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Economic implications of phantom traffic jams: Evidence from traffic experiments*

By Kathrin Goldmann and Gernot Sieg†

Traffic jams occur even without bottlenecks, simply because of interaction of vehicles on the road. From a driver's point of view, the instability of the traffic flow arises stochastically. Because the probability of a traffic jam increases with the number of cars on the road, there is a traffic flow breakdown externality. This paper offers a method to calculate this externality for traffic on a circuit. Ignoring the stochastic nature of traffic flow breakdowns results in congestion charges that are too small.

Keywords: Hypercongestion, congestion costs, circuit, stochastic capacity, external costs, congestion charge, traffic experiments

I. Introduction

Freeway capacity has been defined as the maximum flow rate that can reasonably be expected to traverse a facility under prevailing roadway, traffic and control conditions. The traditional view (Small and Chu, 2003; Button, 2004) is that, with an increasing number of vehicles on the road, vehicles affect each others' speeds and slow each other down. As more traffic enters the road, average speed falls, but up to a point, the flow will continue to rise, because the effect of additional vehicles outweighs the reduction in average speed. This is the congested branch of the speed-flow curve. At the point where increased demand does not increase traffic flow any further, the road's capacity is reached. The flow becomes unstable, with the characteristic stop-and-go conditions which are typical of traffic jams, and constitutes a state called hypercongestion in economics. The reasons for traffic jams could be on the demand side (on-ramps with high inflows, fluctuations in demand) or on the supply side (traffic accidents, construction sites, tunnels or inhomogeneous road design).

However, Sugiyama et al. (2008) showed that even in the absence of supply side reasons, traffic jams (hypercongestion) can occur. For this to happen, it is sufficient that drivers on a street interact with each other to make the traffic flow unstable. There may be deterministic reasons like tailgating, excessively fast reactions to speed changes, slow overtaking by trucks, slow reactions because of inattentiveness or queue-jumping, but within the system, these driving errors occur stochastically (Schönhof and Helbing, 2007). However, some of these factors culminate in a traffic jam, but some do not. The probability of their causing a traffic jam increases with the saturation of the highway, so that capacity is stochastic (Elefteriadou et al., 1995; Brilon et al., 2005).

A driver only considers his own costs, but not the time losses other drivers

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have due to increased traffic. We determine the external costs imposed on other drivers. Drivers can be faced with a free flow or congested traffic on good days, and hypercongested traffic on bad days. Average travel speed and average travel times differ greatly between the two states. For these reasons, a driver entering the road in order to travel a certain distance faces a stochastic travel time, depending on the number of other vehicles on the road. We identify a so-far academically ignored externality of an additional driver on the road. The driver increases the probability of hypercongestion, a state with inefficiently long travel times.

Verhoef (1999) shows that hypercongestion is dynamically infeasible when considering capacity as deterministic. In order to depict hypercongestion in a static model with continuous demand, inflows onto the road must have exceeded the maximum possible inflow at some point in the past, which is inconsistent with the concept of maximum deterministic capacity. Small and Chu (2003) suggest that hypercongestion on a highway entails a queue of cars waiting in front of a bottleneck. Therefore, density within the queue does not exert an effect on the outflow rate from the bottleneck and on travel time, but only on the number of cars waiting. Following this interpretation, Small and Chu (2003, p. 326) state that hypercongestion is irrelevant to users who care only about total travel time.

For bottlenecks and urban streets, there are already models that analyze hypercongestion. A few papers have used the bottleneck model (Vickrey, 1969; Small, 2015) for analyzing hypercongestion, by postulating that bottleneck capacity varies with the length of the queue. Yang and Huang (1997) identify a dynamic externality, that is, the manner in which an additional car influences the queue length and therefore the bottleneck efficiency, but do not consider inefficient hypercongestion states such as stop-and-go traffic, but only queues. Consequently, Yang and Huang (1998) include a queuing externality in the congestion charge and suggest calculating the flow-dependent travel time and the queuing delay separately, with the former being predicted by an analytical delay formula, and the latter determined from network equilibrium conditions. The bottleneck model was extended to stochastic capacity by Lindsey (1994), Arnott et al. (1999) and Fosgerau (2010). Demand and bottleneck capacity is assumed to fluctuate from day to day, but as soon as a given day proceeds, they remain constant for the travel period on this day. For this reason, Lindsey (1994) states that the model can adequately display capacity fluctuations due to roadwork, weather conditions and major truck accidents, but not temporary capacity fluctuations.

The bathtub model (Arnott, 2013; Fosgerau and Small, 2013; Fosgerau, 2015; Arnott et al., 2016) analyzes urban hypercongestion. A backward-bending fundamental diagram of traffic flow also applies at the level of an urban neighborhood, which meets certain conditions (Daganzo et al., 2011). As a result, urban congestion can be analyzed in aggregated form, using a speed-flow relationship.

However, we analyze a different type of congestion technology, a unidirectional flow (on a circuit) without bottlenecks. Those traffic jams are commonly referred to as phantom jams, as they can occur without any sort of bottleneck that reduces road capacity. This congestion technology has not yet been analyzed with an economic model.

To obtain a model that is theoretically consistent, we focus on a predetermined number of homogenous drivers aiming to travel at the same speed on a circuit, that is, a circular street without a beginning or an end. Sugiyama et al. (2008), Nakayama et al. (2009) and Tadaki et al. (2013) performed traffic experiments on just such a circuit to investigate the emergence of a jam without a bottleneck.

Their experiments are depicted in Figure 1. Tadaki et al. (2013) let 10 to 40 homogenous cars enter one by one a circuit of 314 m length, with the driver of the first car requested to drive slowly until all the cars have entered. After that, all drivers are instructed to drive at a homogenous target speed of 30 km/h. When the number of cars N is low, $10 \leq N \leq 25$, they observe free flow. If the number of cars exceeds 32, the flow jams. However, if the number of cars is within a medium range, $26 \leq N \leq 31$, they detect metastable phases of stop-and-go traffic in which cars stop or nearly stop in jam clusters, alternating between escaping from the jam cluster and again catching up with it later on.

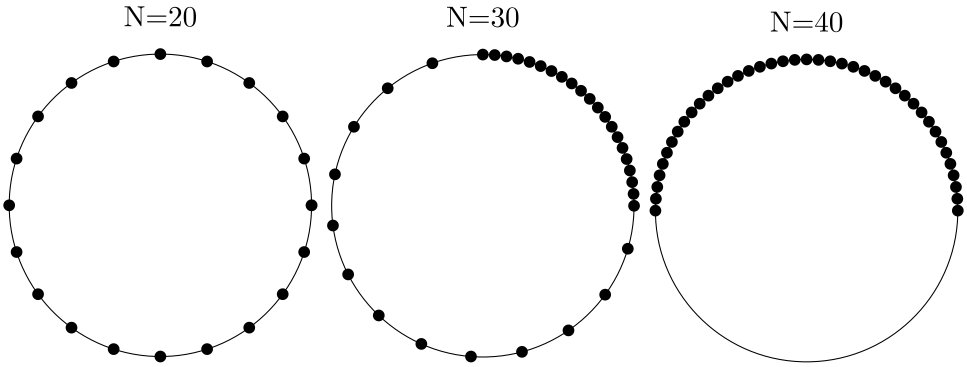

FIGURE 1. DEPICTION OF TRAFFIC EXPERIMENTS ON A CIRCUIT INITIALLY PERFORMED BY TADAKI ET AL. (2013)

II. The Model

The expected travel speed of a driver depends on the traffic situation on the circuit. If the number of vehicles is small, drivers enjoy free flow and can travel at the speed they want. If the number of vehicles is large, drivers are stuck in a traffic jam and travel speed is low. However, in between, there is an area where both states alternate. To calculate the expected travel speed, we consider the two traffic states identified by Tadaki et al. (2013), namely free-flow and jammed. Depending on the number of vehicles N on the circuit, both states alternate, and from the point of view of a driver, the traffic is either fluid and the average travel speed is $\overline{v}(N)$ or jammed at a speed of $\underline{v}(N)$ with the difference of $\Delta_v = \overline{v} - \underline{v}$. The probability of jammed traffic is p which also depends on the number of other drivers on the circuit, because the more vehicles, the larger the probability that the traffic is jammed. A driver on the circuit expects a travel speed[1] of

$$E(v) = p(N)\underline{v}(N) + (1 - p(N))\overline{v}(N).$$

[1] In this calculation, we assume that the probability is calculated in such a way that both the free flow and the jammed traffic holds for a period of time that is long enough for drivers to travel the distance in question, for example, a whole circuit.

If we increase the number of vehicles on the circuit, the expected speed changes and the marginal effect is

$$\begin{aligned}\frac{dv}{dN} &= (1-p)\frac{d\overline{v}}{dN} + p\frac{d\underline{v}}{dN} + \frac{dp}{dN}(\underline{v}(N) - \overline{v}(N)) \\ &= \underbrace{\frac{d\overline{v}}{dN}}_{*} - \underbrace{\left[p\frac{d\Delta_v}{dN} + \frac{dp}{dN}\Delta_v\right]}_{\text{traffic flow breakdown effect}} \quad .\end{aligned} \tag{1}$$

The first term ($*$) on the right hand side of this equation represents the expected loss of speed if an additional car enters the circuit, only if capacity is considered to be deterministic. Because hypercongestion theoretically seems to be unfeasible on the circuit (no excessive entry) and there is no bottleneck on the circuit, the first term ($*$) on the right hand side, i.e. only the function $\bar{v}$, is then used to calculate congestion externalities (Santos and Verhoef, 2011). However, as Tadaki et al. (2013) have shown, hypercongestion does occur and therefore a hypercongestion adjustment to the amount of the traffic-flow breakdown effect is needed to determine the full loss of speed an additional car generates.

The traffic breakdown probability is not always positive. In the experiment of Tadaki et al. (2013) there were no jams but always free flow for small numbers of cars, i.e. $N \leq 25$. If the N is small, the break down probability equals zero and there are no traffic-flow breakdown effects to be considered. If the number of cars exceeds 32, the flow jams. If N is large, the breakdown probability equals one and does not change if additional cars enter the circuit. However, if the number of cars is within a medium range, $26 \leq N \leq 31$, metastable phases of stop-and-go traffic in which cars stop or nearly stop in jam clusters, alternating between escaping from the jam cluster and again catching up with it, it is necessary to include the full adjustment in calculating the effect an additional driver induces when entering the circuit.

III. Calculation of the congestion externality

Travel time costs c depend on the speed, which in turn depends on the number of vehicles on the circuit, and the the expected travel time costs C of a driver are

$$C(N) = p(N)c(\underline{v}(N)) + (1-p(N))c(\bar{v}(N))$$

and when we assume homogenous drivers, these costs are the average costs of all N drivers on the circuit. Social costs are $SC = N \cdot C(N)$ and marginal social costs are $MSC = C + N \cdot C'$. If we were to pay drivers for successfully completing circuits, and if we allowed free entry to the circuit, drivers would take part if their travel time cost C for a circuit is less than the amount we pay. For this decision $N \cdot C'$ is the external effect (on other drivers), which is not taken into account by individual drivers. The marginal external travel time costs are:

$$N\frac{dC}{dN} = N\left[(1-p)\cdot\frac{dc}{dv}\frac{d\overline{v}}{dN} + p\cdot\frac{dc}{dv}\frac{d\underline{v}}{dN} + \frac{dp}{dN}\left(c(\underline{v}(N)) - c(\overline{v}(N))\right)\right].$$

Considering a time value of t, $c(v) = t/v$ and $dc/dv = -t/v^2$, this equation can be written as:

$$N\frac{dC}{dN} = N\left[(1-p)\cdot\frac{(-t)}{\bar{v}^2}\frac{d\bar{v}}{dN} - p\frac{t}{\underline{v}^2}\cdot\frac{d\underline{v}}{dN} + \frac{dp}{dN}\left(\frac{t}{\underline{v}} - \frac{t}{\bar{v}}\right)\right].$$

To summarize, marginal external travel costs on the circuit equal

(2)
$$N\frac{dC}{dN} = \underbrace{-Nt\frac{1}{\bar{v}^2}\frac{d\bar{v}}{dN}}_{\text{Deterministic congestion costs}} \underbrace{-Ntp\left(\frac{1}{\underline{v}^2}\frac{d\underline{v}}{dN} - \frac{1}{\bar{v}^2}\frac{d\bar{v}}{dN}\right) + Nt\frac{dp}{dN}\left(\frac{1}{\underline{v}} - \frac{1}{\bar{v}}\right)}_{\text{Stochastic hypercongestion adjustment}}$$

and we can state:

PROPOSITION 1: *Ignoring the stochastic nature of traffic flow breakdowns by considering capacity as deterministic may underestimate the congestion externality.*

It is worth noting that the number of vehicles on the circuit is fixed and therefore, density on the circuit is constant and does not increase when traffic flow breaks down. Therefore, if we consider traffic on the circuit in Equation 2, N is proportional to the density on the circuit.

IV. Application to traffic data

This section calculates marginal external congestion costs on a circuit with a length of 314 m using the data generated in the experiment of Tadaki et al. (2013). They present the number of cars on the circuit as well as the flow-density data for sessions with free flow, jammed flow and intermediate states. Because we focus on the stochastic nature of traffic flow breakdowns, the intermediate sessions are of particular interest to us. We are able to calculate the expected travel time losses for all sessions (the left hand side of Equation 2) as well as the expected travel time losses for the free flow sessions (deterministic congestion costs in Equation 2). It is then straightforward to deduce the stochastic hypercongestion adjustment needed to calculate the full external costs of an additional vehicle. For numbers of vehicles that induce phantom jams, the stochastic adjustment is about as large as the deterministic component. Therefore, ignoring the stochastic effect underestimates the externality by about one half.

Figure 2 shows the marginal external costs in percent of time costs per hour. If the time costs t are assumed to be 9.19 Euro per hour, the German minimum wage, marginal external costs are between zero and 50 cents per circuit.

In Sessions with 10, 12, and 20, cars traffic flow did not break down and we therefore assume that the probability of a traffic breakdown p equals 0. From Equation 2 we can see that when p equals 0, only the deterministic external marginal congestion costs apply. When the number of cars on the circuit is increased to 25, 28 or 30, we observe both free flow and jammed flow, and the stochastic hypercongestion adjustment becomes positive. Finally, when N exceeds 31, traffic will be jammed and p equals 1. In this case we observe both effects, the marginal speed losses due to additional drivers and the additional speed losses due to stable jam patterns that can be regarded as stop-and-go patterns at a much lower speed than in the intermediate states.

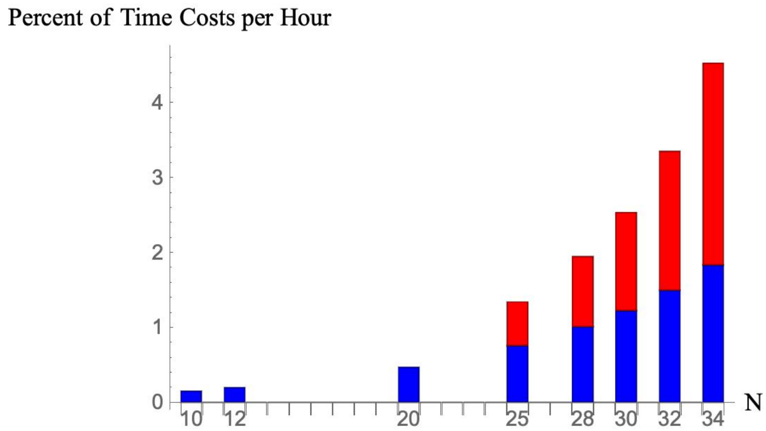

FIGURE 2. DETERMINISTIC CONGESTION COSTS (BLUE) AND STOCHASTIC HYPERCONGESTION ADJUSTMENT (RED) PER CIRCUIT

If we were to pay drivers a premium for successfully completing circuits, and if we allowed free entry to the circuit, drivers would take part if their travel time cost for a circuit were less than the amount we pay. Figure 3 shows the private costs C of completing a circuit, the marginal external costs MEC_d calculated using a deterministic approach, and the correct marginal external costs MEC_s using our stochastic approach. Assumed are 9.19 Euro opportunity costs of one hour driving and a payment of p = 31 cent for each circuit completed.

Free entry of drivers results in a total of 40 vehicles. Drivers do not take into account that they slow down the other cars on the circuit, and an optimal user charge internalizing this effect (MEC_d) could reduce the number of vehicles to 32. Furthermore, each additional vehicle also increases the probability of phantom traffic jams. The welfare-maximizing user charge must therefore internalize the complete (stochastic) marginal external costs (MEC_s), and implementation leads to a total of 28 vehicles.

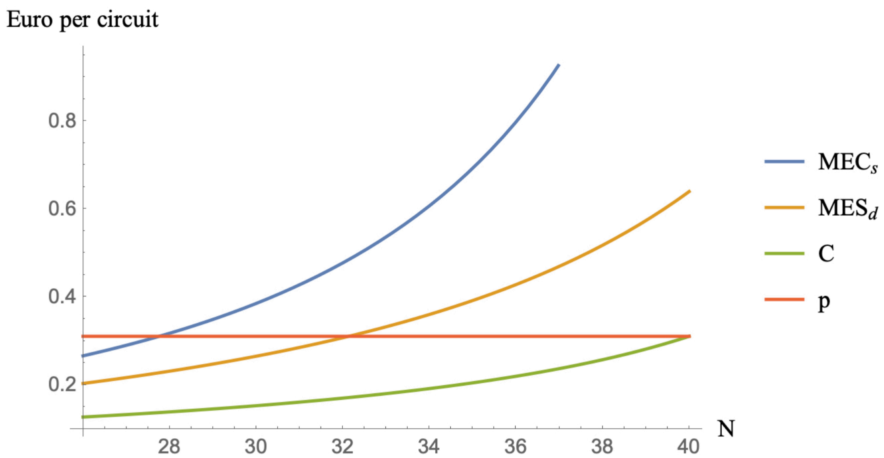

FIGURE 3. COSTS, MARGINAL EXTERNAL COSTS AND FREE ENTRY EQUILIBRIA

V. Conclusion and Discussion

Hypercongestion can occur as a transient response to a demand spike (Arnott, 1990, p. 200), or as a transient reduction in capacity (due, for example, to a traffic accident), or as a queue at a bottleneck (Small and Chu, 2003). Bottleneck models have also been modified with stochastic demand and capacity. We focus on hypercongestion that occurs in a non-linear system without identifiable reasons, and therefore assume stochastic road capacity without bottlenecks.

Departing from traffic experiments and a constant number of drivers, we set up a simple static model with random traffic jam formations. By doing so, congestion and hypercongestion costs can be calculated. We identify a previously ignored externality. An additional vehicle not only reduces the speed of other vehicles, but also increases the probability of a traffic jam developing. This is an important effect that needs to be quantified when calculating the external costs of hypercongestion.

Calculating the deterministic congestion costs and the stochastic hypercongestion costs of phantom jams, we show that the latter costs are roughly as large as the deterministic congestion costs. Ignoring the externality of a phantom jam, will underestimate total costs by approximately 50 %. If we apply this to drivers participating in this traffic experiment, we can see that failing to consider the possibility of phantom jams occurring, leads to too many cars entering the circuit. Applied to highway traffic, ignoring this externality leads to insufficiently high road usage. However, the extent of the phantom jam externality depends on the specific traffic flow conditions on the circuit or on specific highway sections. The impact of phantom jams on a circuit, however, might be larger and more stable than on road sections in reality. Future research should try to quantify the difference.

In experiments, all data that is needed can be collected and it is therefore easy to calculate probabilities of traffic breakdowns, expected speeds and thus the externality an additional driver imposes on the circuit. While in reality, traffic sensors often collect the required flow and speed data, investigated road sections have to meet certain conditions. Most importantly, there should not be a bottleneck directly downstream from the traffic detector, as we would then also measure queues instead of only stochastic jam formations. For this reason, traffic breakdown externalities, as observed in experiments, can be calculated for road sections where traffic detector data is available and that meet the abovementioned conditions.

Our model is a static one based on a specific number of vehicles on the circuit. Real roads differ, because the number of vehicles and the density also change. The outflow of one part of the road is the inflow of the succeeding part. On real roads, bottlenecks occur stochastically, for example, when an accident decreases capacity. Therefore, our approach only captures one aspect of real highway congestion. However, modeling phantom traffic jams as bottlenecks is misleading. Moreover, relatively short time intervals of traffic data can be used when traffic conditions can be assumed to be relatively constant within the intervals. Traffic conditions must be regarded as a chain of steady-states in this model framework. Of course, we are unable to analyze transitions between traffic states, which might be of interest to traffic engineers.

Applying our approach to real roads, for example highways, calculating congestion costs only requires a knowledge of speed-flow functions and flow-dependent breakdown probabilities. This information can easily be extracted from high-frequency traffic data. Furthermore, the capacity drop observed in traffic jams on highways can also be included in an empirical application of our model. Based on the traffic experiments, our model makes the simplifying assumption that vehicles on the road are homogenous. Further research should include vehicle heterogeneity, for instance cars and trucks, as the latter have different acceleration capabilities and thus impact differently the traffic flow conditions. Moreover, we only consider travel time costs and ignore cost components like increased fuel consumption and carbon dioxide emissions, as well as safety issues.

VI. References

REFERENCES

Arnott, R. (1990). Signalized intersection queuing theory and central business district auto congestion. *Economics Letters*, 33:197–201.

Arnott, R. (2013). A bathtub model of downtown traffic congestion. *Journal of Urban Economics*, 76(1):110–121.

Arnott, R., de Palma, A., and Lindsey, R. (1999). Information and time-of-usage decisions in the bottleneck model with stochastic capacity and demand. *European Economic Review*, 43:525–548.

Arnott, R., Kokoza, A., and Naji, M. (2016). Equilibrium traffic dynamics in a bathtub model: A special case. *Economics of Transportation*, 7-8:38–52.

Brilon, W., Geistefeldt, J., and Regler, M. (2005). Reliability of freeway traffic flow: A stochastic concept of capacity. In *Prooceedings of the 16th International Symposium on Transportation and Traffic Theory*, pages 125–144.

Button, K. (2004). The rationale for road pricing: Standard theory and latest advances. *Research in Transportation Economics*, 9:3–25.

Daganzo, C. F., Gayah, V. V., and Gonzales, E. J. (2011). Macroscopic relations of urban traffic variables: bifurcations, multivaluedness and instability. *Transportation Research Part B: Methodological*, 45(1):278–288.

Elefteriadou, L., Roess, R. P., and McShane, W. R. (1995). Probabilistic nature of breakdown at freeway merge junctions. *Transportation Research Record*, 1484:80–89.

Fosgerau, M. (2010). On the relation between the mean and variance of delay in dynamic queues with random capacity and demand. *Journal of Economic Dynamics and Control*, 34:598–603.

Fosgerau, M. (2015). Congestion in the bathtub. *Economics of Transportation*, 4(4):241–255.

Fosgerau, M. and Small, K. A. (2013). Hypercongestion in downtown metropolis. *Journal of Urban Economics*, 76(1):122–134.

Lindsey, R. (1994). Optimal departure scheduling for the morning rush hour when capacity is uncertain. *Presented at the 41st North American meeting of the Regional Science Association, Niagara Falls, Ontario, 17-20 November.*

Nakayama, A., Fukui, M., Kikuchi, M., Hasebe, K., Nishinari, K., Sugiyama, Y., Tadaki, S., and Yukawa, S. (2009). Metastability in the formation of an experimental traffic jam. *New Journal of Physics*, 11(083025).

Santos, G. and Verhoef, E. (2011). Road congestion pricing. *In: A Handbook of Transport Economics*, edited by de Palma, A., Lindsey, R., Quinet, E. and Vickerman, R., 561–585. Cheltenham: Edward Elgar.

Schönhof, M. and Helbing, D. (2007). Empirical features of congested traffic states and their implications for traffic modeling. *Transportation Science*, 41(2):135–166.

Small, K. A. (2015). The bottleneck model: an assessment and interpretation. *Economics of Transportation*, 4(1–2):110–117.

Small, K. A. and Chu, X. (2003). Hypercongestion. *Journal of Transport Economics and Policy*, 37(3):319–352.

Sugiyama, Y., Fukui, M., Kikuchi, M., Hasebe, K., Nakayama, A., Nishinari, K., Tadaki, S.-i., and Yukawa, S. (2008). Traffic jams without bottlenecks — experimental evidence for the physical mechanism of the formation of a jam. *New Journal of Physics*, 10(033001).

Tadaki, S., Kikuchi, M., Fukui, M., Nakayama, A., Nishinari, K., Shibata, A., Sugiyama, Y., Yosida, T., and Yukawa, S. (2013). Phase transition in traffic jam experiments on a circuit. *New Journal of Physics*, 15(103034).

Verhoef, E. T. (1999). Time, speeds, flows and densities in static models of road traffic congestion and congestion pricing. *Regional Science and Urban Economics*, 29(3):341–369.

Vickrey, W. S. (1969). Congestion theory and transport investment. *American Economic Review*, 59:251–261.

Yang, H. and Huang, H. (1997). Analysis of the time-varying pricing of a bottleneck with elastic demand using optimal control theory. *Transportation Research Part B: Methodological*, 31(6):425–440.

Yang, H. and Huang, H. (1998). Principle of marginal-cost pricing: How does it work in a general road network? *Transportation Research Part A: Policy and Practice*, 32(1):45–54.

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