukvar et al. [2016], and Lan et al. [2016] for recent applications at the global level). Both energy footprints and total RUEC take advantage of the Leontief inverse (L), i.e., the total requirement matrix, which shows the total input requirements of each industry per unit of final output. The energy footprint indicator uses energy intensities ($e_q^T \hat{x}^{-1}$) to calcu-

late the corresponding total energy requirement matrix ($e_q^T \hat{x}^{-1} L$), that is, the total (physical) energy requirements of each industry per unit of final output. The RUEC indicator extends it by its monetary flows ($(e_p \odot e_q)^T \hat{x}^{-1} L$). However, contrary to energy footprints, the RUEC indicator does not allocate total energy requirements (neither physical nor monetary) to final demand (y), but it allocates them to sectoral intermediate demand and divides them by its respective value added ($\hat{x} \hat{v}^{-1}$).

Both Equations 1a and 2a, on the one hand, and Equations 1b and 2b, on the other hand, have their own merit. Most researchers will favour the latter versions, as they allow for separation of costs into prices and quantities. However, many databases only provide aggregated information on costs, without underlying quantities and prices, so that only the first pair of equations is of use to researchers. This is generally the case in official single-region input-output tables published by statistical offices, such as Eurostat and the OECD. Even elaborated multi-regional input-output databases, such as the World Input-Output Database used in the case studies below, do not explicitly present harmonised quantities and prices for single energy products. For this reason, the LMDI decomposition framework presented in the next Subsection 2.2 will provide specific LMDI decomposition models for each of the two versions.

2.2 LMDI decomposition framework

The purpose of decomposition analyses is to quantify the effects of underlying factors – in this case particularly costs, prices, quantities, and value added – that lead to changes in the aggregate indicator over time. The method for conducting the decomposition of those changes in this paper is the LMDI method, as it has several desirable properties. The properties of this technique are discussed more extensively in the next Subsection 2.3. At this point it is sufficient to note that it allows for perfect decomposition, i.e., decomposition without residuals. Residuals are undesirable, because a part of the observed effects and changes is then left unexplained. Residuals complicate the interpretation of decomposition results [Ang et al., 2003] and, therefore, run counter to the purpose of the analysis.

The LMDI decomposition framework for unit cost indicators presented in this paper comprises four distinct models in total. This is due to the fact that aggregate changes in unit cost indicators from one period to the other (T_1 to T_2) may be measured in terms of a difference (in our case: $RUEC^{T_2} - RUEC^{T_1}$) or in the form of a ratio ($RUEC^{T_2}/RUEC^{T_1}$). In addition, as defined in Subsection 2.1 above, unit cost indicators may either use aggregated cost data in the numerator or split them up into quantities and corresponding prices (see Equations 1a and 1b), depending on the research question and available data. This results in 2x2 possible combinations or model specifications for decomposition.

In the LMDI literature, factors contributing to the aggregate “difference measure” are studied by expressing the relevant components in the “additive” form, where the aggregate difference is the sum of all underlying changes. Likewise, factors contributing to the aggregate “ratio measure” are studied by expressing the relevant components in the “multiplicative” form, where the aggregate ratio is the product of all underlying changes. The Equations 3a to 3d reflect these relationships in the four models:

$$\begin{aligned}\Delta V_{RUEC} &= RUEC^{T_2} - RUEC^{T_1} \\ &= \Delta V_{EC} + \Delta V_{VA} = \sum_i^n \Delta V_{EC,i} + \sum_j^m \Delta V_{VA,j}\end{aligned}\quad (3a)$$

$$= \Delta V_{EP} + \Delta V_{EQ} + \Delta V_{VA} = \sum_i^n \Delta V_{EP,i} + \sum_i^n \Delta V_{EQ,i} + \sum_j^m \Delta V_{VA,j}\quad (3b)$$

$$\begin{aligned}D_{RUEC} &= \frac{RUEC^{T_2}}{RUEC^{T_1}} \\ &= D_{EC}D_{VA} = \prod_i^n D_{EC,i} \cdot \prod_j^m D_{VA,j}\end{aligned}\quad (3c)$$

$$= D_{EP}D_{EQ}D_{VA} = \prod_i^n D_{EP,i} \cdot \prod_i^n D_{EQ,i} \cdot \prod_j^m D_{VA,j}\quad (3d)$$

In the additive LMDI procedure, ΔV_{RUEC} refers to the aggregate change, measured as a difference, in the RUEC indicator from one period to the other ($RUEC^{T_2} - RUEC^{T_1}$). $\Delta V_{EC,i}$, $\Delta V_{EP,i}$, $\Delta V_{EQ,i}$, and $\Delta V_{VA,j}$ refer to the underlying effects stemming from changes in costs, prices, quantities, and value added. In the same way, in the multiplicative LMDI procedure, D_{RUEC} refers to the aggregate change, measured as a ratio, in the RUEC indicator ($RUEC^{T_2}/RUEC^{T_1}$), and $D_{EC,i}$, $D_{EP,i}$, $D_{EQ,i}$, and $D_{VA,j}$ denote the corresponding underlying multiplicative effects.

Note that the additive decomposition measures percentage-point changes, whereas the multiplicative decomposition measures percentage changes. Which procedure to use, the additive or the multiplicative, depends on which decomposition results can be more easily understood and communicated. Both are fairly easy to interpret and can be used almost interchangeably (see also Subsection 2.3 on desirable properties of LMDI decomposition). The multiplicative procedure allows for presenting decomposition results in indices right away. This might be advantageous when results are derived from time-series data. The additive procedure might be preferable when looking only at selected benchmark years.

From those four models, the LMDI decomposition framework must be derived, which allows quantification of all underlying additive and multiplicative effects. This requires, amongst other things, finding idoneous weight functions ω_i and ω_j that lead to perfect decomposition, i.e., the sum (the product) of all effects in the additive (multiplicative) models is truly amounts to the total effects of the aggregate difference (ratio) measures. All necessary steps to derive the framework are outlined in Annex A for Equation 1a (and resulting model specifications in Equations 3a and 3c) and Annex B for Equation 1b (and resulting model specifications in Equations 3b and 3d). The resulting LMDI framework for unit (energy) cost indicators itself can be found in the summary Table 2.

Table 2: LMDI decomposition framework for unit (energy) cost indicators.

Procedure and method	Additive:	Multiplicative:
LMDI-UC ¹⁾ for Equation 1a	Model 1 for Equation 3a	Model 2 for Equation 3c
Cost effect	$\Delta V_{EC,i} = \sum_i^n \omega_{EC,i} L(RUEC^{T1}, RUEC^{T2}) \ln\left(\frac{EC_i^{T2}}{EC_i^{T1}}\right)$	$D_{EC,i} = \exp\left(\sum_i^n \omega_{EC,i} \ln\left(\frac{EC_i^{T2}}{EC_i^{T1}}\right)\right)$
Value added effect	$\Delta V_{VA,j} = -\sum_j^m \omega_{VA,j} L(RUEC^{T1}, RUEC^{T2}) \ln\left(\frac{VA_j^{T2}}{VA_j^{T1}}\right)$	$D_{VA,j} = 1 / \exp\left(\sum_j^m \omega_{VA,j} \ln\left(\frac{VA_j^{T2}}{VA_j^{T1}}\right)\right)$
	$\omega_{EC,i} = \frac{L\left(\frac{EC_i^{T1}}{\sum_i^n EC_i^{T1}}, \frac{EC_i^{T2}}{\sum_i^n EC_i^{T2}}\right)}{\sum_i^n L\left(\frac{EC_i^{T1}}{\sum_i^n EC_i^{T1}}, \frac{EC_i^{T2}}{\sum_i^n EC_i^{T2}}\right)}$, $\omega_{VA,j} = \frac{L\left(\frac{VA_j^{T1}}{\sum_j^m VA_j^{T1}}, \frac{VA_j^{T2}}{\sum_j^m VA_j^{T2}}\right)}{\sum_j^m L\left(\frac{VA_j^{T1}}{\sum_j^m VA_j^{T1}}, \frac{VA_j^{T2}}{\sum_j^m VA_j^{T2}}\right)}$	
LMDI-UC ¹⁾ for Equation 1b	Model 3 for Equation 3b	Model 4 for Equation 3d
Price effect	$\Delta V_{EP,i} = \sum_i^n \omega_{EP,i} L(RUEC^{T1}, RUEC^{T2}) \ln\left(\frac{EP_i^{T2}}{EP_i^{T1}}\right)$	$D_{EP,i} = \exp\left(\sum_i^n \omega_{EP,i} \ln\left(\frac{EP_i^{T2}}{EP_i^{T1}}\right)\right)$
Quantity effect	$\Delta V_{EQ,i} = \sum_i^n \omega_{EQ,i} L(RUEC^{T1}, RUEC^{T2}) \ln\left(\frac{EQ_i^{T2}}{EQ_i^{T1}}\right)$	$D_{EQ,i} = \exp\left(\sum_i^n \omega_{EQ,i} \ln\left(\frac{EQ_i^{T2}}{EQ_i^{T1}}\right)\right)$
Value added effect	$\Delta V_{VA,j} = -\sum_j^m \omega_{VA,j} L(RUEC^{T1}, RUEC^{T2}) \ln\left(\frac{VA_j^{T2}}{VA_j^{T1}}\right)$	$D_{VA,j} = 1 / \exp\left(\sum_j^m \omega_{VA,j} \ln\left(\frac{VA_j^{T2}}{VA_j^{T1}}\right)\right)$
	$\omega_{EP,i} = \omega_{EQ,i} = \frac{L\left(\frac{EP_i^{T1} EQ_i^{T1}}{\sum_i^n EP_i^{T1} EQ_i^{T1}}, \frac{EP_i^{T2} EQ_i^{T2}}{\sum_i^n EP_i^{T2} EQ_i^{T2}}\right)}{\sum_i^n L\left(\frac{EP_i^{T1} EQ_i^{T1}}{\sum_i^n EP_i^{T1} EQ_i^{T1}}, \frac{EP_i^{T2} EQ_i^{T2}}{\sum_i^n EP_i^{T2} EQ_i^{T2}}\right)}$, $\omega_{VA,j} = \frac{L\left(\frac{VA_j^{T1}}{\sum_j^m VA_j^{T1}}, \frac{VA_j^{T2}}{\sum_j^m VA_j^{T2}}\right)}{\sum_j^m L\left(\frac{VA_j^{T1}}{\sum_j^m VA_j^{T1}}, \frac{VA_j^{T2}}{\sum_j^m VA_j^{T2}}\right)}$	

Notes: 1) LMDI-UC is the abbreviation for LMDI decomposition framework for unit (energy) cost indicators, see also Subsection 2.3; $L(a, b)$ is the logarithmic average of two positive numbers a and b given by $L(a, b) = (b - a) / \ln(b/a)$ with $L(a, a) = a$.

Source: Own elaboration.

2.3 Comparison with traditional LMDI formulae

To avoid confusion, I will refer to the LMDI approach for unit cost decomposition presented in this paper as ‘‘LMDI-UC’’, thereby differentiating it from previous research on LMDI techniques, namely ‘‘LMDI-I’’ and ‘‘LMDI-II’’.

When applying the LMDI approach, users have to select the appropriate variant of the technique. The choice determines the specification of the formulae to compute the effects of underlying factors that lead to changes in the aggregate indicator over time. In line with Ang [2015], but extended to accommodate the four new LMDI-UC variants, decision-making involves several dimensions (or steps): First, if the aggregate indicator is a quantity indicator (such as energy consumption $EQ = \sum_i^n OS_i I_i$) or an intensity indicator (such as energy intensity $EQ/O = \sum_i^n S_i I_i$), LMDI-I/LMDI-II is the right framework. If it is a unit cost indicator (such as $EC/O = \sum_i^n EC_i / \sum_j^m O_j = \sum_i^n EP_i EQ_i / \sum_j^m O_j$), LMDI-UC is applicable.² Second, in the LMDI-I/LMDI-II framework, one must determine which method to use, LMDI-I or LMDI-II. The results given by LMDI-I and LMDI-II are very similar. However, LMDI-I has been adopted far more widely in the literature, partly because of its simpler formulae.³

² $EC = \sum_i^n EC_i$ denotes the sum of energy costs of all industries i , EP_i denotes the energy price in industry i , $EQ = \sum_i^n EQ_i$ denotes the sum of energy quantities of all industries i , $I_i = EQ_i/O_i$ denotes the energy intensity of industry i , $O = \sum_i^n O_i$ denotes sum of outputs of all industries i (whether ‘‘net output’’, i.e. value added, or ‘‘gross output’’), and $S_i = O_i/O$ is the output share of industry i .

³Compared to LMDI-II, LMDI-I has two additional desirable properties that might influence the decision, depending on the research question: it is consistent in aggregation [Ang and Liu, 2001] and perfect in decomposition at the subcategory level [Ang et al., 2009].

In the LMDI-UC framework, either the variants based on Equation 1a or the variants based on Equation 1b must be chosen. Selection depends mainly on the research question and data availability (see Subsection 2.2). Third, both in the LMDI-I/LMDI-II framework and in the LMDI-UC framework, users must choose between the additive and the multiplicative procedure. The choice depends on which results can be more easily understood and communicated (percentage changes or percentage-point changes: again, see Subsection 2.2).

A comparison of the frameworks reveals structural similarities and differences in the specifications of the formulae for quantifying the effects of underlying factors (see Table 2 for LMDI-UC and Table 3 in Annex C for LMDI-I/LMDI-II). All formulae of the four new and the eight traditional models share three basic elements: a summation over categories (\sum), a weight function (ω), and the logarithm of a quotient ($\ln(a^{T_2}/a^{T_1})$), which is related to the change of the underlying factor a over time. Now, the most notable differences between the LMDI-UC and LMDI-I/LMDI-II formulae lie in their respective weight functions. Whereas each of the eight LMDI-I/LMDI-II variants uses just one weight function for all possible effects, the LMDI-UC variants for Equation 1a and 1b use one weight function for the effects in the numerator and another weight function for the effects in the denominator, respectively. Consequently, the LMDI-UC formulae are more complex, but they allow for a summation over categories in the numerator that is independent of the summation in the denominator. In addition, all four LMDI-UC variants require a normalised weight function, which ensures that weights add up to unity (see below). In the LMDI-I/LMDI-II framework, only LMDI-II uses normalised weights. Apart from those differences in the weight functions, note that, in LMDI-UC, effects in the numerator differ in sign from effects in the denominator (additive case), and LMDI-UC requires the calculation of reciprocals (multiplicative case).

Regardless of those structural similarities and differences, LMDI-UC shares all important desirable properties of LMDI-I and LMDI-II. These properties have made them (the most) popular techniques for index decomposition analysis (IDA) and structural decomposition analysis (SDA) [Ang, 2015].⁴ (1) First and foremost, LMDI-UC also allows for perfect decomposition, i.e., decomposition of Equations 3a to 3d without unexplained residuals (i.e., $\Delta V_{RUEC} = \Delta V_{EC} + \Delta V_{VA} + \Delta V_{RSD}$, where $\Delta V_{RSD} \neq 0$, is not desirable). The proof of perfect decomposition is given in Appendix D.1. (2) Additive and multiplicative decomposition results can be linked by a simple formula, as shown in Appendix D.2, and (3) the multiplicative LMDI-UC models possess the additive property in the log form, see Appendix D.3. The last two properties ensure that the choice of procedure (additive versus multiplicative) does not inherently change the results and their interpretation. (4) LMDI-UC also satisfies the time-reversal test, which requires, for each estimated effect, that $\Delta V^{T_1, T_2} = -\Delta V^{T_2, T_1}$ (additive procedure) and that $D^{T_1, T_2} = 1/D^{T_2, T_1}$ (multiplicative procedure), see Appendix D.4. Consequently, the choice of base year also does not inherently change the results. (5) Moreover, LMDI can handle zero values and negative values in the data set (see Ang and Liu [2007a,b]). (6) Also, regardless of the number of categories in i and j that are involved, the formulae neither change nor increase in complexity (see Table 1, and Appendices A and B). Finally, LMDI-UC possesses properties that only either LMDI-I or LMDI-II has: First, LMDI-UC is consistent in aggregation and perfect in decomposition also at the subcategory level (like LMDI-I, not LMDI-II), as shown in Appendix D.5. This property is useful, where multi-level aggregation is performed and decomposition results at the sub-category level are of interest. Second, as mentioned above, LMDI-UC requires normalised weight functions (like LMDI-II, not LMDI-I – see Equation A.12 in Annex A and Equation B.13 in Annex B). Normalisation ensures that the weights add up to unity.

⁴SDA uses input-output models and data, like the case studies in Section 3, whereas IDA uses only aggregate sector information [Hoekstra and van den Bergh, 2003].

This is a desirable property in index construction.

In conclusion, Section 2 makes available to researchers a new LMDI decomposition framework for unit (energy) cost indicators, LMDI-UC, which complements LMDI-I and LMDI-II. Despite its structural differences, LMDI-UC shares all the important desirable properties of traditional LMDI methods, both in theoretical foundation and in practical implementation, including ease of use, ease of adaption, and ease of interpretation.

3 Case studies

In this section, the theoretical LMDI-UC framework is put into practice. It focuses on the global economy (Subsection 3.1) and the global automotive (Subsection 3.3) and manufacturing industries (Subsections 3.2 and 3.4).

3.1 Drivers of total energy costs to GDP ratio in the global economy

Scope: This first analysis provides a top-level overview of how drivers of total energy costs as a percentage of GDP developed in the global economy⁵. The results form the background to the subsequent case studies. The formula subject to decomposition is $RUEC(t) = \sum_i^n EC_i(t)/GDP(t)$, and the LMDI-UC framework for Equation 1a is used. Case study 1 employs an additive decomposition. Consequently, results are presented as percentage-point changes. The analysis covers the period 1995-2009⁶.

Data: Calculations are based on the World Input-Output Database (WIOD, [Timmer et al. \[2015\]](#)), release 2013⁷. WIOD provides world input-output tables and international supply and use tables. World input-output tables give a comprehensive summary of all transactions in the global economy between industries (and final users) across countries, i.e., a comprehensive summary of the complex web of global value chains. Furthermore, world input-output tables and international supply and use tables report (direct) energy costs, distinguishing between four different energy product groups, the output and value added of 35 industries in 40 countries (EU-27 and 13 other major countries; this set covers approximately 85% of world GDP). Output and value added are also available for the residual region (rest of the world).

Results: Between 1995 and 2009, the total energy costs to GDP ratio in the global economy increased by 13.81 percentage points. This represents a remarkable doubling of the ratio (from 15.59% to 29.41%) within just 14 years. Rising total energy costs alone would have increased the ratio by 28.33 percentage points, ceteris paribus, but growth in global GDP decelerated the rise (-14.52 percentage points).

⁵NACE (Statistical classification of economic activities in the European Community) Rev. 1 sections A “agriculture, hunting and forestry” to P “private households with employed persons”.

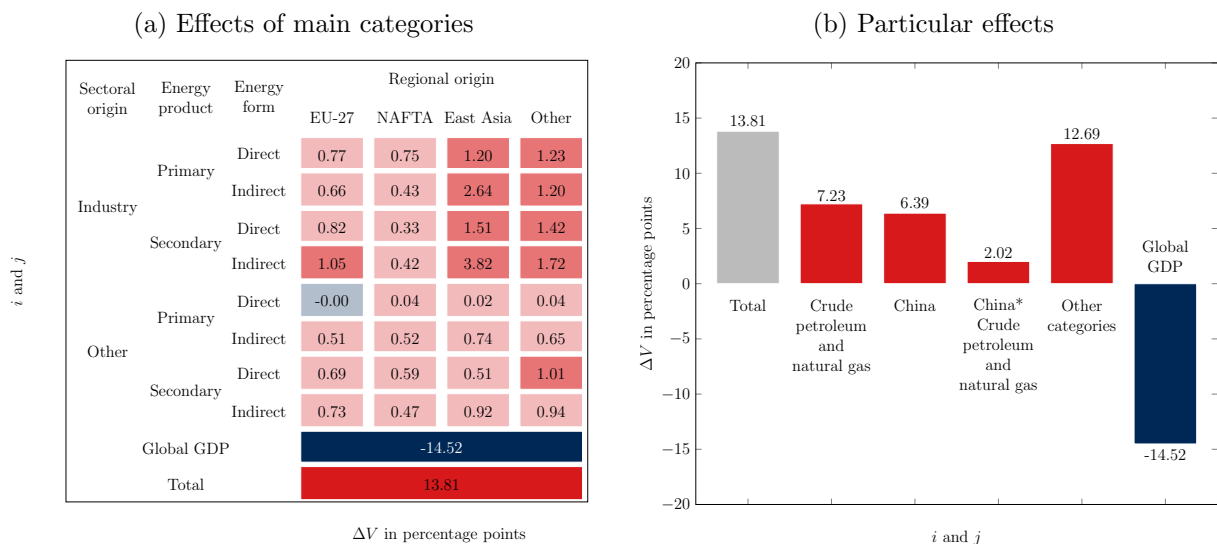
⁶For the sake of consistency with case study 3, the year 2009 was also chosen as the last year of analysis in case studies 1 and 2. The reason for this is that case study 3 builds on WIOD’s Energy Accounts, which only covers the years 1995-2009. For an up-to-date analysis, case study 4 collects additional data for 2009-2016 from energy and economic statistics’ institutions.

⁷WIOD release 2013 was used instead of release 2016, because the 2013 release provides more detail regarding energy products. Whereas WIOD release 2013 compiled four energy-related CPA categories (CPA10 “coal and lignite; peat”, CPA11 “crude petroleum and natural gas; services incidental to oil and gas extraction excluding surveying”, CPA23 “coke, refined petroleum products and nuclear fuels”, and CPA40 “electrical energy, gas, steam and hot water”), release 2016 only compiled two (CPA19 “manufacture of coke and refined petroleum products” and CPA35 “electricity, gas, steam and air conditioning supply”).

Scanning along four selected dimensions (see stylised heatmap in Figure 1a) reveals that total energy costs in East Asia and the other regions were major drivers. Furthermore, it was costs in industry sectors, rather than in agricultural or service sectors, that drove the overall development. Likewise, secondary energy products and indirect energy costs contributed more to the rise in the total energy costs to global GDP ratio than did the initial primary energy products and direct energy costs.

Even though secondary energy products (in sum) had a larger impact than did primary energy products (in sum), a primary energy product category had the largest impact among the four categories studied in this paper (see Figure 1b). The primary energy product category “crude petroleum and natural gas” alone contributed 7.23 percentage points to the global cost to GDP ratio, *ceteris paribus*. At the country level, it was China that had the largest impact by far. The People’s Republic alone increased the ratio by 6.39 percentage points, *ceteris paribus*. Those two factors, crude petroleum/natural gas and China (including their interaction effect), increased the total energy costs to GDP ratio in the global economy more than did all the other possible energy cost effects.

Figure 1: Additive SDA 1995-2009 of total RUEC in the global economy.



Source: Own elaboration based on WIOD.

3.2 Drivers of total RUEC in the global energy and other industry sectors

The [European Commission \[2014\]](#) regards the issue of energy prices and costs as important factors in maintaining and developing a solid and competitive industrial base in the European Union. Case study 1 revealed that, primarily, energy costs in industry sectors, rather than in agricultural or service sectors, drove the doubling of the total energy costs to GDP ratio in the global economy. Therefore, the next three case studies will focus on the secondary sector.

Scope: This case study focuses on the industrial aggregate⁸, distinguishing energy sectors from other sectors. Similar to case study 1, the formula subject to decomposition is $RUEC(t) = \sum_i^n EC_i(t) / \sum_j^m VA_j(t)$, and the LMDI-UC framework for Equation 1a is used. However, this analysis looks deeper into industrial sectors, regions/countries, and factors of production

⁸NACE Rev. 1 sections D “manufacturing” to F “construction”.

(see Table 1) and employs a multiplicative decomposition with results presented as percentage changes. The analysis covers the period 1995-2009.

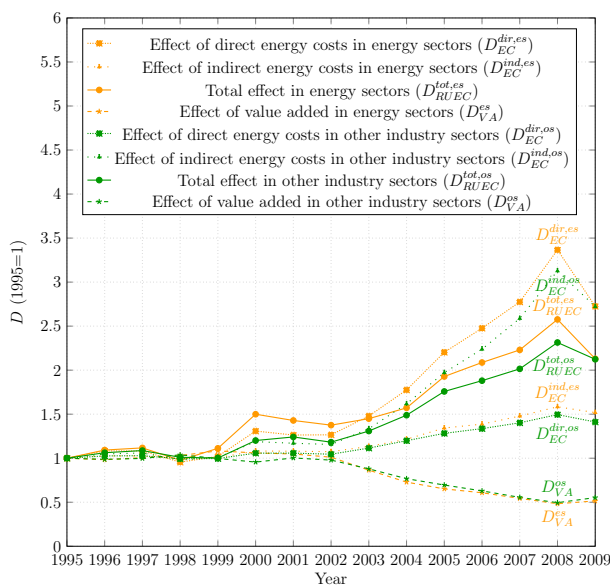
Data: See case study 1.

Results: Before all else, developments in total RUEC are dependent on the type of industry, i.e., energy sectors (refineries and utilities) vs other industry sectors (e.g. chemical, metal, machinery, or textile) (see Figure 2a). Even though total RUEC in energy and other industry sectors increased by the same magnitude between 1995 and 2009 (+113% and +112%), direct and indirect energy costs show different trends. In energy sectors, which convert primary energy products into secondary energy products, direct energy costs were the major drivers. However, in other industry sectors, changes in indirect energy costs were more substantial. For many industries, increases in indirect energy costs were of the same importance to their total RUEC as were direct energy costs for energy sectors.

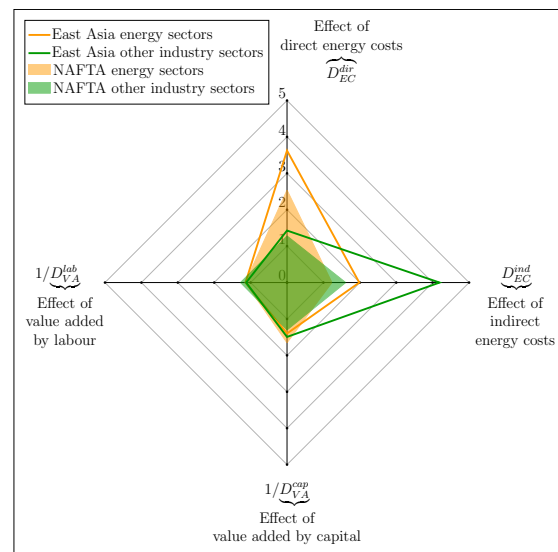
Those general statements for the global level also hold true at the regional and country level, even though there is much variability. For example, direct and indirect energy costs' effects were far more pronounced in East Asia than in the NAFTA regions, both for energy and for other industry sectors (see Figure 2b). Within East Asia, total RUEC in the energy sector increased more in Japan (+214%) than in China (+130%), but direct energy costs were much more important drivers in China than in Japan. Ceteris paribus, direct energy costs in China would have increased total RUEC by 534%, whereas, in Japan, the increase would only be 98% (see Table 3). In the same way, value added generation varied widely. In China, value added by capital decreased total RUEC in industry sectors by 70%, and value added by labour decreased it by 49%, ceteris paribus. However, in Japan, value added shrank considerably, so that value added by capital and labour even added 8% and 29% to total RUEC, ceteris paribus.

Figure 2: Multiplicative SDA 1995-2009 of total RUEC in the global energy and other industry sectors – overview.

(a) Effects in energy sectors vs other industry sectors



(b) Effects in East Asia vs NAFTA



Source: Own elaboration based on WIOD.

Table 3: Multiplicative SDA 1995-2009 of total RUEC in the global industry (energy sectors vs other industry sectors) – detailed results.

Region	Country	Energy sectors					Other industry sectors				
		D_{RUEC}^{tot}	D_{EC}^{dir}	D_{EC}^{ind}	D_{VA}^{cap}	D_{VA}^{lab}	D_{RUEC}^{tot}	D_{EC}^{dir}	D_{EC}^{ind}	D_{VA}^{cap}	D_{VA}^{lab}
		Total effect	Effect of direct energy costs	Effect of indirect energy costs	Effect of value added by capital	Effect of value added by labour	Total effect	Effect of direct energy costs	Effect of indirect energy costs	Effect of value added by capital	Effect of value added by labour
$D^{1995,2009}(1995=1)$											
East Asia	CHN	2.30	6.34	2.80	0.22	0.58	2.23	1.71	8.66	0.30	0.51
	JPN	3.14	1.98	1.41	1.10	1.02	2.22	1.10	1.46	1.08	1.29
	KOR	3.45	3.22	1.59	0.80	0.84	2.93	1.30	3.29	0.85	0.80
	TWN	4.86	2.85	1.49	1.05	1.08	2.31	1.19	2.09	0.84	1.11
East Asia subtotal		4.51	3.62	1.98	0.72	0.88	3.57	1.43	4.19	0.67	0.89
NAFTA	CAN	2.22	2.05	1.82	0.70	0.85	1.26	1.67	1.78	0.64	0.66
	MEX	1.54	3.01	1.35	0.48	0.79	1.09	1.39	2.14	0.46	0.79
	USA	1.61	2.53	1.21	0.60	0.88	1.25	1.23	1.55	0.82	0.80
NAFTA subtotal		1.65	2.54	1.23	0.60	0.87	1.24	1.28	1.61	0.77	0.78
EU-27 subtotal		1.92	2.17	1.39	0.69	0.93	1.57	1.22	1.85	0.91	0.76
Other subtotal		1.52	1.64	2.48	0.37 ¹⁾		1.52	1.64	2.48	0.37 ¹⁾	
WORLD total		2.13	2.73	1.52	0.51 ¹⁾		2.12	1.41	2.72	0.55 ¹⁾	

Notes: 1) D_{VA} , effect of value added.

Source: Own elaboration based on WIOD.

3.3 Drivers of total RUEC in the global automotive industry

Timmer et al. [2015] illustrate the usefulness of the World Input-Output Database by analysing deep changes in global value chains in the automotive industry. This sector has been particularly affected by the increasing opportunities for offshoring and international fragmentation of production.

Scope: This case study analyses the global automotive industry⁹. The formula subject to decomposition is $RUEC(t) = \sum_i^n EP_i(t)EQ_i(t) / \sum_j^m VA_j(t)$. The LMDI-UC framework for Equation 1b is used, which allows for separating energy price from energy quantity effects. In a second step, the energy quantity effect is decomposed into $EQ_r(t) = [EQ_r(t)/GO_r(t)] \cdot GO_r(t)$, where r denotes the region, to separate the effects of energy efficiency improvements and industry growth. Case study 3 covers the period 1995-2009 and employs an additive decomposition (results are presented as percentage-point changes).

Data: See case studies 1 and 2. However, separating energy price effects from energy quantity effects requires splitting up energy costs' data in WIOD's supply and use tables. WIOD's energy accounts provide data on gross energy use in terajoules for all WIOD countries and sectors [Genty et al., 2012]. Matching those two data sets allows estimation of energy prices. It must be added that bringing together those two data sets to estimate implicit energy prices has major shortcomings. For example, CPA categories in WIOD's supply and use tables might include

⁹NACE Rev. 1 sectors 34 and 35 "transport equipment".

other goods and services apart from energy products (provided by WIOD’s energy accounts). Also, both data sets have, in part, been processed independently. Those shortcomings and the fact that other researchers and institutions might want to use additional data provide sufficient reasons for collecting data from other sources in case study 4.

Results: In 2009, among the top seven automobile producers, total RUEC in China and Korea were substantially higher than was the global average (53.07%), but they stayed below the global average in the United States, Japan, and Europe (Germany, France, and the United Kingdom) (see Table 4). Whereas German car manufacturers had the lowest total RUEC among the top seven in 1995, fourteen years later car manufacturers in the United States and in the United Kingdom had lower ratios than did German producers. For all top seven automobile producers, energy prices were drivers of their total RUEC, while energy quantities showed mixed effects: in 2009, some producers consumed more and some consumed less energy in their production than in 1995, both directly and indirectly.

A detailed comparison between China and the United States illustrates parallel and divergent developments of the top automobile producers (see Figure 3a). Total RUEC of Chinese car manufacturers increased by 63.53 percentage points, and they increased by only by 8.75 percentage points for American car manufacturers. While the expansion of energy consumption in the Chinese case drove total RUEC (+113.37 percentage points), reductions in energy consumption decelerated total RUEC in the American case (-5.82 percentage points), ceteris paribus. In both countries, indirect effects were larger than were direct effects. This is true both for prices and quantities and across all energy products.

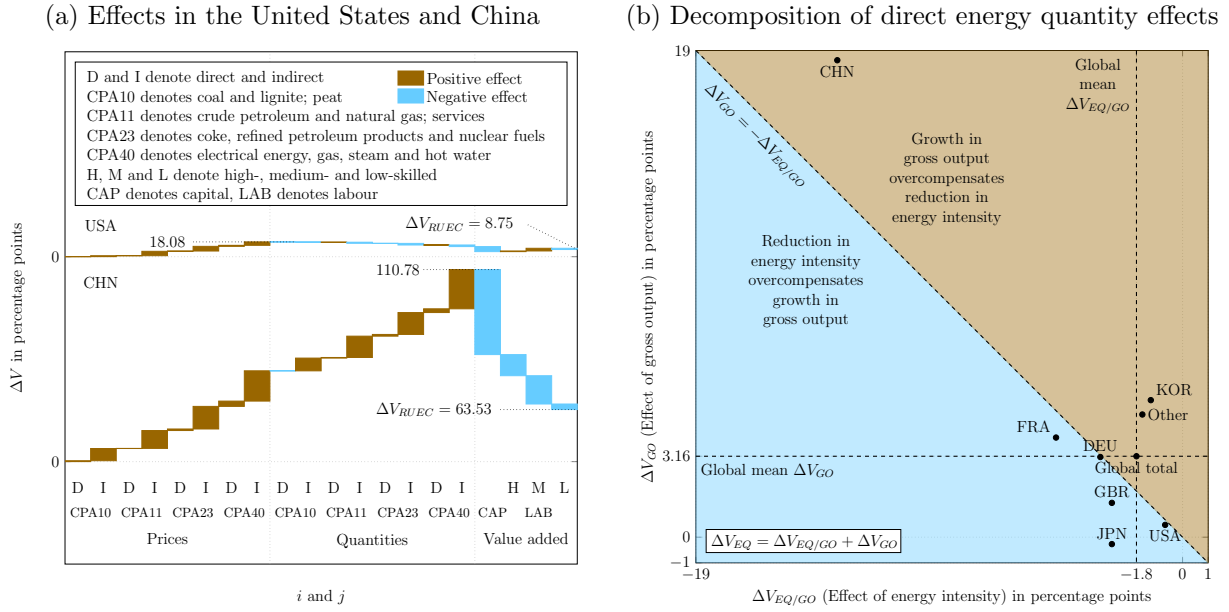
A subsequent decomposition of the direct energy quantity effect (over all energy products) separates the effects of energy efficiency improvements and industry growth (see Figure 3b). Though the top automobile producer, the United States, increased neither energy efficiency nor gross output between 1995 and 2009 (to a significant degree), the second largest automobile producer, China, displayed strong increases. During that period, growth in gross output in China overcompensated for reductions in energy intensity.

Table 4: Additive SDA 1995-2009 of total RUEC in the global automotive industry.

Countries	$RUEC(1995)$	$RUEC(2009)$	ΔV_{RUEC}	ΔV_{EP}^{dir}	ΔV_{EP}^{ind}	ΔV_{EQ}^{dir}	ΔV_{EQ}^{ind}	ΔV_{VA}
(Top 7 sorted by value added in 2009)	Total real unit energy costs in 1995	Total real unit energy costs in 2009	Total effect	Effect of energy prices (direct)	Effect of energy prices (indirect)	Effect of energy quantities (direct)	Effect of energy quantities (indirect)	Effect of value added
	Percent		Percentage points					
USA	18.58	27.32	8.75	1.70	16.45	-0.20	-5.62	-3.59
CHN	53.06	116.59	63.53	12.23	108.74	5.14	108.23	-170.80
JPN	23.99	49.80	25.81	4.33	17.91	-3.04	1.77	4.84
DEU	15.23	34.71	19.49	3.18	16.60	-0.09	6.37	-6.57
KOR	29.20	85.33	56.13	2.72	41.40	4.11	47.56	-39.67
FRA	35.10	52.11	17.01	0.96	24.65	-1.05	4.84	-12.38
GBR	20.07	32.68	12.62	3.45	16.80	-1.42	-3.28	2.93
Other	30.19	47.67	17.48	1.92	28.80	3.23	6.62	-23.08
WORLD total	24.43	53.07	28.64	2.61	27.36	1.36	12.58	-15.27

Source: Own elaboration based on WIOD.

Figure 3: Subsequent additive SDA 1995-2009 (based on Table 4).



Source: Own elaboration based on WIOD.

3.4 Drivers of (direct) RUEC in the European industry

The World Input-Output Database, its energy accounts in particular, does not provide data beyond the year 2009. Also, case study 3 revealed important shortcomings when estimating energy prices based on WIOD data. That is why case study 4 now collects additional data from energy and economic statistics' institutions for an up-to-date analysis.

Data: Appendix E describes the compilation of the data for this case study. The time series data set covers the period 2009-2016. It comprises data on energy quantities and prices for six energy products. While energy consumption data are collected from IEA's energy balances, price data are sourced from a variety of statistics' collecting institutions (e.g., OECD, IEA, Eurostat, and national ministries and associations). Gaps in price data, particularly outside the EU region, are closed by estimates. Value added data are from the World Bank. The data set includes all 40 WIOD countries and the residual region (rest of the world). However, it distinguishes only between industry¹⁰ and other sectors.

Scope: This last analysis focuses on recent (direct) RUEC trends in the European industry. It focuses on direct RUEC instead of total RUEC because of the sectoral aggregation¹¹ chosen in this case study, which does not allow reliable calculation of indirect energy costs. Furthermore, it focuses on the European industry because of quality of available data. The formula subject to decomposition is $RUEC(t) = \sum_i^n EP_i(t)EQ_i(t) / \sum_j^m VA_j$, and the LMDI-UC framework for Equation 1b is used. Case study 4 employs a multiplicative decomposition, and the results are presented as percentage changes.

Results: Taking advantage of the data from case studies 1 to 3 and the additional data mentioned above, Figure 4 shows the development of the global direct and indirect energy costs to GDP ratio for the period 1995-2016. The ratio of direct energy costs to global GDP doubled

¹⁰NACE Rev. 2 sections B "mining and quarrying" to F "construction".

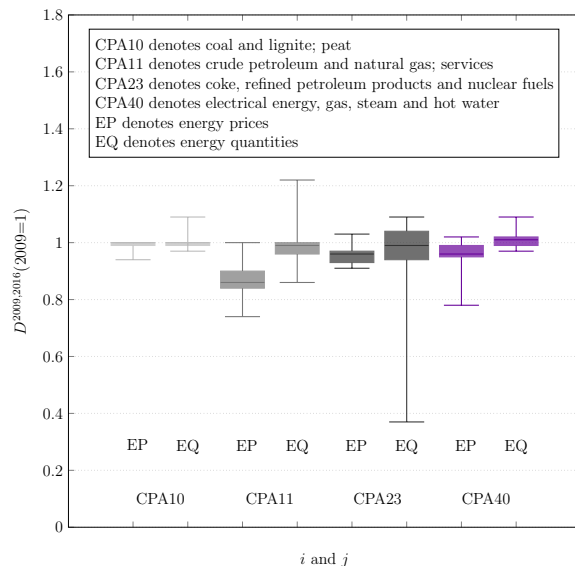
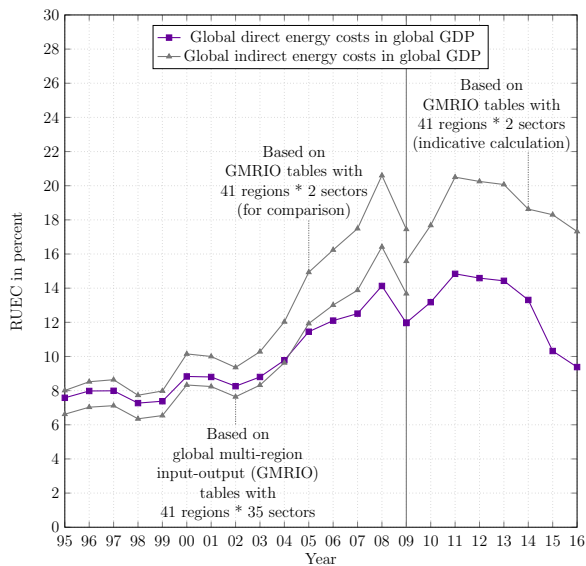
¹¹Sectoral breakdown into two broad sectors, industry and other sectors, instead of 35 sectors in the WIOD release 2013. This high level of aggregation leads to a considerable loss of information.

between 1998 and 2011, the years of its lowest and highest values (7.27% and 14.84%). In the five years after 2011, the ratio declined sharply (to 9.38% in 2016). Compared to its direct counterpart, the indirect energy cost to global GDP ratio rose more strongly. Even though the calculations for indirect energy costs before and after 2009 are not exactly comparable (see explanation above and details in Annex E), global indirect RUEC after 2009 are depicted in the figure as indicative information.

The decline of the direct energy cost to global GDP ratio is due to the fall in energy prices and the expansion of value added generation. At the global level, between 2009 and 2016, direct RUEC fell by 24% (see Table 5). Value added generation alone would have reduced the ratio by 20%, *ceteris paribus*. Prices for crude petroleum/natural gas would have reduced it by 15%, and prices for refined petroleum products would have reduced it by 3%. However, increases in energy consumption had offsetting effects on the decline.

Those global trends are generally also true for the member countries of the EU-27, where medium energy consumption remained rather stable, but prices for crude petroleum, natural gas, petroleum products, electricity, and produced gases declined (see Figure 5). However, there is much variability at the country level. For example, between 2009 and 2016 the (multiplicative) electricity price effect in Germany, France, the United Kingdom, Portugal, Greece, Bulgaria, and Malta is approximately 1.00, whereas it is approximately 0.90 in Austria, the Czech Republic, Hungary, Slovakia, and Slovenia (see Table 5).

Figure 4: Global RUEC between 1995-2016. Figure 5: Boxplots of multiplicative SDA 2009-2016 (direct RUEC in EU-27, by CPA categories).



Source: Own elaboration based on WIOD and additional data.

Table 5: Multiplicative SDA 2009-2016 of direct RUEC in the European industry – detailed results.

Region	Country	D_{RUEC}^{tot} Total effect	CPA10		CPA11		CPA23		CPA40		D_{VA} Effect of value added
			D_{EP} Effect of energy prices	D_{EQ} Effect of energy quantities	D_{EP} Effect of energy prices	D_{EQ} Effect of energy quantities	D_{EP} Effect of energy prices	D_{EQ} Effect of energy quantities	D_{EP} Effect of energy prices	D_{EQ} Effect of energy quantities	
$D^{2009,2016}(2009=1)$											
EU-27	AUT	0.83	1.00	1.00	0.91	0.99	0.93	1.03	0.90	1.00	1.07
	BEL	0.92	1.00	1.00	0.83	1.00	0.97	1.09	0.97	1.02	1.07
	BGR	0.79	0.98	0.99	0.81	1.06	0.97	0.94	1.00	0.99	1.05
	CYP	1.68	1.00	1.00	1.00	1.00	1.03	0.81	0.98	0.99	2.07
	CZE	0.63	0.98	0.99	0.90	0.96	0.92	0.87	0.89	1.02	1.04
	DEU	0.72	0.99	1.00	0.87	0.98	0.93	1.00	1.00	1.03	0.89
	DNK	0.67	0.99	0.97	0.86	0.90	0.97	0.96	0.97	0.98	1.03
	ESP	0.92	1.00	1.00	0.84	1.00	0.94	0.88	0.95	0.97	1.44
	EST	0.86	0.94	1.09	0.97	0.95	0.97	1.07	1.02	1.02	0.85
	FIN	1.03	0.99	1.00	0.90	0.98	0.98	1.07	0.96	1.00	1.16
	FRA	0.86	1.00	1.00	0.86	0.95	0.94	0.97	0.99	1.02	1.14
	GBR	0.62	1.00	0.98	0.86	0.90	0.95	0.97	1.00	0.99	0.90
	GRC	1.42	0.99	0.97	0.75	1.22	0.96	0.94	0.99	0.98	1.84
	HUN	0.81	1.00	1.00	0.84	0.99	0.93	1.06	0.91	1.09	1.01
	IRL	0.47	0.99	1.00	0.88	1.03	0.95	1.02	0.95	1.04	0.54
	ITA	0.80	1.00	1.00	0.85	0.93	0.98	0.91	0.97	0.98	1.20
	LTU	0.69	1.00	1.00	0.74	1.07	0.97	1.05	0.98	1.02	0.85
	LUX	0.54	1.00	1.00	0.94	0.86	0.93	1.04	0.78	1.02	0.86
	LVA	0.85	1.00	1.00	0.82	0.95	0.93	1.05	0.96	1.09	1.08
	MLT	0.36	1.00	1.00	1.00	1.00	0.97	0.37	0.99	0.97	1.07
NLD	0.98	1.00	1.01	0.81	1.00	0.96	1.04	0.96	1.01	1.26	
POL	0.80	0.97	0.99	0.86	1.08	0.96	1.02	0.96	1.01	0.93	
PRT	0.96	1.00	1.00	0.86	1.12	0.95	0.89	1.00	0.98	1.22	
ROM	0.90	0.99	0.98	0.88	0.98	1.00	0.99	0.96	1.02	1.11	
SVK	0.69	0.99	1.00	0.86	1.00	0.92	0.98	0.89	1.04	0.97	
SVN	0.85	0.99	0.99	0.96	0.97	0.91	0.97	0.91	1.05	1.10	
SWE	0.74	1.00	1.00	0.87	0.99	1.01	1.00	0.94	1.00	0.91	

Memo items:

EU-27 subtotal	0.78	0.99	1.00	0.85	0.98	0.95	0.97	0.97	1.01	1.04
East Asia subtotal	0.74	0.96	1.04	0.92	1.08	0.95	1.05	1.04	1.14	0.64
NAFTA subtotal	0.66	0.99	0.99	0.78	1.08	0.99	0.99	1.00	1.00	0.82
Other subtotal	0.78	0.99	1.01	0.85	1.07	0.97	1.08	0.99	1.02	0.82
WORLD total	0.76	0.99	1.01	0.85	1.06	0.97	1.04	1.00	1.05	0.80

Notes: CPA10="coal and lignite; peat"; CPA11="crude petroleum and natural gas; services incidental to oil and gas extraction excluding surveying"; CPA23="coke, refined petroleum products and nuclear fuels"; CPA40="electrical energy, gas, steam and hot water".

Source: Own elaboration based on WIOD.

4 Discussion

This paper is an extension of the LMDI research carried out by Ang [2015] and his many colleagues. The novel LMDI-UC framework contributes four models to the existing LMDI model family.

The derivation of the new set of models was necessary, because the decomposition identities of interest in this paper differ structurally from the traditional decomposition identities of quantity and intensity indicators. Compare the following three (typical) formulae: (1) quantity indicator, e.g., energy consumption $EQ = \sum_i^n O \cdot O_i/O \cdot EQ_i/O_i = \sum_i^n OS_iI_i$, (2) intensity indicator, e.g., energy intensity $EQ/O = \sum_i^n O_i/O \cdot EQ_i/O_i = \sum_i^n S_iI_i$, and (3) unit cost indicator, e.g., unit energy costs $EC/O = \sum_i^n EC_i/\sum_j^m O_j = \sum_i^n EP_iEQ_i/\sum_j^m O_j$ (for notation, see Subsection 2.3). The main structural difference in the identities is that (1) and (2) are sums over multiple factors, whereas unit cost indicators (typically) are fractions of a sum in the numerator and a sum in the denominator. This difference drives the distinct characteristics of the LMDI-UC and traditional models (e.g., distinct number of weight functions, see Subsection 2.2, see also Annex C). Regardless of those distinct characteristics, the LMDI-UC framework retains all desirable properties of the traditional LMDI models (e.g., perfect decomposition, see Subsection 2.3, see also Annex D).

Going back to the research question, what then drove global total RUEC?

The empirical Section 2 reveals major drivers: Between 1995 and 2009, rising costs for crude petroleum/natural gas and the rise of China in the global economy were the largest drivers (increasing global total RUEC by 7.23 percentage points, *ceteris paribus*). In energy sectors, which convert primary energy products into secondary energy products, direct energy costs were the major drivers (direct energy costs alone would have increased total RUEC by 173%, indirect energy costs only by 53%). However, in other industry sectors, changes in indirect energy costs (embedded in intermediate inputs along global value chains) were more significant (+172%, compared to +41% in the case of direct energy costs). During this period, total RUEC of Chinese car manufacturers increased by 63.53 percentage points, and it increased by only 8.75 percentage points for American car manufacturers. Between 2009 and 2016, global direct RUEC fell by 24%. Value added generation alone would have reduced the ratio by 20%, *ceteris paribus*. Prices for crude petroleum/natural gas would have reduced it by 15%, and prices for refined petroleum products would have reduced it by 3%. However, increases in energy consumption had offsetting effects on the decline.

Note, in this paper, energy costs refer to costs of energy products that are used (or might be used¹²) as a source of energy (see principle 3.146 in SEEA 2012, UN et al. [2014]). This means that (total real unit energy) costs, as calculated above, do not include investments in energy efficiency. However, national energy transitions may trigger substantial energy efficiency investments in the future. This would reduce the need for direct energy inputs and, in this way, bring down energy costs. The current methodology does not capture those countervailing effects.

Note also, that the current methodology implicitly assumes a cost pass-through rate of 100%, as indirect energy costs are calculated using the Leontief inverse (see Equations 2a and 2b). This means, 100% of energy costs incurred at one stage of the production chain are passed on downstream subsequently. Even though (total) RUEC are regarded as indicators of energy cost competitiveness, an in-depth analysis of the impact of energy costs on industrial competitiveness should take (true) cost pass-through rates and other relevant factors (e.g., labour costs, infrastructure, and human capital) into account. In this regard, e.g., Schenker et al. [2018] show that unilateral environmental regulation can have effects on multi-stage production structures of industries (i.e., global value chains).

¹²Actually, some energy products are used for non-energy purposes, e.g., as feedstock in the chemical industry.

The case studies give a first impression of the wide range of possible applications of the LMDI-UC framework. LMDI-UC formulae can be applied to all kind of categories and sub-categories (Table 1 provides a non-exhaustive list) that might be of interest in structural and index decomposition analyses. This includes any combination of categories (e.g., see case study 1, which first combines regions, sectors, energy products, and energy forms, and later singles out particular effects). At the same time, categories in the numerator can be chosen (rather) independently of the categories in the denominator (e.g., see case study 2, where the numerator distinguishes between energy forms and the denominator distinguishes between factors of production). To analyse quantity effects in more detail, researchers may also combine LMDI-UC models with LMDI-I and LMDI-II models (see case study 3, where the quantity effect was first determined by using a LMDI-UC model and was then further separated into an energy efficiency effect and an industry growth effect by using an LMDI-II model). Also, LMDI-UC formulae decompose indicators irrespective of whether the indicator is either value-added based or gross-output based¹³, and irrespective of whether indirect costs are taken into account (see case study 4, which focuses primarily on direct RUEC). Of course, the LMDI-UC formulae are not limited to unit energy costs, but they can be applied to any structurally similar environmental (e.g., CO₂ unit costs or water unit costs) or other indicator (e.g., unit labour costs, business investment rates, or R&D intensities).

In essence, the theoretical Section 2 and the empirical Section 3 can be regarded as an application of the LMDI approach to the work by Kaltenecker et al. [2017a] to take their analysis one step further. In their paper, the authors only described trends in (total real unit) energy costs. Now, the LMDI-UC models presented in this paper actually allow for the quantification of underlying drivers.

5 Conclusions

The purpose of this research was to present a novel LMDI decomposition framework that is tailor-made for unit cost indicators (see Section 2, see also Annexes A and B). It adds four new models to the existing LMDI model family (see also Annex C). The main novelty lies in the separation of quantity and price effects captured in unit cost indicators, while retaining the same desirable properties of traditional models (see also Annex D). In the empirical section, four case studies apply the LMDI framework to the total RUEC indicator (see Section 3, see also Annex E).

The empirical section shows how the new LMDI formulae enrich the possibilities for monitoring the energy transitions around the world. The global perspective was chosen deliberately. Case study 4 illustrates how an up-to-date monitoring with current data might look like. Of course, there are many limitations regarding the collected current data (e.g., price concepts that are not harmonised and no sectoral breakdown), but they are sufficient to provide an example. Generally, more detailed and harmonised data are available at the national level. For example, the German Expert Commission on the Energy of the Future Monitoring Process [Löschel et al., 2012] (since 2012), and, recently, the German Government itself [Bundesministerium für Wirtschaft und Energie, 2018] use “end-user spending on electricity in terms of GDP” as an indicator for affordability of the energy transition of the German electricity sector. The indicator uses economy-wide data and includes a detailed breakdown of market-driven and regulated

¹³In fact, it can be argued that unit energy costs should be expressed as a percentage of gross output (not as a percentage of value added), because energy costs are only part of gross output and not part of value added.

elements, such as taxes, fees, and charges on electricity. Another example is the energy cost index for the German industry by [Matthes et al. \[2016\]](#), which uses firm-level data to track RUEC of high, middle, and low energy-intensive German industries. The LMDI-UC framework can also be applied to those two examples, and it is not limited to structural decomposition analyses based on input-output data as shown in the case studies.

Beyond mere monitoring, the new framework enhances the possibilities for future economic analyses. It does so by explicitly separating price effects from quantity effects, both captured in unit cost indicators. In the context of the chosen indicator in this paper, total RUEC, questions of competitiveness have been of primary concern. However, as discussed in [Section 4](#), the analysis of impacts of total RUEC on competitiveness requires additional comprehensive research.

Surely, economic analyses are also of interest in the broader context of sustainable development. UN Sustainable Development Goal 7 (SDG7) aims to deliver affordable, reliable, sustainable, and modern energy for all. More specifically, by 2030, universal access to “affordable” energy services should be ensured. Affordability is clearly linked to costs and prices. At the household level, this might imply considering disposable income (instead of value added). Also, by 2030, “investment in energy infrastructure and clean energy technology” should be promoted. To capture those energy system costs, it might be useful to extend the RUEC indicator appropriately (again, see discussion in [Section 4](#)). Of course, economic questions arise also in the interplay between SDG7 and the other SDGs (e.g., with SDG8, which promotes sustained, inclusive and sustainable economic growth, full and productive employment and decent work for all). This is in line with the call by [Taylor et al. \[2017\]](#) for researchers in this field to create better energy indicators for sustainable development and to address the key challenge to improve current indicator methodologies.

It should be added that environmental unit cost indicators will likely become more widespread in the future. The System of Environmental-Economic Accounting (SEEA Central Framework, [UN et al. \[2014\]](#)) was adopted by the United Nations Statistical Commission in 2012 and is the first international statistical standard for environmental-economic accounting. It aims explicitly at organising both physical and monetary data that have common scope, definitions, and classifications into combined presentations (see principles 1, 1.51, 1.54, and 2.81 in SEEA 2012). On this basis, researchers can readily apply the approach presented above for the case of energy to other natural resources, such as water, land, and materials. From the point of view of energy transition research, an application to CO₂ costs is of particular interest, as carbon pricing will become increasingly important in future energy markets. Therefore, similar analyses might inform the debate on comparing emission mitigation efforts across countries under the Paris Agreement (see [Aldy and Pizer \[2016\]](#) for a starting point).

In conclusion, this paper provides a suitable tool for scientists who want to build on unit cost indicators in their research in general and for all policy-oriented institutions concerned with monitoring and analysing the energy transition in particular.

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A LMDI-UC framework for Equation 1a

Real unit energy costs (RUEC), whether direct, indirect, or total, are generally defined as in the following equation:

$$RUEC(t) = \frac{EC(t)}{VA(t)} = \frac{\sum_i^n EC_i(t)}{\sum_j^m VA_j(t)} \quad (\text{A.1})$$

The Divisia index is a weighted sum of logarithmic growth rates. Derivation with respect to t and rearrangement of terms gives:

$$\begin{aligned} \frac{dRUEC(t)}{dt} &= \frac{\left(\sum_i^n \frac{dEC_i(t)}{dt}\right) \left(\sum_j^m VA_j(t)\right) - \left(\sum_j^m \frac{dVA_j(t)}{dt}\right) \left(\sum_i^n EC_i(t)\right)}{\left(\sum_j^m VA_j(t)\right)^2} \\ &= \sum_i^n \frac{EC_i(t)}{\sum_j^m VA_j(t)} \frac{dEC_i(t)}{EC_i(t)} - \sum_j^m \frac{VA_j(t)}{\left(\sum_j^m VA_j(t)\right)^2} \sum_i^n EC_i(t) \frac{dVA_j(t)}{VA_j(t)} \end{aligned} \quad (\text{A.2})$$

Applying the theorem of instantaneous growth rates and rearrangement of terms leads to:

$$\frac{\frac{dRUEC(t)}{dt}}{RUEC(t)} = \sum_i^n \underbrace{\frac{EC_i(t)}{\sum_i^n EC_i(t)}}_{\omega_{EC,i}(t)} \frac{dEC_i(t)}{EC_i(t)} - \sum_j^m \underbrace{\frac{VA_j(t)}{\sum_j^m VA_j(t)}}_{\omega_{VA,j}(t)} \frac{dVA_j(t)}{VA_j(t)} \quad (\text{A.3})$$

$$\frac{d \ln RUEC(t)}{dt} = \sum_i^n \omega_{EC,i}(t) \frac{d \ln EC_i(t)}{dt} - \sum_j^m \omega_{VA,j}(t) \frac{d \ln VA_j(t)}{dt} \quad (\text{A.4})$$

Integration over T_1 and T_2 :

$$\int_{T_1}^{T_2} \frac{d \ln RUEC(t)}{dt} dt = \int_{T_1}^{T_2} \sum_i^n \omega_{EC,i}(t) \frac{d \ln EC_i(t)}{dt} dt - \int_{T_1}^{T_2} \sum_j^m \omega_{VA,j}(t) \frac{d \ln VA_j(t)}{dt} dt \quad (\text{A.5})$$

$$\int_{T_1}^{T_2} \frac{d \ln RUEC(t)}{dt} dt = \ln RUEC^{T_2} - \ln RUEC^{T_1} = \ln \left(\frac{RUEC^{T_2}}{RUEC^{T_1}} \right) \quad (\text{A.6})$$

$$\frac{RUEC^{T_2}}{RUEC^{T_1}} = \exp \left(\int_{T_1}^{T_2} \sum_i^n \omega_{EC,i}(t) \frac{d \ln EC_i(t)}{dt} dt - \int_{T_1}^{T_2} \sum_j^m \omega_{VA,j}(t) \frac{d \ln VA_j(t)}{dt} dt \right) \quad (\text{A.7})$$

Equation A.7 may be expressed in the additive form:

$$\begin{aligned} \Delta V_{RUEC} &= \Delta V_{EC} + \Delta V_{VA} \\ &= \sum_i^n \Delta V_{EC,i} + \sum_j^m \Delta V_{VA,j} \\ &= \Delta V_{EC,1} + \dots + \Delta V_{EC,n} \\ &\quad + \Delta V_{VA,1} + \dots + \Delta V_{VA,m} \end{aligned} \quad (\text{A.8})$$

Where:

$$\Delta V_{RUEC} = RUEC^{T_2} - RUEC^{T_1} \quad (\text{A.9})$$

$$\Delta V_{EC} = \int_{T_1}^{T_2} \sum_i^n \omega_{EC,i}(t) L(RUEC^{T_1}, RUEC^{T_2}) \frac{d \ln EC_i(t)}{dt} dt \quad (\text{A.10})$$

$$\Delta V_{VA} = - \int_{T_1}^{T_2} \sum_j^m \omega_{VA,j}(t) L(RUEC^{T_1}, RUEC^{T_2}) \frac{d \ln VA_j(t)}{dt} dt \quad (\text{A.11})$$

The insertion of $L(RUEC^{T_1}, RUEC^{T_2})$ in the additive procedure becomes clear in Annex D.2.

In empirical studies, the discrete versions of Equations A.10 and A.11 are needed because the data available are discrete. In the spirit of Ang and Choi [1997] and Sato [1976], who proposed $L(a, b) = (b - a) / \ln(b/a)$ with $L(a, a) = a$ as a weight function, LMDI-UC uses the normalised¹⁴ logarithmic weight function for a perfect decomposition (that leaves no residual):

$$\omega_k^*(t) = \frac{L(\omega_k^{T_1}, \omega_k^{T_2})}{\sum_k^l L(\omega_k^{T_1}, \omega_k^{T_2})} \quad (\text{A.12})$$

Hence, the formulae for the effects in Equations A.10 and A.11 result in:

$$\Delta V_{EC} = \sum_i^n \omega_{EC,i}^*(t) L(RUEC^{T_1}, RUEC^{T_2}) \ln \left(\frac{EC_i^{T_2}}{EC_i^{T_1}} \right) \quad (\text{A.13})$$

with $\omega_{EC,i}^*(t) = \frac{L\left(\frac{EC_i^{T_1}}{\sum_i^n EC_i^{T_1}}, \frac{EC_i^{T_2}}{\sum_i^n EC_i^{T_2}}\right)}{\sum_i^n L\left(\frac{EC_i^{T_1}}{\sum_i^n EC_i^{T_1}}, \frac{EC_i^{T_2}}{\sum_i^n EC_i^{T_2}}\right)}$, see also A.3.

$$\Delta V_{VA} = - \sum_j^m \omega_{VA,j}^*(t) L(RUEC^{T_1}, RUEC^{T_2}) \ln \left(\frac{VA_j^{T_2}}{VA_j^{T_1}} \right) \quad (\text{A.14})$$

with $\omega_{VA,j}^*(t) = \frac{L\left(\frac{VA_j^{T_1}}{\sum_j^m VA_j^{T_1}}, \frac{VA_j^{T_2}}{\sum_j^m VA_j^{T_2}}\right)}{\sum_j^m L\left(\frac{VA_j^{T_1}}{\sum_j^m VA_j^{T_1}}, \frac{VA_j^{T_2}}{\sum_j^m VA_j^{T_2}}\right)}$, see also A.3.

Alternatively, Equation A.7 may be expressed in the multiplicative form:

$$\begin{aligned} D_{RUEC} &= D_{EC} D_{VA} \\ &= \prod_i^n D_{EC,i} \cdot \prod_j^m D_{VA,j} \\ &= D_{EC,1} \cdots D_{EC,n} \cdot D_{VA,1} \cdots D_{VA,m} \end{aligned} \quad (\text{A.15})$$

¹⁴Normalisation is necessary so that a basic property of weight functions is fulfilled, namely that the sum of the weights is unity. Normalisation is achieved by dividing $L(\omega_k^{T_1}, \omega_k^{T_2})$ by $\sum_k^l L(\omega_k^{T_1}, \omega_k^{T_2})$.

Where:

$$D_{RUEC} = \frac{RUEC^{T_2}}{RUEC^{T_1}} \quad (\text{A.16})$$

$$D_{EC} = \exp\left(\int_{T_1}^{T_2} \sum_i^n \omega_{EC,i}(t) \frac{d \ln EC_i(t)}{dt} dt\right) \quad (\text{A.17})$$

$$D_{VA} = \exp\left(-\int_{T_1}^{T_2} \sum_j^m \omega_{VA,j}(t) \frac{d \ln VA_j(t)}{dt} dt\right) \quad (\text{A.18})$$

Again, Equation A.12 is used to derive the discrete versions of Equations A.17 and A.18:

$$D_{EC} = \exp\left(\sum_i^n \omega_{EC,i}^*(t) \ln\left(\frac{EC_i^{T_2}}{EC_i^{T_1}}\right)\right) \quad (\text{A.19})$$

$$D_{VA} = 1 / \exp\left(\sum_j^m \omega_{VA,j}^*(t) \ln\left(\frac{VA_j^{T_2}}{VA_j^{T_1}}\right)\right) \quad (\text{A.20})$$

Note, $\omega_{EC,i}^*(t)$ and $\omega_{VA,j}^*(t)$ in A.13 and A.14 also apply in A.19 and A.20.

B LMDI-UC framework for Equation 1b

Alternatively, by separating $EC(t)$ into $EP(t)EQ(t)$, real unit energy costs can be defined as in the following equation:

$$RUEC(t) = \frac{EP(t)EQ(t)}{VA(t)} = \frac{\sum_i^n EP_i(t)EQ_i(t)}{\sum_j^m VA_j(t)} \quad (\text{B.1})$$

The Divisia index is a weighted sum of logarithmic growth rates. Derivation with respect to t and rearrangement of terms gives:

$$\begin{aligned} \frac{dRUEC(t)}{dt} &= \frac{\left(\sum_i^n \frac{dEP_i(t)}{dt} EQ_i(t) + \sum_i^n EP_i(t) \frac{dEQ_i(t)}{dt} \right) \left(\sum_j^m VA_j(t) \right)}{\left(\sum_j^m VA_j(t) \right)^2} \\ &\quad - \frac{\left(\sum_j^m \frac{dVA_j(t)}{dt} \right) \left(\sum_i^n EP_i(t)EQ_i(t) \right)}{\left(\sum_j^m VA_j(t) \right)^2} \\ &= \frac{\left(\sum_i^n \frac{\frac{dEP_i(t)}{dt}}{EP_i(t)} EP_i(t)EQ_i(t) + \sum_i^n EP_i(t) \frac{\frac{dEQ_i(t)}{dt}}{EQ_i(t)} EQ_i(t) \right) \left(\sum_j^m VA_j(t) \right)}{\left(\sum_j^m VA_j(t) \right)^2} \\ &\quad - \frac{\left(\sum_j^m \frac{\frac{dVA_j(t)}{dt}}{VA_j(t)} VA_j(t) \right) \left(\sum_i^n EP_i(t)EQ_i(t) \right)}{\left(\sum_j^m VA_j(t) \right)^2} \\ &= \sum_i^n \frac{EP_i(t)EQ_i(t)}{\sum_j^m VA_j(t)} \frac{\frac{dEP_i(t)}{dt}}{EP_i(t)} + \sum_i^n \frac{EP_i(t)EQ_i(t)}{\sum_j^m VA_j(t)} \frac{\frac{dEQ_i(t)}{dt}}{EQ_i(t)} \\ &\quad - \sum_j^m \frac{VA_j(t) \sum_i^n EP_i(t)EQ_i(t)}{\left(\sum_j^m VA_j(t) \right)^2} \frac{\frac{dVA_j(t)}{dt}}{VA_j(t)} \end{aligned} \quad (\text{B.2})$$

Applying the theorem of instantaneous growth rates and rearrangement of terms leads to:

$$\begin{aligned} \frac{\frac{dRUEC(t)}{dt}}{RUEC(t)} &= \sum_i^n \underbrace{\frac{EP_i(t)EQ_i(t)}{\sum_i^n EP_i(t)EQ_i(t)}}_{\omega_{EP,i}(t)} \frac{\frac{dEP_i(t)}{dt}}{EP_i(t)} + \sum_i^n \underbrace{\frac{EP_i(t)EQ_i(t)}{\sum_i^n EP_i(t)EQ_i(t)}}_{\omega_{EQ,i}(t)} \frac{\frac{dEQ_i(t)}{dt}}{EQ_i(t)} \\ &\quad - \sum_j^m \underbrace{\frac{VA_j(t)}{\sum_j^m VA_j(t)}}_{\omega_{VA,j}(t)} \frac{\frac{dVA_j(t)}{dt}}{VA_j(t)} \end{aligned} \quad (\text{B.3})$$

$$\begin{aligned} \frac{d \ln RUEC(t)}{dt} &= \sum_i^n \omega_{EP,i}(t) \frac{d \ln EP_i(t)}{dt} + \sum_i^n \omega_{EQ,i}(t) \frac{d \ln EQ_i(t)}{dt} \\ &\quad - \sum_j^m \omega_{VA,j}(t) \frac{d \ln VA_j(t)}{dt} \end{aligned} \quad (\text{B.4})$$

Integration over T_1 and T_2 :

$$\int_{T_1}^{T_2} \frac{d \ln RUEC(t)}{dt} dt = \int_{T_1}^{T_2} \sum_i^n \omega_{EP,i}(t) \frac{d \ln EP_i(t)}{dt} dt + \int_{T_1}^{T_2} \sum_i^n \omega_{EQ,i}(t) \frac{d \ln EQ_i(t)}{dt} dt - \int_{T_1}^{T_2} \sum_j^m \omega_{VA,j}(t) \frac{d \ln VA_j(t)}{dt} dt \quad (\text{B.5})$$

$$\int_{T_1}^{T_2} \frac{d \ln RUEC(t)}{dt} dt = \ln RUEC^{T_2} - \ln RUEC^{T_1} = \ln \left(\frac{RUEC^{T_2}}{RUEC^{T_1}} \right) \quad (\text{B.6})$$

$$\frac{RUEC^{T_2}}{RUEC^{T_1}} = \exp \left(\int_{T_1}^{T_2} \sum_i^n \omega_{EP,i}(t) \frac{d \ln EP_i(t)}{dt} dt + \int_{T_1}^{T_2} \sum_i^n \omega_{EQ,i}(t) \frac{d \ln EQ_i(t)}{dt} dt - \int_{T_1}^{T_2} \sum_j^m \omega_{VA,j}(t) \frac{d \ln VA_j(t)}{dt} dt \right) \quad (\text{B.7})$$

Equation B.7 may be expressed in the additive form:

$$\begin{aligned} \Delta V_{RUEC} &= \Delta V_{EP} + \Delta V_{EQ} + \Delta V_{VA} \\ &= \sum_i^n \Delta V_{EP,i} + \sum_i^n \Delta V_{EQ,i} + \sum_j^m \Delta V_{VA,j} \\ &= \Delta V_{EP,1} + \cdots + \Delta V_{EP,n} \\ &\quad + \Delta V_{EQ,1} + \cdots + \Delta V_{EQ,n} \\ &\quad + \Delta V_{VA,1} + \cdots + \Delta V_{VA,m} \end{aligned} \quad (\text{B.8})$$

Where:

$$\Delta V_{RUEC} = RUEC^{T_2} - RUEC^{T_1} \quad (\text{B.9})$$

$$\Delta V_{EP} = \int_{T_1}^{T_2} \sum_i^n \omega_{EP,i}(t) L(RUEC^{T_1}, RUEC^{T_2}) \frac{d \ln EP_i(t)}{dt} dt \quad (\text{B.10})$$

$$\Delta V_{EQ} = \int_{T_1}^{T_2} \sum_i^n \omega_{EQ,i}(t) L(RUEC^{T_1}, RUEC^{T_2}) \frac{d \ln EQ_i(t)}{dt} dt \quad (\text{B.11})$$

$$\Delta V_{VA} = - \int_{T_1}^{T_2} \sum_j^m \omega_{VA,j}(t) L(RUEC^{T_1}, RUEC^{T_2}) \frac{d \ln VA_j(t)}{dt} dt \quad (\text{B.12})$$

The insertion of $L(RUEC^{T_1}, RUEC^{T_2})$ in the additive procedure becomes clear in Annex D.2.

In empirical studies, the discrete versions of Equations B.10, B.11, and B.12 are needed because the data available are discrete. In the spirit of Ang and Choi [1997] and Sato [1976], who proposed $L(a, b) = (b - a) / \ln(b/a)$ with $L(a, a) = a$ as a weight function, LMDI-UC uses the

normalised¹⁵ logarithmic weight function for a perfect decomposition (that leaves no residual):

$$\omega_k^*(t) = \frac{L(\omega_i^{T_1}, \omega_i^{T_2})}{\sum_k^l L(\omega_k^{T_1}, \omega_k^{T_2})} \quad (\text{B.13})$$

Hence, the formulae for the effects in Equations B.10, B.11, and B.12 result in:

$$\Delta V_{EP} = \sum_i^n \omega_{EP,i}^*(t) L(RUEC^{T_1}, RUEC^{T_2}) \ln \left(\frac{EP_i^{T_2}}{EP_i^{T_1}} \right) \quad (\text{B.14})$$

with $\omega_{EP,i}^*(t) = \frac{L\left(\frac{EP_i^{T_1} EQ_i^{T_1}}{\sum_i^n EP_i^{T_1} EQ_i^{T_1}}, \frac{EP_i^{T_2} EQ_i^{T_2}}{\sum_i^n EP_i^{T_2} EQ_i^{T_2}}\right)}{\sum_i^n L\left(\frac{EP_i^{T_1} EQ_i^{T_1}}{\sum_i^n EP_i^{T_1} EQ_i^{T_1}}, \frac{EP_i^{T_2} EQ_i^{T_2}}{\sum_i^n EP_i^{T_2} EQ_i^{T_2}}\right)}$, see also B.3.

$$\Delta V_{EQ} = \sum_i^n \omega_{EQ,i}^*(t) L(RUEC^{T_1}, RUEC^{T_2}) \ln \left(\frac{EQ_i^{T_2}}{EQ_i^{T_1}} \right) \quad (\text{B.15})$$

with $\omega_{EQ,i}^*(t) = \omega_{EP,i}^*(t)$.

$$\Delta V_{VA} = - \sum_j^m \omega_{VA,j}^*(t) L(RUEC^{T_1}, RUEC^{T_2}) \ln \left(\frac{VA_j^{T_2}}{VA_j^{T_1}} \right) \quad (\text{B.16})$$

with $\omega_{VA,j}^*(t) = \frac{L\left(\frac{VA_j^{T_1}}{\sum_j^m VA_j^{T_1}}, \frac{VA_j^{T_2}}{\sum_j^m VA_j^{T_2}}\right)}{\sum_j^m L\left(\frac{VA_j^{T_1}}{\sum_j^m VA_j^{T_1}}, \frac{VA_j^{T_2}}{\sum_j^m VA_j^{T_2}}\right)}$, see also B.3.

Alternatively, Equation B.7 may be expressed in the multiplicative form:

$$\begin{aligned} D_{RUEC} &= D_{EP} D_{EQ} D_{VA} \\ &= \prod_i^n D_{EP,i} \cdot \prod_i^n D_{EQ,i} \cdot \prod_j^m D_{VA,j} \\ &= D_{EP,1} \cdots D_{EP,n} \cdot D_{EQ,1} \cdots D_{EQ,n} \cdot D_{VA,1} \cdots D_{VA,m} \end{aligned} \quad (\text{B.17})$$

Where:

$$D_{RUEC} = \frac{RUEC^{T_2}}{RUEC^{T_1}} \quad (\text{B.18})$$

$$D_{EP} = \exp \left(\int_{T_1}^{T_2} \sum_i^n \omega_{EP,i}(t) \frac{d \ln EP_i(t)}{dt} dt \right) \quad (\text{B.19})$$

$$D_{EQ} = \exp \left(\int_{T_1}^{T_2} \sum_i^n \omega_{EQ,i}(t) \frac{d \ln EQ_i(t)}{dt} dt \right) \quad (\text{B.20})$$

¹⁵Normalisation is necessary so that a basic property of weight functions is fulfilled, namely that the sum of the weights is unity. Normalisation is achieved by dividing $L(\omega_i^{T_1}, \omega_i^{T_2})$ by $\sum_k^l L(\omega_k^{T_1}, \omega_k^{T_2})$.

$$D_{VA} = \exp\left(-\int_{T_1}^{T_2} \sum_j^m \omega_{VA,j}(t) \frac{d \ln VA_j(t)}{dt} dt\right) \quad (\text{B.21})$$

Again, Equation B.13 is used to derive the discrete versions of Equations B.19, B.20 and B.21:

$$D_{EP} = \exp\left(\sum_i^n \omega_{EP,i}^*(t) \ln\left(\frac{EP_i^{T_2}}{EP_i^{T_1}}\right)\right) \quad (\text{B.22})$$

$$D_{EQ} = \exp\left(\sum_i^n \omega_{EQ,i}^*(t) \ln\left(\frac{EQ_i^{T_2}}{EQ_i^{T_1}}\right)\right) \quad (\text{B.23})$$

$$D_{VA} = 1 / \exp\left(\sum_j^m \omega_{VA,j}^*(t) \ln\left(\frac{VA_j^{T_2}}{VA_j^{T_1}}\right)\right) \quad (\text{B.24})$$

Note, $\omega_{EP,i}^*(t)$, $\omega_{EQ,i}^*(t)$, and $\omega_{VA,j}^*(t)$ in B.14, B.15, and B.16 also apply in B.22, B.23, and B.24.

C Traditional LMDI models

The LMDI approach involves variations in three dimensions: (1) aggregate indicator (quantity indicator versus intensity indicator), (2) decomposition procedure (additive versus multiplicative), and (3) decomposition method (LMDI-I versus LMDI-II). Table 6, based on the guidelines for implementing LMDI decomposition by Ang [2015], shows the three dimensions and corresponding eight LMDI decomposition models.

Table 6: Formulae of the eight traditional LMDI decomposition models.

Indicator	Quantity indicator, e.g. energy consumption: $E = \sum_i^n Q S_i I_i$		Intensity indicator, e.g. energy intensity: $V = E/Q = \sum_i^n S_i I_i$	
Procedure and method	Additive: $\Delta E_{tot} = E^{T_2} - E^{T_1} = \Delta E_{act} + \Delta E_{str} + \Delta E_{int}$	Multiplicative: $D_{tot} = E^{T_2}/E^{T_1} = D_{act} D_{str} D_{int}$	Additive: $\Delta V_{tot} = V^{T_2} - V^{T_1} = \Delta V_{str} + \Delta V_{int}$	Multiplicative: $U_{tot} = V^{T_2}/V^{T_1} = U_{str} U_{int}$
LMDI-I	Model 1 Source: Ang et al. [1998]	Model 2 Source: Ang and Liu [2001]	Model 5 Source: Ang and Zhang [2000]	Model 6 Source: Choi and Ang [2003]
Activity effect	$\sum_i^n \omega_i \ln\left(\frac{Q^{T_2}}{Q^{T_1}}\right)$	$exp\left(\sum_i^n \omega_i \ln\left(\frac{Q^{T_2}}{Q^{T_1}}\right)\right)$	N.A.	N.A.
Structure effect	$\sum_i^n \omega_i \ln\left(\frac{S_i^{T_2}}{S_i^{T_1}}\right)$	$exp\left(\sum_i^n \omega_i \ln\left(\frac{S_i^{T_2}}{S_i^{T_1}}\right)\right)$	$\sum_i^n \omega_i \ln\left(\frac{S_i^{T_2}}{S_i^{T_1}}\right)$	$exp\left(\sum_i^n \omega_i \ln\left(\frac{S_i^{T_2}}{S_i^{T_1}}\right)\right)$
Intensity effect	$\sum_i^n \omega_i \ln\left(\frac{I_i^{T_2}}{I_i^{T_1}}\right)$	$exp\left(\sum_i^n \omega_i \ln\left(\frac{I_i^{T_2}}{I_i^{T_1}}\right)\right)$	$\sum_i^n \omega_i \ln\left(\frac{I_i^{T_2}}{I_i^{T_1}}\right)$	$exp\left(\sum_i^n \omega_i \ln\left(\frac{I_i^{T_2}}{I_i^{T_1}}\right)\right)$
	$\omega_i = L(E_i^{T_1}, E_i^{T_2})$	$\omega_i = \frac{L(E_i^{T_1}, E_i^{T_2})}{L(E^{T_1}, E^{T_2})}$	$\omega_i = L\left(\frac{E_i^{T_1}}{Q^{T_1}}, \frac{E_i^{T_2}}{Q^{T_2}}\right)$	$\omega_i = \frac{L\left(\frac{E_i^{T_1}}{Q^{T_1}}, \frac{E_i^{T_2}}{Q^{T_2}}\right)}{L(V^{T_1}, V^{T_2})}$
LMDI-II	Model 3 Source: Ang et al. [2003]	Model 4 Ang et al. [2003] Source:	Model 7 Source: Ang et al. [2003]	Model 8 Source: Ang and Choi [1997]
Activity effect	$\sum_i^n \omega_{ij} L(E^{T_1}, E^{T_2}) \ln\left(\frac{Q^{T_2}}{Q^{T_1}}\right)$	$exp\left(\sum_i^n \omega_{ij} \ln\left(\frac{Q^{T_2}}{Q^{T_1}}\right)\right)$	N.A.	N.A.
Structure effect	$\sum_i^n \omega_{ij} L(E^{T_1}, E^{T_2}) \ln\left(\frac{S_i^{T_2}}{S_i^{T_1}}\right)$	$exp\left(\sum_i^n \omega_{ij} \ln\left(\frac{S_i^{T_2}}{S_i^{T_1}}\right)\right)$	$\sum_i^n \omega_{ij} L(V^{T_1}, V^{T_2}) \ln\left(\frac{S_i^{T_2}}{S_i^{T_1}}\right)$	$exp\left(\sum_i^n \omega_{ij} \ln\left(\frac{S_i^{T_2}}{S_i^{T_1}}\right)\right)$
Intensity effect	$\sum_i^n \omega_{ij} L(E^{T_1}, E^{T_2}) \ln\left(\frac{I_i^{T_2}}{I_i^{T_1}}\right)$	$exp\left(\sum_i^n \omega_{ij} \ln\left(\frac{I_i^{T_2}}{I_i^{T_1}}\right)\right)$	$\sum_i^n \omega_{ij} L(V^{T_1}, V^{T_2}) \ln\left(\frac{I_i^{T_2}}{I_i^{T_1}}\right)$	$exp\left(\sum_i^n \omega_{ij} \ln\left(\frac{I_i^{T_2}}{I_i^{T_1}}\right)\right)$
	$\omega_{ij} = \frac{L\left(\frac{E_i^{T_1}}{E^{T_1}}, \frac{E_i^{T_2}}{E^{T_2}}\right)}{\sum_j^m L\left(\frac{E_j^{T_1}}{E^{T_1}}, \frac{E_j^{T_2}}{E^{T_2}}\right)}$	$\omega_{ij} = \frac{L\left(\frac{E_i^{T_1}}{E^{T_1}}, \frac{E_i^{T_2}}{E^{T_2}}\right)}{\sum_j^m L\left(\frac{E_j^{T_1}}{E^{T_1}}, \frac{E_j^{T_2}}{E^{T_2}}\right)}$	$\omega_{ij} = \frac{L\left(\frac{E_i^{T_1}}{E^{T_1}}, \frac{E_i^{T_2}}{E^{T_2}}\right)}{\sum_j^m L\left(\frac{E_j^{T_1}}{E^{T_1}}, \frac{E_j^{T_2}}{E^{T_2}}\right)}$	$\omega_{ij} = \frac{L\left(\frac{E_i^{T_1}}{E^{T_1}}, \frac{E_i^{T_2}}{E^{T_2}}\right)}{\sum_j^m L\left(\frac{E_j^{T_1}}{E^{T_1}}, \frac{E_j^{T_2}}{E^{T_2}}\right)}$

Notes: $E = \sum_i E_i$ is the sum of energy consumption of all industries i ; $Q = \sum_i Q_i$ is sum of industrial activity; $S_i = Q_i/Q$ is the activity share of industry i ; $I_i = E_i/Q_i$ is the energy intensity of industry i ; subscripts *tot*, *act*, *str*, and *int* denote the total effect and the effects associated with overall activity level, activity structure, and sectoral energy intensity, respectively; $L(a, b)$ is the logarithmic average of two positive numbers a and b given by $L(a, b) = (b - a)/\ln(b/a)$ with $L(a, a) = a$.

Source: Own elaboration based on Ang [2015].

D Desirable properties of LMDI-UC

The proof of perfect decomposition (see D.1), the link between the additive and multiplicative form (see D.2), the additive property in the log form (see D.3), the fulfilment of the time-reversal test (see D.4), and the property of consistency in aggregation (see D.5) are shown only for $RUEC(t) = \sum_i^n EP_i(t)EQ_i(t) / \sum_j^m VA_j(t)$ in this Appendix, i.e., for the Equation 1b. Of course, all proofs and examples can be reduced easily to the (simpler) case of $RUEC(t) = \sum_i^n EC_i(t) / \sum_j^m VA_j(t)$ in Equation 1a.

D.1 Perfect decomposition

From Equations B.9 and B.14 to B.16:

$$\begin{aligned}
 \Delta V_{RUEC} &= RUEC^{T_2} - RUEC^{T_1} \\
 &= \sum_i^n \omega_{EP,i}^*(t) L(RUEC^{T_1}, RUEC^{T_2}) \ln\left(\frac{EP_i^{T_2}}{EP_i^{T_1}}\right) \\
 &\quad + \sum_i^n \omega_{EQ,i}^*(t) L(RUEC^{T_1}, RUEC^{T_2}) \ln\left(\frac{EQ_i^{T_2}}{EQ_i^{T_1}}\right) \\
 &\quad - \sum_j^m \omega_{VA,j}^*(t) L(RUEC^{T_1}, RUEC^{T_2}) \ln\left(\frac{VA_j^{T_2}}{VA_j^{T_1}}\right)
 \end{aligned} \tag{D.1}$$

Factoring out $L(RUEC^{T_1}, RUEC^{T_2})$ yields:

$$\begin{aligned}
 \Delta V_{RUEC} &= \frac{RUEC^{T_2} - RUEC^{T_1}}{\ln(RUEC^{T_2}) - \ln(RUEC^{T_1})} \\
 &\quad \cdot \left[\sum_i^n \omega_{EP,i}^*(t) \ln\left(\frac{EP_i^{T_2}}{EP_i^{T_1}}\right) + \sum_i^n \omega_{EQ,i}^*(t) \ln\left(\frac{EQ_i^{T_2}}{EQ_i^{T_1}}\right) - \sum_j^m \omega_{VA,j}^*(t) \ln\left(\frac{VA_j^{T_2}}{VA_j^{T_1}}\right) \right]
 \end{aligned} \tag{D.2}$$

LMDI-UC uses normalised weight functions, which ensure that weights add up to unity. Also, $\omega_{EP,i}^*(t) = \omega_{EQ,i}^*(t)$, i.e.:

$$\begin{aligned}
 \Delta V_{RUEC} &= \frac{RUEC^{T_2} - RUEC^{T_1}}{\ln(RUEC^{T_2}) - \ln(RUEC^{T_1})} \\
 &\quad \cdot \left[\ln\left(\frac{\sum_i^n EP_i^{T_2} EQ_i^{T_2}}{\sum_i^n EP_i^{T_1} EQ_i^{T_1}}\right) - \ln\left(\frac{\sum_j^m VA_j^{T_2}}{\sum_j^m VA_j^{T_1}}\right) \right]
 \end{aligned} \tag{D.3}$$

Applying simple logarithm rules gives:

$$\begin{aligned}
\Delta V_{RUEC} &= \frac{RUEC^{T_2} - RUEC^{T_1}}{\ln(RUEC^{T_2}) - \ln(RUEC^{T_1})} \cdot \left[\ln\left(\frac{\sum_i^n EP_i^{T_2} EQ_i^{T_2}}{\sum_j^m VA_j^{T_2}}\right) - \ln\left(\frac{\sum_i^n EP_i^{T_1} EQ_i^{T_1}}{\sum_j^m VA_j^{T_1}}\right) \right] \\
&= \frac{RUEC^{T_2} - RUEC^{T_1}}{\ln(RUEC^{T_2}) - \ln(RUEC^{T_1})} \cdot \ln(RUEC^{T_2}) - \ln(RUEC^{T_1}) \\
&= RUEC^{T_2} - RUEC^{T_1} \\
&= \Delta V_{RUEC}
\end{aligned} \tag{D.4}$$

D.2 Link between additive and multiplicative form

Additive and multiplicative decomposition results can be linked by a simple formula:

$$\frac{\Delta V_{RUEC}}{\ln(D_{RUEC})} = \frac{\Delta V_{EP}}{\ln(D_{EP})} = \frac{\Delta V_{EQ}}{\ln(D_{EQ})} = \frac{\Delta V_{VA}}{\ln(D_{VA})} = L(RUEC^{T_1}, RUEC^{T_2}) \tag{D.5}$$

For example, dividing Equation B.9 by Equation B.18 in log form gives:

$$\frac{\Delta V_{RUEC}}{\ln(D_{RUEC})} = \frac{RUEC^{T_2} - RUEC^{T_1}}{\ln(RUEC^{T_2}) - \ln(RUEC^{T_1})} = L(RUEC^{T_1}, RUEC^{T_2}) \tag{D.6}$$

The same holds true for dividing Equation B.14 by Equation B.22 in log form:

$$\frac{\Delta V_{EP}}{\ln(D_{EP})} = \frac{\sum_i^n \omega_{EP,i}^*(t) L(RUEC^{T_1}, RUEC^{T_2}) \ln\left(\frac{EP_i^{T_2}}{EP_i^{T_1}}\right)}{\ln\left[\exp\left(\sum_i^n \omega_{EP,i}^*(t) \ln\left(\frac{EP_i^{T_2}}{EP_i^{T_1}}\right)\right)\right]} = L(RUEC^{T_1}, RUEC^{T_2}) \tag{D.7}$$

Of course, dividing Equation B.15 by Equation B.23 in log form and dividing Equation B.16 by Equation B.24 in log form yields the same results.

The practical implication of this property is that the choice between multiplicative form and additive form is inconsequential for the measurement of relative effect sizes.

D.3 Additive property in the log form

Equation B.17 can be expressed both as a product and as a sum:

$$D_{RUEC} = \prod_i^n D_{EP,i} \cdot \prod_i^n D_{EQ,i} \cdot \prod_j^m D_{VA,j} \tag{D.8a}$$

$$\ln(D_{RUEC}) = \sum_i^n \ln(D_{EP,i}) + \sum_i^n \ln(D_{EQ,i}) + \sum_j^m \ln(D_{VA,j}) \tag{D.8b}$$

Note, substituting expressions B.18 and B.22 to B.24 into Equation D.8b again yields the result in Equation D.6.

D.4 Fulfilment of the time-reversal test

The time-reversal test requires, in the additive procedure, that $\Delta V^{T_1, T_2} / \Delta V^{T_2, T_1} = -1$ or, in the multiplicative procedure, that $D^{T_1, T_2} / D^{T_2, T_1} = 1$.

Note that $(b - a) / (a - b) = -1$ and that $(\ln(b) - \ln(a)) / (\ln(a) - \ln(b)) = -1$. Consequently, $L(a, b) / L(b, a) = 1$ and $\omega_i^{*, T_1, T_2}(t) / \omega_i^{*, T_2, T_1}(t) = 1$.

Applied to e.g. $\Delta V_{EP, i}$ in Equation B.8 in the additive procedure, it follows that:

$$\Delta V_{EP, i}^{T_1, T_2} / \Delta V_{EP, i}^{T_2, T_1} = \underbrace{\frac{\omega_{EP, i}^{*, T_1, T_2}(t)}{\omega_{EP, i}^{*, T_2, T_1}(t)}}_{=1} \cdot \underbrace{\frac{L(RUEC^{T_1}, RUEC^{T_2})}{L(RUEC^{T_2}, RUEC^{T_1})}}_{=1} \cdot \underbrace{\frac{\ln\left(\frac{EP_i^{T_2}}{EP_i^{T_1}}\right)}{\ln\left(\frac{EP_i^{T_1}}{EP_i^{T_2}}\right)}}_{=-1} = -1 \quad (\text{D.9})$$

$\omega_i^{*, T_1, T_2}(t) = \omega_i^{*, T_2, T_1}(t)$. Applied to e.g. $D_{EP, i}$ in Equation B.17 in the multiplicative procedure, it follows that:

$$D_{EP, i}^{T_1, T_2} / D_{EP, i}^{T_2, T_1} = \exp\left(\underbrace{\omega_{EP, i}^{*, T_1, T_2}(t) \ln\left(\frac{EP_i^{T_2}}{EP_i^{T_1}}\right) + \omega_{EP, i}^{*, T_2, T_1}(t) \ln\left(\frac{EP_i^{T_1}}{EP_i^{T_2}}\right)}_{=0}\right) = 1 \quad (\text{D.10})$$

D.5 Consistency in aggregation

I closely follow the procedure and notation in Ang and Liu [2001]. According to Ang and Liu [2001], Diewert [1978] and Vartia [1976], consistency in aggregation means that the set of commodities $A = \{a_1, a_2, \dots, a_n\}$ is a union of disjoint subsets A_k , i.e. $A = \bigcup_k^K A_k$, where $k = 1, 2, \dots, K$. For that reason, (multiplicative) decomposition is consistent in aggregation if the aggregate effect (A) calculated in a one-step decomposition is equal to the product of sub-effects ($\prod_k^K A_k$) calculated in a two-step decomposition performed on disjoint subsets.

Consider the effects $A_{EP*EQ} = D_{EP} D_{EQ}$ and $A_{VA} = D_{VA}$ (see also Equation B.17) in one-step decomposition (e.g., applied to k WIOD sectors, which are collapsed into one comprehensive sector, or the total economy):

$$\begin{aligned} A_{EP*EQ} &= \left[D_{EP} D_{EQ} \right]^{one-step} = \exp\left(\ln\left(\frac{EP^{T_2}}{EP^{T_1}}\right)\right) \cdot \exp\left(\ln\left(\frac{EQ^{T_2}}{EQ^{T_1}}\right)\right) \\ &= \exp\left(\ln\left(\frac{EP^{T_2}}{EP^{T_1}}\right) + \ln\left(\frac{EQ^{T_2}}{EQ^{T_1}}\right)\right) \\ &= \exp\left(\ln\left(\frac{EP^{T_2} EQ^{T_2}}{EP^{T_1} EQ^{T_1}}\right)\right) \end{aligned} \quad (\text{D.11})$$

$$A_{VA} = \left[D_{VA} \right]^{one-step} = 1 / \exp\left(\ln\left(\frac{VA^{T_2}}{VA^{T_1}}\right)\right) \quad (\text{D.12})$$

In the first step of the two-step decomposition, sub-effects $A_{EP*EQ, k} = D_{EP, k} D_{EQ, k}$ and $A_{VA, k} = D_{VA, k}$ are calculated on k disjoint subsets:

$$A_{EP*EQ,k} = \left[D_{EP,k} D_{EQ,k} \right]^{two-step(1)} = \exp\left(\omega_{EP,k}(t) \ln\left(\frac{EP_k^{T_2}}{EP_k^{T_1}}\right)\right) \cdot \exp\left(\omega_{EQ,k}(t) \ln\left(\frac{EQ_k^{T_2}}{EQ_k^{T_1}}\right)\right) \quad (D.13)$$

$$A_{VA,k} = \left[D_{VA,k} \right]^{two-step(1)} = 1 / \exp\left(\omega_{VA,k}(t) \ln\left(\frac{VA_k^{T_2}}{VA_k^{T_1}}\right)\right) \quad (D.14)$$

where:

$$\omega_{EP,k}(t) = \omega_{EQ,k}(t) = \frac{L\left(\frac{EP_k^{T_1} EQ_k^{T_1}}{\sum_k^K EP_k^{T_1} EQ_k^{T_1}}, \frac{EP_k^{T_2} EQ_k^{T_2}}{\sum_k^K EP_k^{T_2} EQ_k^{T_2}}\right)}{\sum_k^K L\left(\frac{EP_k^{T_1} EQ_k^{T_1}}{\sum_k^K EP_k^{T_1} EQ_k^{T_1}}, \frac{EP_k^{T_2} EQ_k^{T_2}}{\sum_k^K EP_k^{T_2} EQ_k^{T_2}}\right)} \quad (D.15)$$

$$\omega_{VA,k}(t) = \frac{L\left(\frac{VA_k^{T_1}}{\sum_k^K VA_k^{T_1}}, \frac{VA_k^{T_2}}{\sum_k^K VA_k^{T_2}}\right)}{\sum_k^K L\left(\frac{VA_k^{T_1}}{\sum_k^K VA_k^{T_1}}, \frac{VA_k^{T_2}}{\sum_k^K VA_k^{T_2}}\right)} \quad (D.16)$$

In the second step, we have:

$$\begin{aligned} \prod_k^K A_{EP*EQ,k} &= \left[\prod_k^K D_{EP,k} D_{EQ,k} \right]^{two-step(2)} = \prod_k^K \left(\exp\left(\omega_{EP,k}(t) \ln\left(\frac{EP_k^{T_2}}{EP_k^{T_1}}\right)\right) \cdot \exp\left(\omega_{EQ,k}(t) \ln\left(\frac{EQ_k^{T_2}}{EQ_k^{T_1}}\right)\right) \right) \\ &= \exp\left(\sum_k^K \omega_{EP,k}(t) \ln\left(\frac{EP_k^{T_2}}{EP_k^{T_1}}\right)\right) \cdot \exp\left(\sum_k^K \omega_{EQ,k}(t) \ln\left(\frac{EQ_k^{T_2}}{EQ_k^{T_1}}\right)\right) \\ &= \exp\left(\sum_k^K \omega_{EP,k}(t) \ln\left(\frac{EP_k^{T_2}}{EP_k^{T_1}}\right) + \sum_k^K \omega_{EQ,k}(t) \ln\left(\frac{EQ_k^{T_2}}{EQ_k^{T_1}}\right)\right) \\ &= \exp\left(\ln\left(\frac{\sum_k^K EP_k^{T_2} EQ_k^{T_2}}{\sum_k^K EP_k^{T_1} EQ_k^{T_1}}\right)\right) \\ &= \exp\left(\ln\left(\frac{EP_k^{T_2} EQ_k^{T_2}}{EP_k^{T_1} EQ_k^{T_1}}\right)\right) = \left[D_{EP} D_{EQ} \right]^{one-step} = A_{EP*EQ} \end{aligned} \quad (D.17)$$

$$\begin{aligned} \prod_k^K A_{VA,k} &= \left[\prod_k^K D_{VA,k} \right]^{two-step(2)} = \prod_k^K \left[1 / \exp\left(\omega_{VA,k}(t) \ln\left(\frac{VA_k^{T_2}}{VA_k^{T_1}}\right)\right) \right] \\ &= 1 / \exp\left(\sum_k^K \omega_{VA,k}(t) \ln\left(\frac{VA_k^{T_2}}{VA_k^{T_1}}\right)\right) \\ &= 1 / \exp\left(\ln\left(\frac{\sum_k^K VA_k^{T_2}}{\sum_k^K VA_k^{T_1}}\right)\right) \\ &= 1 / \exp\left(\ln\left(\frac{VA_k^{T_2}}{VA_k^{T_1}}\right)\right) = \left[D_{VA} \right]^{one-step} = A_{VA} \end{aligned} \quad (D.18)$$

The same holds true for the corresponding additive effects (see Equation B.8).

E Compilation of case study 4 data

The compilation of time series data for case study 4 covers the period 2009-2016 for all 40 World Input-Output Database release 2013 countries and the residual region (rest of the world). According to the formula subject to decomposition, $RUEC(t) = \sum_i^n EP_i(t)EQ_i(t) / \sum_j^m VA_j$, data on energy quantities, energy prices, and value added have to be collected.

Classification of products: The dataset comprises data on energy quantities and prices for six energy products with a total share of more than 80% in global total primary energy supply in 2016. Those six energy products are “coal”, “crude oil”, “natural gas”, “oil products”, “electricity”, and “heat”. Table 7 shows an energy product correspondence table: whereas case studies 1 to 3 are based on product classifications in the World Input-Output Database release 2013 (left and middle column of the correspondence table), case study 4 uses IEA definitions (right column of the table).

Table 7: Correspondence table for energy products.

CPA 2002 ¹⁾ codes and descriptions in WIOD release 2013 supply and use tables	Energy product codes and descriptions in WIOD release 2013 energy accounts	Energy product codes in IEA world energy balances
10 “Coal and lignite; peat”	BCOAL “Lignite and derivatives” HCOAL “Hard coal and derivatives”	Coal
11 “Crude petroleum and natural gas ²⁾ ”	CRUDE “Crude oil, NGL and feedstocks” NATGAS “Natural gas”	Crude oil; natural gas ³⁾
23 “Coke, refined petroleum products and nuclear fuels”	COKE “Coke” DIESEL “Diesel oil for road transport” GASOLINE “Motor gasoline” JETFUEL “Jet fuel (kerosene and gasoline)” LFO “Light fuel oil” HFO “Heavy fuel oil” NAPHTA “Naphtha” OTHGAS “Derived gas” OTHPETRO “Other petroleum products” BIODIESEL “Biodiesel” BIOGASOL “Biogasoline”	Oil products
40 “Electrical energy, gas, steam and hot water”	ELECTR “Electricity” HYDRO “Hydroelectric” NUCLEAR “Nuclear” SOLAR “Solar” WIND “Wind power” BIOGAS “Biogas” GEOTHERM “Geothermal” HEATPROD “Heat”	Electricity; hydro; geotherm./solar/etc.; heat; nuclear ⁴⁾
Energy products not elsewhere classified	WASTE “Industrial and municipal waste” OTHRENEW “Other combustible renewables” OTHSOURC “Other sources” LOSS “Distribution losses”	Biofuels/waste ⁵⁾

Notes: 1) Statistical classification of products by activity in the European Economic Community, 2002 version; 2) Includes services incidental to oil and gas extraction excluding surveying; 3) Deliveries of natural gas can be accounted either in the resource (CPA11), or in the distribution service (CPA40); 4) In the sense of nuclear power, not nuclear fuels; 5) IEA world energy balances combine biofuels and waste into a single category. Source: Own elaboration based on [Genty et al. \[2012\]](#), [IEA \[2018e\]](#), and [Timmer et al. \[2012\]](#).

Classification of sectors: Case study 4 distinguishes only between industry and other sectors.

The definition of “industry” corresponds to the one used by the [World Bank \[2018b\]](#), i.e., NACE Rev. 2 sections B “mining and quarrying” to F “construction”, including C “manufacturing”, D “electricity, gas, steam and air conditioning supply” and E “water supply, sewerage, waste management and remediation activities”.

Data sources: (1) Energy quantities: Energy consumption data are collected from [IEA \[2018d\]](#)’s world energy balances, which provide statistics for 150 countries and regions, including all OECD countries, as well as the world totals.

A country’s industrial energy consumption is estimated as the sum of energy consumption in the transformation sector, plus final energy consumption in industry (including consumption for non-energy use), plus proportionate final energy consumption in road transport¹⁶. Energy consumption in other sectors is calculated as total final energy consumption, minus final energy consumption in industry (including consumption for non-energy use), minus residential final consumption, minus proportionate final energy consumption in road transport of industry and households.

(2) Energy prices: (Nominal) prices are sourced from a variety sources. [Table 8](#) gives an overview.

Table 8: Data sources of energy prices.

Energy product codes in IEA world energy balances	Sources (amongst others ¹⁾)
Coal	BP [2018] , World Bank [2018a]
Crude oil	IEA [2018b] , OECD [2018]
Natural gas	Department for Business, Energy and Industrial Strategy [2018b] , Eurostat [2018b]
Oil products ²⁾	European Commission [2018b] , IEA [2018c] , German Agency for International Cooperation (GIZ) [2018]
Electricity	Eurostat [2018a] , Department for Business, Energy and Industrial Strategy [2018a]
Heat	Eurostat [2018c]

Notes: 1) Other data sources, particularly outside the EU region, are national ministries of energy, national statistical offices, country reports, surveys and news sites; Gaps in price data are closed by estimates; 2) Simple approximation using diesel prices.

Source: Own elaboration.

(3) Value added: GDP and industry value added generation (in current prices) are sourced from the [World Bank \[2018b\]](#). Its database covers more than 250 countries and regions, including all OECD countries, and the world totals.

Supplementary calculations for [Figure 4](#): The title of this paper promises the analysis of total RUEC, i.e., direct plus indirect RUEC. Surely, the data set described above allows for calculating (direct) RUEC. However, the high level of sectoral aggregation, distinguishing only two sectors (industry and other sectors), leads to a loss of information and makes the calculation of indirect RUEC problematic. Nevertheless, indirect RUEC trends are included in the figure as (indicative) trends. The actual computation of indirect energy costs requires world input-output tables with the same sectoral breakdown as the energy quantities and prices. For this

¹⁶Final energy consumption in road transport was roughly allocated in the ratio of final energy consumption in industry to total final energy consumption.

purpose, the world input-output tables 2009-2014 of the WIOD release 2016 (44 regions, 56 sectors per country) are aggregated into 41 regions (the ones of the WIOD release 2013) and two sectors (“industry” and “other sectors”). To cover the entire period from 2009 to 2016, the most recent world input-output table for 2014 is updated with [World Bank \[2018b\]](#) data and rebalanced with the widely-accepted RAS procedure [[Miller and Blair, 2009](#)]. For the sake of comparison, the world input-output tables for 1995-2009 of the WIOD release 2013 (41 regions, 35 sectors per country) are aggregated in the same way (41 regions, 2 sectors).

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