A Quantitative Liquidity Portfolio Model for Credit Facilities

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Abstract

As of late, regulators expect financial institutions to explicitly account for portfolio liquidity risk effects of contingent off-balance sheet items such as credit lines, as part of their liquidity management framework. Consequently, risks arising from (i) the development of underlying risk factors as well as (ii) potentially unobservable triggering events, need to be considered. We specify and implement a portfolio model for the assessment of portfolio funding risk for credit facilities, which directly addresses these portfolio risk components in a liquidity risk framework: We explicate the admission of multiple liquidity risk drivers for single credit facilities based on a factor modeling approach and further admit a portfolio correlation structure at the draw-down level. The model setup is implemented for a unique dataset of revolving credit facilities from a Europe-based bank. To the best of our knowledge, we are the first to specify and implement a portfolio model which allows an explicit calibration of the stated risk components. We demonstrate the model's capability to handle both business as usual applications as well as a number of economically suggestive stress tests. The proposed model provides a further step towards a quantitative liquidity risk management framework in financial institutions.

Key words: Credit Facility, Line of Credit, Loan Commitment, Liquidity Risk, Funding Risk, Portfolio Model

JEL Classification: C53, G21, G32.

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1 Introduction

As a consequence of the financial crisis, regulatory requirements for the management of liquidity risk were strengthened and extended by regulators around the world. The aftermath of the financial meltdown revealed that in the past numerous banks had not even considered the most basic principles for a sound liquidity management framework. Consequently, banks are now (finally) required to establish a robust liquidity management framework to withstand a range of stress events and ensure sufficiently high liquidity reserves.[1] Regulators have defined quantitative measures (Pillar 1 of Basel) as well as qualitative requirements (Pillar 2 of Basel) for the management of liquidity risk and in particular funding risk.[2] Qualitative requirements also refer to the management of off-balance sheet products including (but not constrained to) credit lines which may trigger further funding demand.[3] In that regard, a portfolio model for credit and liquidity facilities serves as an important building block for a robust internal liquidity risk management framework:

- As part of the business as usual liquidity management framework (going concern perspective), the expected volume of future draw-downs of facilities[4] should be considered and needs to be taken into account within a forecast of future cash flows under normal conditions.[5]
- As a minimum requirement, financial institutions need to consider potential deviations from expected future draw-downs of facilities, i.e., *contingent future draw-downs*[6], as part of their (liquidity) stress testing framework.[7] Considering that risks

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[1] See BIS (2008).

[2] It is very common to distinguish liquidity risk in terms of funding liquidity risk and market liquidity risk. "Funding liquidity risk is the risk that the firm will not be able to meet efficiently both expected and unexpected current and future cash flow and collateral needs without affecting either daily operations or the financial condition of the firm. Market liquidity risk is the risk that a firm cannot easily offset or eliminate a position at the market price because of inadequate market depth or market disruption." (BIS (2008)).

[3] See BIS (2008).

[4] In the following, we will refer to *expected future draw-downs*.

[5] See BIS (2008), p.11.

[6] In addition, the need for business as usual considerations of contingent future draw-downs depends on the management framework.

[7] See BIS (2008), p.12 and CEBS (2010b), p. 8.

arising from (potential) future draw-downs[8] *exceeding* the combined level of expected and contingent future draw-downs remain unfunded, an institution's risk tolerance is directly reflected in the calibration of contingent future draw-downs.

- Explicit consideration of risk factors (e.g. the credit worthiness of the counterparts) as well as concentrations of the underlying portfolio (e.g. geographical sectors and industries) are to be taken into account, as required for Pillar 2 models.[9]

Regulatory requirements can be divided into quantitative and qualitative measures. Quantitative measures give a first feeling for the regulators' view of funding risk on a portfolio level with respect to specific products. Notably, Basel III liquidity ratios include drawing percentages for committed credit and liquidity facilities.[10] Thereby, we define liquidity and credit facilities on the basis of BIS (2010). These facilities are distinguished according to the purpose for which a customer holds such a line. Consequently, a liquidity facility is defined as a back-up facility purely put in place for the purpose of refinancing debt in tense situations, i.e., when the customer is unable to attract funding in the normal course of business from financial markets. Borderd to that, credit facilities are defined as general working capital facilities (e.g. revolving credit facilities in place for general and/or working capital purposes).[11] Qualitative requirements (Pillar 2 of Basel) offer general guidelines for the management of funding risk regarding specific products. Principles for a sound liquidity management framework were laid out on a European level in detail and in comprehensive form in BIS (2008). According to CEBS (2009) and CEBS (2010a), credit and liquidity facilities (as part of contingent off-balance sheet items) play a major role in the implementation of liquidity risk management systems. In particular, liquidity effects arising from these products have to be taken into account in a going concern perspective as well as in liquidity stress tests. Additionally, liquidity risk concentrations (e.g. regarding counterparts, industries, products or geographical regions) as well as interrisk concentrations need to be considered. Hence, the modeling of correlations and diversification effects for a portfolio of facilities is of major importance. On the basis of such models, banks have to derive both the relevant volume of liquidity reserves to be held and the necessary

[8] Expected and contingent future draw-downs need to be covered by a financial institution through her funding. While expected future draw-downs are generally (pre-)funded, contingent future draw-downs could be met via a sound liquidity reserve (see CEBS (2010b), p. 8.).

[9] See BIS (2008), p. 12 and CEBS (2010a).

[10] Additionally, in the past (some) national regulatory authorities have defined liquidity ratios to be included in regulatory reports. For instance, the German regulator defines liquidity ratios in its liquidity regulation LiqV (2006). These include the assumption of a liquidity outflow of 20% for the total undrawn volume of committed credit and liquidity facilities, based on a time-horizon of one year.

[11] The terms credit facility, credit line and line of credit are used synonymously.

liquidity inflows to meet expected liquidity outflows (see e.g., CEBS (2010a)). Since the aforementioned requirements are guidelines on a European level, further transformation to a national level is needed. For instance, German requirements are phrased in BaFin (2010).[12]

Aside from regulatory rules and guidelines, current (theoretical) literature on models regarding credit and liquidity facilities can be broadly divided into explanatory models which analyze characteristics driving the utilization of single facilities (i.e., analyses of risk factors underlying specific datasets) and forecast models for the utilization of such facilities on a portfolio level. While much literature is available for the former strand,[13] only a few papers address the latter topic. Duffy *et al.* (2005) are the first to employ credit portfolio modeling techniques for the measurement of funding risk for credit facilities and implement these on a real dataset. However, their model mainly relies on the credit rating of a customer as single driver of credit line utilization. Furthermore, dependencies between draw-down distributions between different lines are not reflected. Heidorn *et al.* (2008) also specify and implement a portfolio model for specific credit facilities, but complectly omit portfolio diversification effects, due to various data constraints. Consequently, this model is not suitable to adequately capture liquidity risk on a portfolio level. Klein and Sarin (2012) establish a theoretical portfolio environment admitting a number of environmental parameter constellations. They analyze the performance of various portfolio approaches for a range of plausible environmental scenarios and conclude that a specific Monte-Carlo simulation approach dominates other forecast models currently discussed in the literature.[14]

While Duffy *et al.* (2005) and Heidorn *et al.* (2008) do not resolve or consider all critical issues as they calibrate their model to real data, Klein and Sarin (2012) derive their results from a purely theoretical setup. We will close this gap in the current literature and resolve practical issues arising from specification and implementation of a funding risk model for a real portfolio based on the results of Klein and Sarin (2012). First, we introduce *more than one* risk factor driving the utilization of credit lines on an single deal level and on the portfolio level. Risk factors are simulated on the basis of models for the

[12] The latest revision of MaRisk has been conducted on December 15th, 2010, and includes the previously mentioned aspects of the cited CEBS-Guidelines.A recent consultation paper regarding MaRisk was published on April 26th, 2012 (see BaFin (2012)).

[13] Introductions to explanatory models for credit line behavior are, e.g., the works by Agarwal *et al.* (2006) and Sufi (2009).

[14] For a more detailed overview see Section 2. We consider the works of Duffy *et al.* (2005), Heidorn *et al.* (2008) and Klein and Sarin (2012) as most relevant for our research.

management of (portfolio) credit risk such as Credit Metrics^{TM}.[15] Given the realization of such risk factors per credit line, we generate draw-downs from utilization distributions which follow a dependency structure that has to the best of our knowledge not been considered in theoretical literature regarding funding risk. To handle the complexity of such a dependency structure, we propose an innovative complexity reduction algorithm. Once the model is specified, we present practical applications for both business as usual and stress testing assignments. Consequently our model provides a further step towards a quantitative liquidity risk management framework for financial institutions.

The content of this paper is organized as follows: In Section 2 we propose risk factors driving the utilization of credit lines both on a single line level as well as on the portfolio level. On the basis of these results we develop a Monte-Carlo simulation approach for the assessment of funding risk characteristics, explicitly taking the risk factors and possible additional dependency structures of the portfolio into account. We discuss the results of the approach for normal as well as stressed (market) conditions in Section 3. Section 4 winds up with some concluding remarks.

[15] There exists a vast body of literature concerning credit risk models. Seminal papers include Gupton *et al.* (1997), Crouhy *et al.* (2000) and Bluhm *et al.* (2002).

2 Model

This section aims to provide a method for deriving an estimate for future portfolio draw-down distributions, based on a 'real' data set. Theoretical literature on such forecast models is rare.

Duffy *et al.* (2005) are the first to derive a model for the measurement of potential funding risk on a portfolio level, but only for liquidity facilities connected to investment accounts at Merrill Lynch. A Monte-Carlo simulation approach on the basis of a Mark-to-Market(MtM)-Credit Portfolio Model is used to derive potential portfolio liquidity effects. Given the nature of the portfolio, which mainly consists of liquidity facilities particularly used as 'back-up facilities', Duffy *et al.* (2005) utilize the credit rating as major driver for their model. Further risk drivers for the utilization of facilities are not derived in their work. It is therefore questionable, in how far the proposed model could generate useful results for facilities with different properties, such as credit lines. Moreover, Duffy *et al.* (2005) do not consider dependencies between the draw-down distributions of single credit lines. Additionally, the impact of specific model parameters is not analyzed.

In analogy to Duffy *et al.* (2005), Paulsen (2007) formulates a model for liquidity risk regarding liquidity and credit facilities. The draw-downs of these facilities are based on the realization of *one* risk factor (the credit spread or the rating of the corresponding customer), a method commonly applied for MtM-Credit Portfolio Models. While Duffy *et al.* (2005) allow for flexible independent drawings on the basis of the realization of such risk factors, Paulsen (2007) assumes either a full draw-down or zero utilization per credit line. In addition, he analyzes the impact of some input parameters on the portfolio distribution. The model is neither tested against real data sets nor does it address the need for stress testing at financial institutions.

Heidorn *et al.* (2008) analyze how to set up and calibrate models for specific portfolios of facilities (i.e., specific sub-portfolios of term and rolling credit facilities) in the context of an industry project, suffering the typical data constraints. Due to incomplete data histories, the authors employ the historical volatility of the respective single facilities as the basis for the measurement of portfolio volatility, omitting diversification effects of draw-downs between single facilities. The portfolio model approach is hence not driven by specific risk drivers or other characteristics of the facilities.

All these models have in common that they do not combine methods for the development of a portfolio measure and methods for the analysis of the characteristics of a specific data set (i.e., explanatory models to identify risk drivers of the underlying portfolio). We construct a forecast model based on the results from Klein and Sarin (2012) and will explicitly consider the risk drivers.

Klein and Sarin (2012) review these models and conclude that a Monte-Carlo simulation approach yields the most favorable results in a fairly general theoretical setup. In particular their setup aims to evaluate the forecast quality of such models via out-of the sample tests for different types of constructed data sets. However, their model does not rely on real data. We will highlight and resolve the practical issues arising from the specification and implementation on real data sets and refer to Klein and Sarin (2012) for theoretical tests regarding the explicit forecast quality of our model.

2.1 General setup

We consider a time horizon of 1 month, which is a natural building block for a sound liquidity risk management framework according to regulators.[16] Credit line information in the data set is stored on a monthly basis with data availability starting on June 30th, 2009. The latest entries used for the calibration of our model dates from July 30th, 2010, resulting in a data history containing 14 dates. For our purpose, we assign a unique number to each credit line. *If* credit line no. i exists at the t-th date[17], the documented draw-down is denoted by $d_{i,t}$.

Credit line information[18] includes (historical) credit line draw-downs, each line's granted volume, customer ratings, collateral values and information regarding the industry sector of the customer. Based on the information available at $t = 14$, we are interested in estimating the portfolio draw-down $\sum_i \widetilde{d}_{i,15}$.[19] We simulate the distribution of the total portfolio draw-down in the following five steps, which will be described and motivated in detail in sections 2.3 through 2.7:

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16 See BaFin (2010), BTR 3.2. and BIS (2010).

17 According to the chronological order of all 14 dates.

18 A detailed description of the data set is given in section 2.2.

19 Note that $\widetilde{d}_{i,15}$ is a random variable describing the draw-down of line i on August 31^{st}, 2010 ($t = 15$).

1. For each credit line belonging to the portfolio, the corresponding customer's future[20] rating is simulated based on the initial rating, a migration matrix as well as a vector of weights for systematic factors associated with the customer's business.

2. For each credit line, a future collateral status is simulated based on the initial collateral status and a corresponding migration matrix. The collateral status denotes whether a credit line is (at least partly) collateralized.

3. Draw-down distributions for each credit line are generated according to the simulated 'rating/collateral status' combination.

4. The calibration of cluster-specific draw-down distributions is based on historical data. We introduce different families of distributions for the approximation of historical draw-downs (i.e., the actual historical distribution as well as normal and gamma distributions are applied) to analyze whether the choice of a potentially unrealistic distributional form has a significant impact on the portfolio measures.

5. We estimate a correlation structure from historical line utilizations. A portfolio draw-down is generated from the marginal distributions obtained in step 4, according to this correlation structure.

Steps $1.-5.$ provide us with *one* value for the portfolio draw-down at $t = 15$. (Independent) repetition of steps $1.-5.$ via Monte-Carlo simulation yields a distribution for the portfolio. Before we outline our model in detail, we describe the underlying data set and motivate the utilization of the risk factors 'rating' and 'collateral status' in the following section.

2.2 Data description and motivation of risk factors

We employ a comprehensive data set of specific credit lines from a Europe-based bank. The data contains single deal information for credit and liquidity facilities. We extract the specific product category of revolving lines of credit. Most of these lines have a short-term maturity or are granted 'until further notice'.[21] Usually the customer pays a variable interest rate (i.e. following a swap rate plus a fixed add-on) for utilizing these credit lines.

[20] *Future* always refers to the following period, in this case $t = 15$.

[21] Credit lines are granted 'until further notice' if the unutilized amount associated with a line may generally be terminated by the committing bank without further notice. It is at least questionable, whether this is feasible in times of stress for a huge portfolio of credit lines, given the possible impact on the reputation of the bank. A similar line of argumentation is provided by regulators (see CEBS (2010b), p. 8).

The data set is cleansed according to the following steps in order to tailor for our research. To account for irregular data points, all credit line data points with a utilization of more than 250% or an abnormal collateral status (i.e., a collateralization of more than 250% in relation to the line's granted volume) are deleted. In addition, credit line data points with a total granted volume of less than 500 € or more than 100,000,000 € are omitted.[22] In total, approximately 3.2 mio. data points are included. Data availability is very uniform, with no month falling short of at least 225,000 available data points. Furthermore, the average total granted volume on the portfolio level is approximately €17 bn, with less than 2.5% fluctuation over time. However, the *utilization* on the portfolio level fluctuates up to 7% from the average utilization. The bank has a rating master scale with more than ten (internal) rating classes, where rating class 1 is associated with the best credit-worthiness. Out of all data points, 34% are attached to a rating class of 3 or better. In total, 88% of the data points are attached to a rating class of 6 or better. Furthermore, we will use information regarding the collateralization of a credit line. Approximately 82% of all credit lines in our sample are uncollateralized.[23] The industrial representation of the customers in the data set is quite heterogenous and covers more or less all big industry sectors, including financials (banks, insurance companies, etc.), the public sector, manufacturing industry, commercial enterprizes as well as retail customers. Most of the customers are domiciled in Europe.

The estimation of the portfolio draw-down $\sum_i \tilde{d}_{i,15}$ will be based on risk factors driving the utilization of the underlying credit lines, which we will derive in the following:

- An extensive academic discussion documents the importance of a customer's rating as a key driver for credit line utilization.[24] A good rating quality is generally associated with a broad availability of various funding sources, many of which are supposedly cheaper than comparable funding through credit lines. Such customers may therefore tend not to draw on their available credit lines. If credit quality *worsens*, the costs for alternative funding consequently rise. Funding conditions for credit lines may not react immediately to a deterioration in a customer's rating quality, ultimately resulting in a rising relative attractiveness as a funding source.[25] More-

[22] The mentioned data cleansing steps result in the omission of about approximately 3.2% of all exploitable data points. Whenever a 'data set' is mentioned in the remainder of this work, we always refer to the present, 'cleansed' data.

[23] Most of the collateralized credit lines are fully collateralized.

[24] See e.g. Agarwal *et al.* (2006).

[25] Compare Avery and Berger (1991) for the same line of argumentation as well as Norden and Weber (2010) for the customer behavior in case of bad creditworthiness.

over, overall availability of external funding choices may decrease for lower rating segments. In the extreme case, if the credit quality worsens even more, no other sources of funding may be available, turning existing credit lines into a 'funding source of last resort'. Following this line of argumentation and an in-depth analysis of our data set in Sarin (2012), we include the risk factor 'rating' into our forecast model.

- In general, banks may react to an increase in riskiness of a client by cutting down existing credit lines (if possible).[26] Expecting that effect, clients may only utilize their credit lines and use them as reliable source of funding in case the probability of a cut down is rather low. Schertler *et al.* (2010) find that firms with moderate recall risk for their credit lines use these lines less extensively and use cash instead,[27] in order to prevent a possible cut-down. substitute banks ma als To lower the recall risk and ultimately employ credit lines as a reliable long-term funding source, customers can opt to pledge collateral as a signal of quality.[28] Therefore, collateralized credit lines should be utilized in a different manner than uncollateralized credit lines.[29] Furthermore, once a customer has pledged collateral for a credit line, he should include these costs for (future) funding decisions. Customers will draw heavily on their credit lines for (short-term) funding, if additional costs for a draw-down of *collateralized* credit lines amount to less than comparative costs for alternative (short-term) *unsecured* funding. The risk factor 'collateral' is therefore applied as risk driver for credit line utilization.

We come to the conclusion that both rating and collateral status constitute significant risk drivers and should hence be included in the prediction of credit line utilization for the management of funding risk.

[26] Huang (2010) lined out that banks may e.g. opt out e.g financial covenants to cut down credit lines.

[27] This argumentation is underpinned by Ferreira and Vilela (2004). They find also a negative correlation between cash and bank debt.

[28] Klapper (2001) survey a sample of publicly traded U.S. manufacturing firms, amongst which firms associated with a high degree of borrowing risk are permitted to borrow larger amounts, if they offer collateral. Sufi (2009) states that firms, who can pledge a higher degree of collateral, can obtain more debt. The same line of argumentation should therefore hold for the utilization of a credit line.

[29] Compare also Jiménez *et al.* (2009) who use the collateralization of a credit line as a control variable influencing the utilization of a credit line.

2.3 Modeling of rating migrations

For the simulation of a customer's rating, we rely on the methodology provided by factor models which is both well-established in financial theory and accepted by practitioners. Furthermore, in this modeling class correlations between different customers are acknowledged through the correlations of their respective asset value processes and can therefore be estimated via external data.

Following the framework first introduced by Merton (1974), the survival of each customer i at time t is determined through the value $A_{i,t}$ of the customer's assets at that time, which is described by the dynamics[30]

$$dA_{i,t} = \mu_i A_{i,t} dt + \sigma_i A_{i,t} dB_{i,t}. \tag{1}$$

A solution for (1) is given through

$$A_{i,t} = A_{i,0} \cdot exp\left((\mu_i - \frac{\sigma_i^2}{2})t + \sigma_i \sqrt{t} W_{i,t} \right), \tag{2}$$

with $W_{i,t} := \frac{B_{i,t}}{\sqrt{t}}$. The asset value realizations for a portfolio containing m customers (at $t = T$) therefore only depend on the realizations of $W_{1,T}, ..., W_{m,T}$. Due to $W_{i,T} \sim \mathcal{N}(0,1)$, we find a representation

$$W_{i,T} = \sqrt{\frac{R_i^2}{\sum_{k,l=1}^{n} \omega_{ik}\omega_{il}}} \sum_{j=1}^{n} \omega_{ij} \cdot \psi_j + \sqrt{1 - R_i^2} \cdot \epsilon_i, \tag{3}$$

where the vector $\Psi = (\psi_1, ..., \psi_n)$ is composed of systematic factors, ω_{ij} denotes the weight according to which customer i depends on the systematic factor j, and ϵ_i corresponds to the realization of customer i's idiosyncratic risk component.[31] The extent to which $W_{i,T}$ is determined through the realization of idiosyncratic components is captured by $1 - R_i^2$ (vice versa for systematic influence). All random variables are assumed to be Gaussian, such that $\Psi \sim \mathcal{N}(0, \Sigma)$ for a correlation matrix Σ and $\epsilon_i \sim \mathcal{N}(0,1)$ for all i. The factor $\sqrt{\frac{R_i^2}{\sum_{k,l=1}^{n} \omega_{ik}\omega_{il}}}$ is included for standardization of $W_{i,T}$.

The realizations of all asset values associated with the portfolio therefore only depend on the joint realizations of n systemic factors as well as the customer specific idiosyncratic

[30] $(B_{i,t})_{t \in [0,\infty)}$ denotes a Brownian motion.

[31] Estimates for the weights $(\omega_{i,j})_{i,j}$ are provided along with the data set.

risk components.[32] The determination of a rating for customer i at the considered time horizon is now straightforward: After calculation of $W_{i,T}$, the relative outcome of the return, $\Phi(W_{i,T})$,[33] is compared to the corresponding row (representing the customer's initial rating) in a suitable $s \times s$ rating migration matrix:

- Let $p_{\gamma\to\delta}$ denote the probability by which a customer, who starts out with an initial rating of $\gamma \in \{1, \cdots, s\}$, will receive a rating of δ at $t = T$.
- Suppose customer i has initial rating γ_0. A rating of $\gamma_T \in \{1, ..., s\}$ will then be assigned at $t = T$ if

$$\sum_{\gamma=1}^{\gamma_T} p_{\gamma_0\to\gamma} \leq \Phi(r_i) < \sum_{\gamma=1}^{\gamma_T+1} p_{\gamma_0\to\gamma}. \tag{4}$$

Rating migration probabilities are derived from internal ratings included in the data set by application of a cohort method:[34]
Let $\mathcal{M} = \{(i,t)|\text{customer } i \text{ holds a credit line at date } t \in \{1, ..., 14\}\}$ denote the subset of all observable customer-date combinations in our data set. We estimate the rating transition probability $p_{\gamma\to\delta}$ via

$$\hat{p}_{\gamma\to\delta} = \frac{|\mathcal{M}_{\gamma\to\delta}|}{|\mathcal{M}_*|}, \tag{5}$$

where

$$\mathcal{M}_{\gamma\to\delta} = \{(i,t) \in \mathcal{M}|(i,t+1) \in \mathcal{M}, Rat(i,t) = \gamma, Rat(i,t+1) = \delta\}, \tag{6}$$

$$\mathcal{M}_* = \{(i,t) \in \mathcal{M}|(i,t+1) \in \mathcal{M}\} \tag{7}$$

and $Rat(i,t)$ denotes the observed rating of customer i at date t. For the duty of secrecy, we do not publish the estimated migration matrix $(\hat{p}_{\gamma\to\delta})_{\gamma,\delta}$.[35]

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[32] For an extensive discussion how correlation of asset values and diversification in such a portfolio will be affected by systemic risks, we refer to e.g. Gersbach and Lipponer (2003).

[33] As usual, Φ denotes the standard normal distribution function.

[34] An introduction to cohort methods can be found in e.g. Engelmann and Rauhmeier (2011).

[35] As a substitute for the proposed cohort approach, rating agencies such as Moody's publish rating migration matrices for various industry segments. Due to the fact that most migration matrices are based on one-year transitions, a 'scaling' is necessary to derive a one-month migration matrix. Altrock and Hakenes (2001) describe a methodology for a decomposition of a one-year matrix A into a one-month equivalent A_{1month}, such that $(A_{\mathrm{1month}})^{12} = A$.

2.4 Modeling of a collateral status

An adaptation of a factor modeling approach (as applied for the simulation of customer ratings in the last section) to assess collateral values on a portfolio basis may provide valuable insights. Although the consideration of collateral value uncertainty has already been addressed in literature by Jokivuolle and Peura (2003) a *full* adaption for the assessment of liquidity risk clearly surpasses the scope of this work. Nevertheless, general inclusion of information regarding a customer's collateral yields three major advantages: First, we documented the importance of collateral as a risk driver for credit line utilization in section 2.2. Second, the simultaneous inclusion of two drivers for credit line utilization could be adapted to fit other or additional risk drivers. Finally, stress testing collateral developments seems economically desirable, due to the fact that collateral quotas may vary significantly under stressed market conditions.

We therefore include a very basic simulation of a credit line's collateral status, which does not capture correlation effects. In analogy to the calculation of rating migration probabilities in section 2.3, we calculate

$$\hat{p}_{c_1 \to c_2} = \frac{|\mathcal{M}_{c_1 \to c_2}|}{|\mathcal{M}_*|}, \tag{8}$$

where

$$\mathcal{M}_{c_1 \to c_2} = \{(i,t) \in \mathcal{M} | (i,t+1) \in \mathcal{M}, Col(i,t) = c_1, Col(i,t+1) = c_2\} \tag{9}$$

and $Col(i,t) \in \{0,1\}$ denotes the observed collateral status of customer i at date t. If collateral of *any* value > 0 is documented for (i,t), Col values one, otherwise zero. As before, we do not publish the calculated collateral migration matrix for confidentiality reasons. A customer's future collateral status is simulated in a Markov fashion, solely based on the current rating and the two corresponding migration probabilities.

2.5 Calibration of line draw-downs

So far, we simulated a rating as well as a collateral status for each credit line in the portfolio. Let (rat_j, col_j) denote the pair of simulated values for credit line j. To generate a draw-down $\widetilde{d}_j$, we first define a cluster $C_{(rat_j,col_j)}$ for the simulated rating/collateral combination via

$$C_{(rat_j,col_j)} = \{(i,t) \in \mathcal{M} | Rat(i,t) = rat_j, Col(i,t) = col_j\}. \tag{10}$$

We obtain an empirical draw-down distribution based on C_{rat_j,col_j}, by adjusting the observable draw-downs according to the granted volume $V_{i,t}$ of the respective credit line (at date t):

$$Hist_{(rat_j,col_j)} = \{\left(\frac{d_{i,t}}{V_{i,t}}\right) |(i,t) \in C_{(rat_j,col_j)}\}. \tag{11}$$

For each set $Hist_{(\gamma,c)}$ (for $\gamma = 1,...,s, c \in \{0,1\}$), let $F_{(\gamma,c)}$ denote the corresponding empirical distribution. Based on $F_{(rat_1,col_1)},...,F_{(rat_n,col_n)}$, we generate draw-downs for all n credit lines by recalibrating each simulated (relative) draw-down according to the line's granted volume. The distribution of the portfolio draw-down, $\widetilde{d}_{PF}= \sum_j \widetilde{d}_{j,15}$, is then given through

$$\widetilde{d}_{PF}= \sum_{j=1}^{n} V_{j,14} \cdot F^{-1}_{(rat_j,col_j)}(\widetilde{X}_j), \tag{12}$$

where all $\widetilde{X}_j$ are uniformly distributed[36] on $[0,1]$.

To study the impact of distributional assumptions for the $2 \cdot s$ marginal distributions $F_{1,0}, F_{1,1}, F_{2,0}, F_{2,1},..., F_{s,0}, F_{s,1}$, we estimate parameters to fit the relative historical distributions to a normal as well as a gamma distributional form. The definition of *all* $3 \cdot 2 \cdot s$ marginal distributions (historical, normal and gamma) does not depend on any results that were established during the Monte-Carlo simulation and therefore do not need to be re-constructed during every simulation run.

Normal and gamma distribution families are applied to analyze whether fitting historical data by means of a (potentially unrealistic) parametric distribution has a significant impact on the portfolio measure. A portfolio draw-down based on any of the three marginal distribution types is generated via (12). The only remaining 'piece' is the dependency structure for the multivariate random variable $(\widetilde{X}_1,...,\widetilde{X}_n)$.

36 The dependency structure between the random variables $\widetilde{X}_1,...,\widetilde{X}_j$ is subject of Section 2.6.

2.6 Calibration of a draw-down dependency structure

We explicitly consider the dependency structure between individual credit line utilizations. Figure 1 shows histograms for the empirical correlation coefficients for all credit lines from the data set, which exhibited a specific high quality rating (left side) or a specific low quality rating (right side) for all 14 dates.[37]

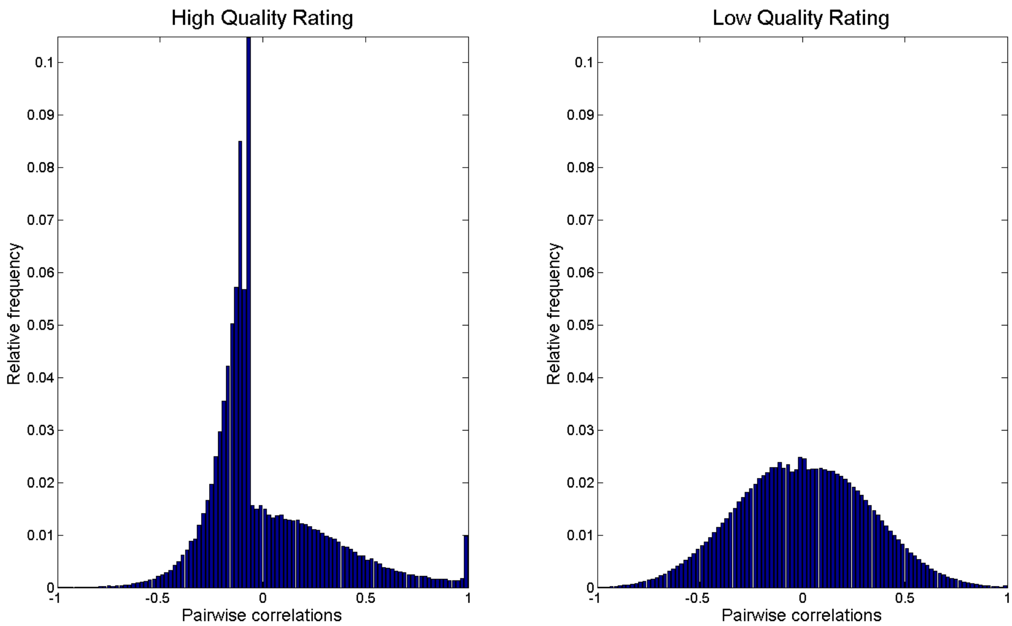

Figure 1: Rating-specific correlation histograms

The *mean* correlation is very close to zero for both groups. The extreme skewness for the correlation histogram of lines attached to the high quality rating results from a low number of non-zero line utilizations. Since customers with a high credit quality are assumed to quickly find (comparatively cheap) access to alternative funding via capital markets or other sources, drawn amounts are often repaid in a short interval.[38] On the other hand,

[37] Correlations are calculated via Pearson's linear correlation coefficient which is ill defined, if one of the considered histories shows a constant utilization. Such lines are therefore not part of the representation in Figure 1.

[38] The Pearson correlation coefficient for such credit lines, which exhibit utilizations exclusively for disjoint dates, are generally negative and close to zero. For instance, Pearson's correlation coefficient for two draw-down histories $(d_{j,1}, ..., d_{j,14}) = (a, 0, ..., 0)$ and $(d_{k,1}, ..., d_{k,14}) = (0, b, 0, ..., 0)$ with $a, b > 0$ computes $\rho_{j,k} = -0.0769$.

customers associated with a comparatively 'low' rating class suffer from the typical funding constraints described in Section 2.2, which lead to (on average) longer non-zero draw-down intervals. Elimination of all credit facilities with exactly one or two draw-downs over the full observation period supports this line of argument:

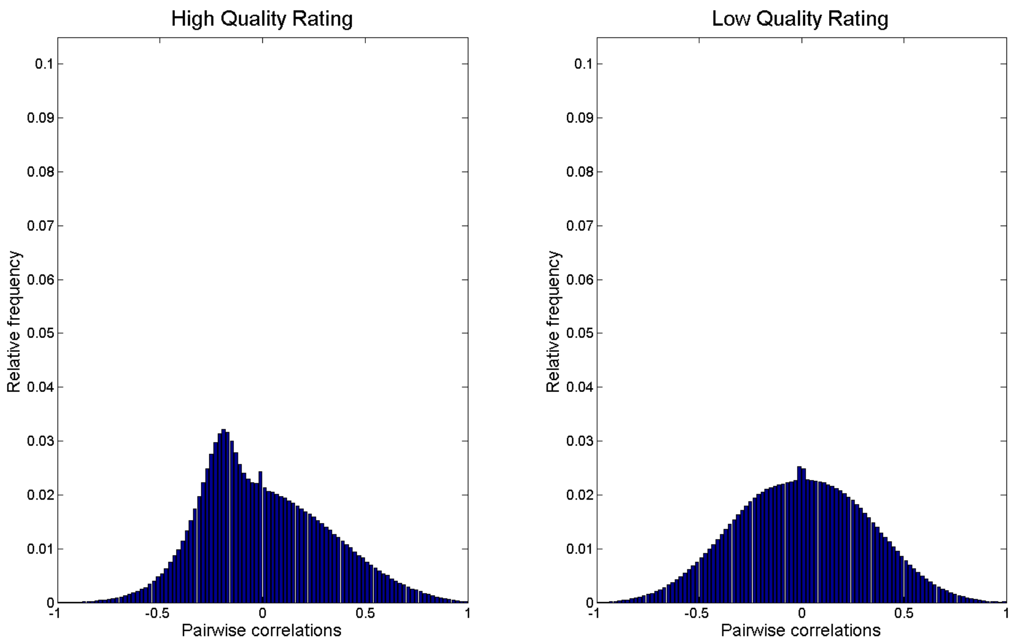

Figure 2: Rating-specific correlation histograms, adjusted

Whereas the observation of diverse correlation structures within different clusters makes the inclusion of such structures into a full-scale forecast model sound desirable, subsequent implementation suffers from a number of challenges. Most obvious among these is the necessary inclusion of line histories which migrate in between clusters during the observation period. Consequently, we apply a correlation structure for the *entire* portfolio to calculate simultaneous draw-downs for all credit lines. Cluster-specific utilization effects are therefore not considered in this work. We will estimate a correlation structure based on historical data, which is then applied to set up the dependency structure between the random variables $\widetilde{X}_1, ..., \widetilde{X}_n$ in (12). This poses the following problems:

- Pearson's correlation coefficient is ill defined, if one of the considered draw-down histories is constant (compare footnote 37).

- Correlations must be computed on the basis of utilization histories containing identical observation dates.
- Given the large number of credit lines included in the data set, it is technically not feasible to compute and apply a correlation matrix for the full data set.

The most straight-forward solution to the first two problems is to only consider complete, non-constant credit line histories. Due to the large number of such histories in the portfolio[39], we estimate a correlation structure from these credit lines without neglecting large parts of potentially existing portfolio correlation. To estimate the correlation structure, we transform all considered data points to fit the model (12):

$$u_{j,t} = F_{(rat_j,col_j)}(\frac{d_{j,t}}{V_{j,t}}) \tag{13}$$

for all $d_{j,t} \in Hist_{comp}$ with

$$Hist_{comp} = \{d_{j,t} | d_{i,t} \in \mathcal{M} \text{ for } i = 1, ..., 14 \text{ and } \exists\ t_1, t_2 \text{ such that } d_{j,t_1} \neq d_{j,t_2}\}. \tag{14}$$

Pearson's linear correlation coefficients are calculated based on the resulting relative draw-downs.[40]

Due to technical reasons, it is not possible to compute and apply the full historical correlation structure. Hence, we draw a random sample of $10,000$ credit lines with complete data histories from $Hist_{comp}$ and compute the respective correlation matrix from the relative draw-downs in (13). While the resulting matrix is symmetrical by definition, the necessary condition of positive definiteness is not guaranteed. We apply a quadratically convergent Newton method to compute a 'similar' positive definite correlation matrix Γ, according to a pre-specified eigenvector.[41] We simulate random numbers

$$x_j = \Phi^{-1}(y_j) \text{ for } j = 1, ..., 10,000, \tag{15}$$

where $(y_1, ..., y_{10,000})$ are realizations for random variables $(\widetilde{Y}_1, ..., \widetilde{Y}_{10,000}) \sim \mathcal{N}(0, \Gamma)$, which are simulated via MATLAB. To sustain a dependency structure for all credit lines,

---

39 The data set contains more than $100,000$ complete, non-constant credit line histories. The majority of lines is excluded because they exhibit a constant draw-down for the full horizon ($t = 1, ..., 14$).

40 Compare Klein and Sarin (2012) for a more formal description of the approach.

41 Let $A = (a_{ij}), B = (b_{ij})$ two Matrices with identical size. The distance between A and B is measured via the Frobenius norm: $\|A - B\|_F = \sqrt{\sum_{i,j} |a_{ij} - b_{ij}|^2}$. See Houduo and Defeng (2006) for a theoretical background for the applied Newton algorithm, as well as a numerical illustration.

independent samples are drawn from $(x_1, ..., x_{10,000})$ and redistributed (independently) to the remaining credit lines.

The proposed approach preserves the initial correlation structure reasonably.[42] The mechanism applies existing (relative) draw-downs to multiple customers, therefore leading to a perfect correlation between a number of lines in the portfolio. This effect will later be offset, due to the calculation of individual (absolute) draw-downs from the individual marginal distributions $F_{rat_1,col_1}, \cdots, F_{rat_1,col_n}, F_{rat_2,col_1}, \cdots, F_{rat_n,col_n}$.

2.7 Monte-Carlo simulation

As mentioned earlier, a simple repetition of steps 1-5 as outlined in Section 2.1 now easily yields an approximation for the portfolio distribution. In analogy to Klein and Sarin (2012), we propose to fund expected future draw-downs according to the size of the mean (estimated) draw-down, whereas contingent future draw-downs should be addressed via a suitable portfolio quantile (minus the expected future draw-down).

The presented Monte-Carlo approach allows us to explicitly model a number of stochastic influences within the portfolio, including the rating migration process, cluster-specific draw-down distributions, collateral quotas and the portfolio correlation structure. We will study the impact of a 'shocked' rating migration matrix on the estimated portfolio draw-down in section 3.2. Additionally, the effects of a systematic factor as well as amplified draw-down distributions are examined in the same section.

[42] To understand in how far this approach affects the portfolio correlation structure, we run a control simulation not for our real data set rather than for a small, randomly generated set of data histories. Hence, the results from this control simulation are not overruled from specific structural elements within the real data set: Based on a randomly generated correlation matrix of dimension 200, we simulate 200 multivariate normally distributed random variables, according to the given correlation structure. Variables are simulated in a Monte-Carlo setting, generating draw-down histories for $1,000$ dates for all 200 random variables. Subsequently, draw-down values for 200 additional lines are chosen by assignment of already existing draw-downs to these 'new' lines. Resulting correlation coefficients are calculated and compared to the original correlation structure. We explored that effect in more detail by adding 200, 400, 800, 1,600 and 3,200 more 'copied' draw-down histories. A comparison of the original correlation structure to the ex post estimated (via Pearson's correlation coefficient) correlation structures for the portfolios containing 400, 600, 1,000, 1,800 and 3,400 credit lines, shows no significant deviations.

3 Results

We present the results of the model setup discussed in chapter 2. We simulate a portfolio draw-down distribution for a single time step, from $t = 14$ (30.07.2010) to $t = 15$ (31.08.2010). All results in Section 3 are based on $20,000$ simulation steps ('iterations') of a Monte-Carlo simulation. First we discuss the going concern setting in section 3.1, before the parameter setting is stressed in section 3.2.

3.1 Normal market condition results

We first compare the portfolio measures for the **three different families of distributions** for future draw-downs on a single deal level, as outlined in Section 2.5. The simulation results presented below display the ratio of the simulated portfolio utilization at $t = 15$ to the total granted volume of the portfolio at $t = 14$.[43] While the results for the expected future draw-downs $\hat{\mu}$ are almost identical for all distribution types, the contingent future draw-downs[44] at a tolerance level of 95% ($\hat{q}_{95\%}$) and a tolerance level of 99% ($\hat{q}_{99\%}$) vary significantly. Consistently, the standard deviations $\hat{\sigma}$ also vary considerably. We illustrate the resulting contingent future draw-downs via a mean adjusted cumulative distribution function plot.[45] The graph clearly shows that the contingent future draw-downs are (for any reasonable quantile) lowest for the family of normal distributions, whereas the highest contingent future draw-downs are observed for the family of gamma distributions. The following Table 1 and Figure 3 summarize these results:[46]

Draw-down distribution	$\hat{\mu}$	$\hat{\sigma}$	$\hat{q}_{95\%}$	$\hat{q}_{99\%}$
Historical distribution	26.44	3.60	6.59	11.09
Gamma distribution	26.43	4.87	9.06	15.32
Normal distribution	26.46	2.05	3.35	4.85

Table 1: Descriptive statistics for the simulated distribution of future portfolio draw-downs. Draw-downs are derived for three different distribution types.

43 We define this ratio as 'portfolio utilization' and we subduct a fixed factor from the portfolio utilizations for the duty of secrecy. This operation has no impact on the relative results but simply shifts the absolute mean of all distributions.

44 Note that we define the contingent future draw-down as add-on onto the expected future draw-downs.

45 I.e., we plot the contingent future draw-downs (in percentage points) on the x-axis and the related quantiles (tolerance levels) on the y-axis.

46 For the determination of draw-downs, the actual correlation structure (via the estimated matrix Γ) is applied.

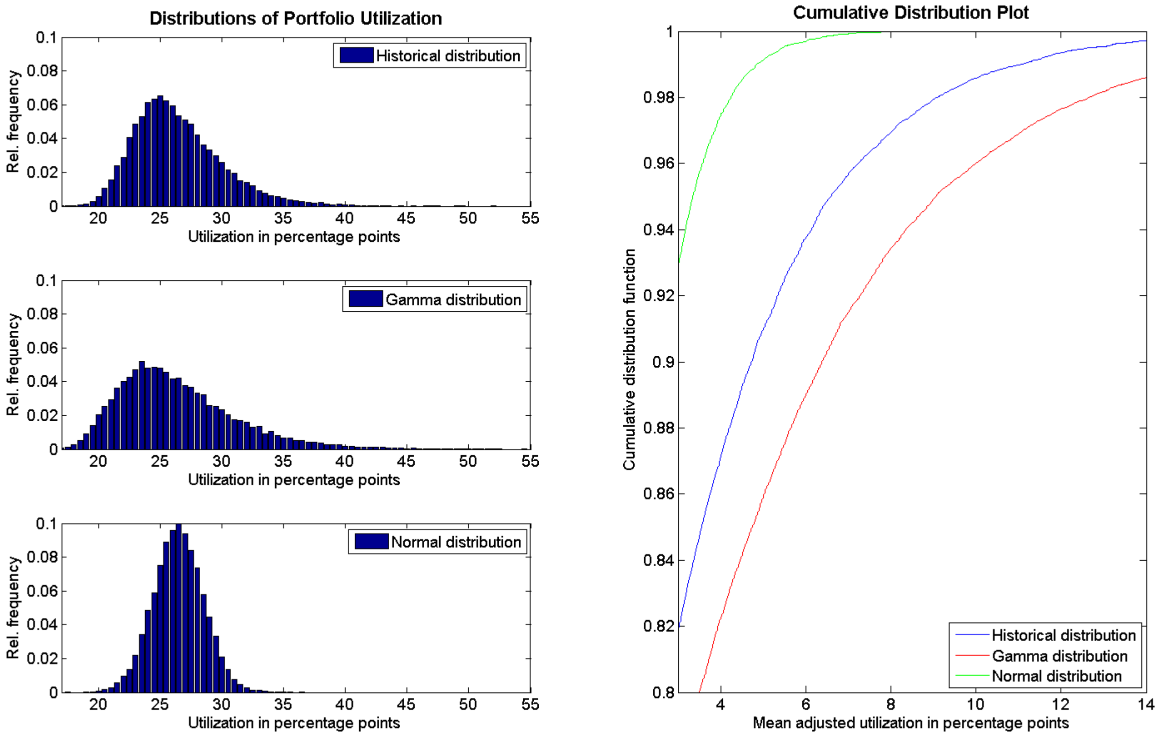

Figure 3: Distribution of portfolio utilization at $t = 15$ for different families of distributions for the respective draw-downs

Normal, as well as gamma distributions are fitted according to the underlying historical data, leading to identical expected values for all three distribution types. Additionally, we apply the same correlation structure in all three cases, leading to (almost) identical values for resulting mean portfolio utilizations. The chosen fitting mechanism clearly impacts the shape of the resulting portfolio distribution.[47] Moreover, the magnitude of resulting discrepancies seems to be driven by the general shape of the applied distribution family: The estimated historical distributions have significant peaks at specific utilization levels, which may be approximated somewhat more accurately by a more skewed distribution type, as can be seen in the results for the gamma distribution fit.[48] Application of a

[47] If independent draw-downs were assumed for single credit lines, the choice of a 'fitted' (parametric) distribution would result in no major impact for the resulting portfolio distribution. Due to the law of large numbers, the mean draw-down for credit lines in cluster $C_{(\gamma,c)}$ converges to the mean of the respective marginal distribution, for large cluster sizes. In Section 2.5 we calibrated both normal and gamma cluster-specific distributions, such that the mean for all three distribution types remains the same for all $2 \cdot s$ clusters. For a sufficiently large portfolio (such as the present one), the total portfolio draw-down in the independent case is therefore almost identical for all three distribution types.

[48] For the duty of secrecy, we are not able to promulgate a sample of (historical) utilizations from our data set.

(symmetrical) normal distribution consequently yields a very inaccurate result. Hence, we favor the historical distribution approach. The practical implication of the observed fitting shortcomings is obvious: while it might be appropriate to assume (log)-normally distributed returns in various applications of finance (since the assumption is based on well-founded theoretical models and broadly backed-up via empirical analysis), such an assumption is highly questionable for draw-downs of single credit lines and in particular for a portfolio distribution as indicated by our results.

Since the dependency structure is of such importance, we analyze its impact in some more detail by comparing portfolio distributions simulated on the basis of **three different dependency structures**. As described in section 2.6 we estimated pairwise correlations for $10,000$ randomly chosen credit lines and simulated correlated draw-downs for the entire portfolio, according to this structure. We compare this approach (i) with fully independent draw-downs for all credit lines and (ii) apply the identity-matrix instead of the estimate Γ, i.e. we simulate $10,000$ independent relative draw-downs according to the method outlined in Section 2.6 and redistribute these to the remaining lines. The results show, that expected future draw-downs are (again) of similar magnitude, whereas contingent future draw-downs differ substantially. For a comparison of the latter, we supply values for $\hat{\sigma}$, $\hat{q}_{95\%}$ and $\hat{q}_{99\%}$. The following Table 2 and Figure 4 summarize these results:[49]

Correlation structure	$\hat{\mu}$	$\hat{\sigma}$	$\hat{q}_{95\%}$	$\hat{q}_{99\%}$
Independent draw-downs	26.68	1.35	2.25	3.39
Partially independent draw-downs	26.47	1.33	2.20	3.21
Correlated draw-downs	26.44	3.60	6.59	11.09

Table 2: Descriptive statistics for the simulated distribution of future portfolio draw-downs. Draw-downs are derived on the basis of three different dependency structures.

[49] For the determination of draw-downs, 'historical distributions' are applied.

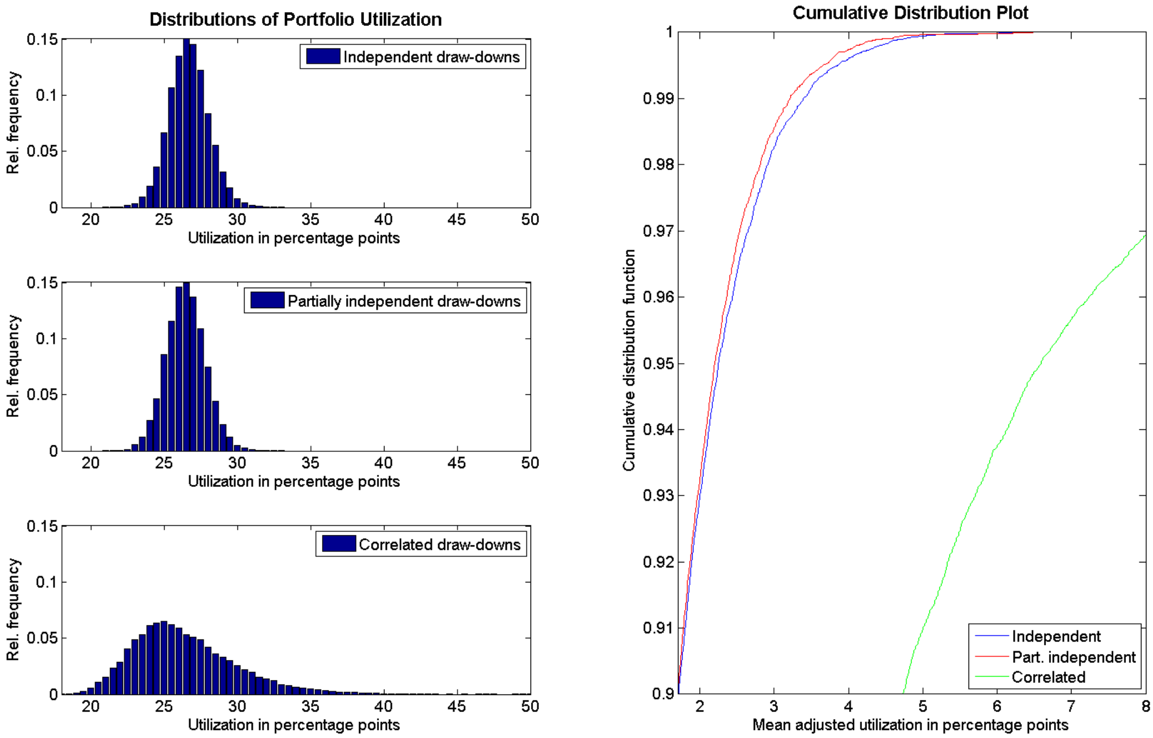

Figure 4: Distribution of portfolio utilization at $t = 15$ for different correlation structures

The application of a non-trivial (historical) correlation structure results in a much higher volatility of the portfolio utilization, as well as increased contingent future draw-downs. The differences between independent and partially independent draw-downs are a good indicator for possible errors resulting from the complexity-reducing approach for the portfolio correlation structure outlined in Section 2.6. Examination of the cumulative distribution plot shows that the approach for partially independent draw-downs results in slightly higher contingent future draw-downs, as the redistribution of $10,000$ relative draw-downs to a portfolio comprising more than $200,000$ lines creates some additional high correlations. Nevertheless, the results for independent and partially independent draw-downs remain in the same range. We conclude that it is crucial to acknowledge a realistic correlation structure. The development of a theoretically reasonable and technically feasible method to adequately capture the portfolio dependency structure is a challenging task, but the assumption of independent draw-downs may deliver highly polluted results, as observed in Table 2.

Since we are the first to simultaneously rely on more than one risk factor driving the utilization of single credit lines by forming 'rating/collateral' clusters, we wish to quantify benefits resulting from the inclusion of such additional information, possibly leading to further calibration improvements in future models. We therefore compare simulation results derived from the application of the 'full' **rating/collateral** cluster structure, with simulation results based on 'rough' **rating** clusters. In the latter case, information regarding the collateral status is neither taken into account during the calibration nor in the simulation, i.e., historical distributions are calibrated by each rating cluster (using both collateralized and uncollateralized credit lines). We evaluate the expected future draw-downs $\hat{\mu}$ and the contingent future draw-downs $\hat{q}_{95\%}$ and $\hat{q}_{99\%}$. The expected future draw-down $\hat{\mu}$ changes significantly if only rating clusters are used. Additionally, the standard deviation $\hat{\sigma}$ increases. The following Table 3 and Figure 5 summarize the results:[50]

Cluster	$\hat{\mu}$	$\hat{\sigma}$	$\hat{q}_{95\%}$	$\hat{q}_{99\%}$
Rating/collateral	26.44	3.60	6.59	11.09
Rating	26.31	3.70	6.80	11.51
Difference	0.13	-0.10	-0.11	-0.42

Table 3: Descriptive statistics for the simulated distribution of future portfolio draw-downs. Draw-downs are derived for rating/collateral clusters and (sole) rating clusters.

[50] For the determination of draw-downs, the 'historical distributions' and the actual correlation structure (via the estimated matrix Γ) are applied.

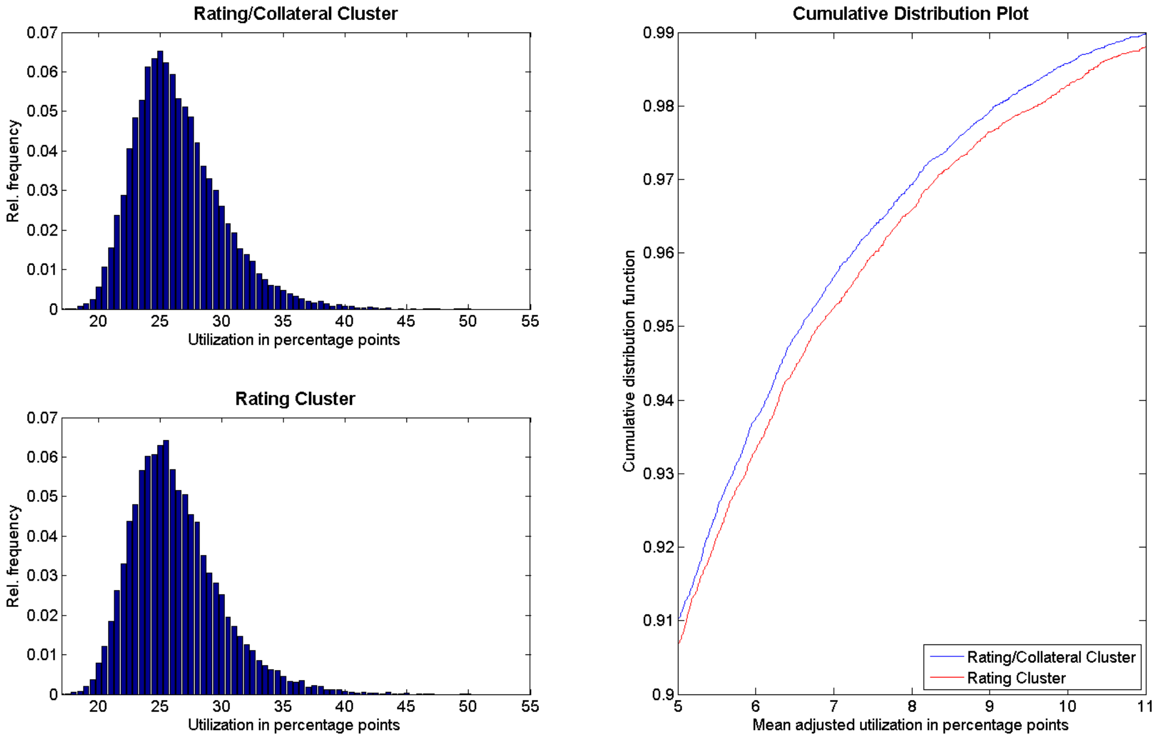

Figure 5: Distribution of portfolio utilizations at $t = 15$ for rating/collateral clusters and (sole) rating clusters

The results indicate that using more than one relevant risk-factor (i.e., rating) has a material impact on the resulting portfolio measures, in particular on expected future draw-downs. This effect is driven by our calibrated collateral status migration matrix. While in the 'Rating' setup the information regarding changes in the collateral status is ignored, the 'Rating/Collateral' setup includes these. A change in the collateral status for the latter scenario results (on average) in a utilization decrease/increase of approx. 4% per single credit line.[51] The estimated migration matrix is not perfectly symmetric, leading to a slight deviation in the average collateralization for the portfolio, which in turn accounts for the observed difference in expected future draw-downs.

The standard deviation for the 'Rating/Collateral' approach is lower, since a customer's

[51] This effect was estimated via a robust OLS regression model.

utilization, contingent on the respective rating, is drawn from two less volatile draw-down distributions (instead of one more volatile draw-down distribution for the 'Rating' approach). We conclude that the inclusion of the risk driver 'collateral status' is highly advisable from an economic perspective, as relevant additional information regarding the systematically changing collateral status is included.

3.2 Stress testing results

Liquidity stress testing is an integral pillar of a sound liquidity risk management framework. Using a Monte-Carlo simulation approach for credit lines as part of the stress testing environment features the advantage that economically reasonable scenarios can be translated through stressed parameter settings, which in turn lead to economically reasonable results as an output of the simulation. Consequently, we analyze the effects of stressing the rating migration matrix as well as the cluster-specific draw-down distributions.

We first analyze the effect of a **rating migration shock**. For this, we assume that the probability of migrating into the next inferior rating class increases by 10 percentage points while the probability of staying in the same rating class decreases by 10 percentage points.[52] This shift has a significant impact on the portfolio distribution. Expected future draw-downs rise significantly (i.e., by 1.3%). Additionally, the standard deviation $\hat{\sigma}$ of simulated portfolio utilizations increases. The following Table 4 and Figure 6 summarize the results:[53]

Scenario	$\hat{\mu}$	$\hat{\sigma}$	$\hat{q}_{95\%}$	$\hat{q}_{99\%}$
Normal	26.44	3.60	6.59	11.09
Stress	27.70	3.68	6.64	11.05
Difference	-1.26	-0.08	-0.05	0.04

Table 4: Descriptive statistics for the simulated distribution of future portfolio draw-downs. Draw-downs are derived for normal and stressed rating migration matrices

[52] We assume that the probability of staying in the default class does not change over time.

[53] For the determination of draw-downs, the 'historical distributions' and the actual correlation structure (via the estimated matrix Γ) are applied.

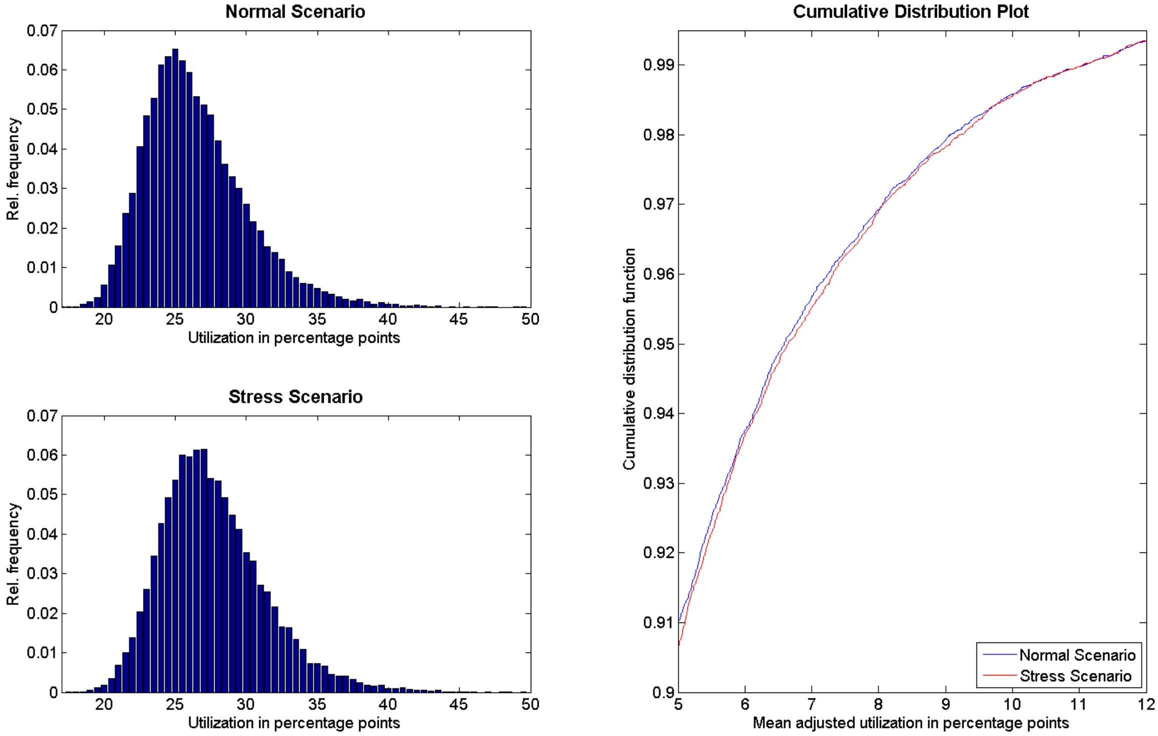

Figure 6: Distribution of portfolio utilizations at $t = 15$ for normal and stressed rating migration matrices

Times of stressed market conditions are usually associated with a deterioration of rating quality. As we change the expected rating quality (by shifting the rating migration matrix), the expected future draw-down rises by 1.3%. To estimate the impact of a change in a rating class onto the utilization of a single credit line, we use a robust OLS regression. The results indicate that the average utilization per credit line increases by approx. 13% if the rating decreases by one class. Since we decreased the expected rating quality on average by 10%, the resulting expected future draw-down should rise for approx. $13\% \times 10\% = 1.3\%$.

Last, we analyze the effect of shifted **draw-down distributions**. The cluster-specific draw-down distributions $F_{1,0}, F_{1,1}, F_{2,0}, F_{2,1}, ...F_{s,0}, F_{s,1}$ are restrained at the 10% (50%)-level.[54] Hence, we assume that under stress, clients are drawing more heavily on their credit lines. This shift has a material impact on the simulated portfolio distribution.

[54] I.e., all realizations for $\widetilde{X}_1, ..., \widetilde{X}_n$ falling short of 0.1 (0.5) in (12), are rejected.

Expected future draw-downs as well as contingent draw-downs rise strongly. Additionally, the standard deviation $\hat{\sigma}$ of simulated portfolio utilizations increases. The following Table 5 and Figure 7 summarize the results:[55]

Scenario	$\hat{\mu}$	$\hat{\sigma}$	$\hat{q}_{95\%}$	$\hat{q}_{99\%}$
Normal	26.38	3.57	6.63	11.12
Stress 10% Quantile	29.98	3.71	6.87	11.84
Stress 50% Quantile	26.85	4.08	7.52	12.76
Difference	0	0	0	0

Table 5: Descriptive statistics for the simulated distribution of future portfolio draw-downs. Draw-downs are derived for stressed cluster-specific draw-down distributions

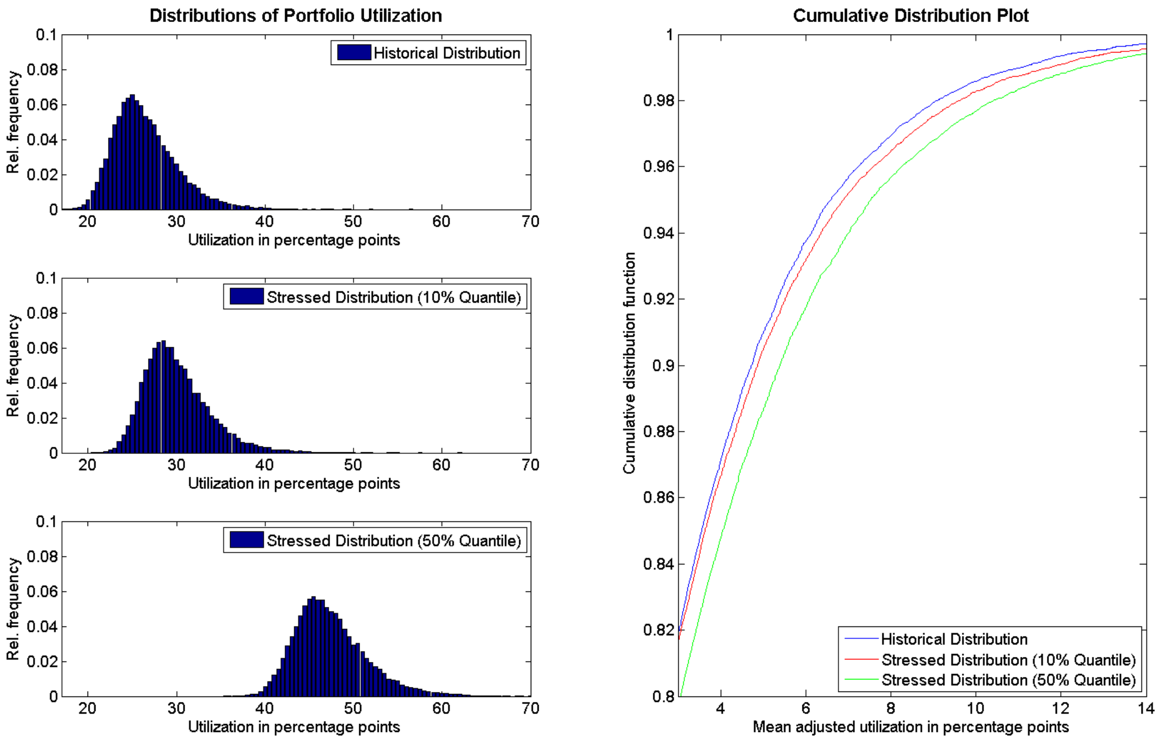

Figure 7: Distribution of portfolio utilizations at $t = 15$ for stressed cluster-specific distributions

[55] For the determination of draw-downs, the 'historical distributions' and the actual correlation structure (via the estimated matrix Γ) are applied.

The expected draw-down for the portfolio ($\hat{\mu}$) increases as a direct result of the elevated expected values for all cluster-specific marginal distributions. For most of the 'rating/collateral' clusters, a significant part of the probability mass is concentrated at an utilization level of exactly 0%, which is cut down in our stress testing application. As we mitigate the concentration of the marginal distributions at the (lower) extreme of zero utilization, $\hat{\sigma}$ consequently rises for the 10% (50%) stress. Although $\hat{q}_{95\%}$ and $\hat{q}_{99\%}$ increase as well, the effect is less severe than for $\hat{\mu}$. This results from the fact, that only low utilizations are directly manipulated (i.e., rejected), whereas higher utilization levels are only impacted indirectly through the redistribution of the remaining probability mass.

We demonstrated how the presented model setup allows for easy and consistent implementations of economically reasonable stress testing applications. The exact determination of relevant parameter combinations as well as the size of the stress itself are to be evaluated in the context of a consistent stress testing scenario as stated through regulatory requirements.[56] As anecdotal evidence suggests that a majority of stress testing applications regarding liquidity risk is based on expert opinion, our model can provide a further step towards a quantitatively oriented liquidity risk management framework.

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[56] See BIS (2008).

4 Conclusion

The development of a portfolio model applicable for funding risk measurement of credit facilities is of major importance as part of a robust internal liquidity risk management framework, especially (but not exclusively) to meet regulatory requirements. We resolved practical issues arising while specifying and implementing a Monte-Carlo simulation approach. We introduced the risk factors rating *and* collateralization as drivers for the utilization of single credit lines. Further, we are the first who consider an explicit dependency structure between the draw-downs of single credit lines *and* also proposed a complexity reduction algorithm for the dependency structure. Our approach allows for simulation and calibration of a number of stochastic influences within the portfolio. In addition, the model allows for a transparent prediction of expected future draw-downs and for a precise and adequate definition of contingent future draw-downs in line with an assumed level of risk tolerance. In particular, the approach can be utilized consistently in a holistic stress testing framework.

Due to the fact that regulators may require an adequate liquidity cost allocation on a single deal level, the model may provide a basis for the development of an allocation mechanism, which tracks liquidity costs attached to expected and contingent future portfolio draw-downs back to single lines of credit. Furthermore, the approach can be enhanced to simultaneously address different classes of contingent products, such as term facilities, guarantees and credit lines. Additionally, the applied stress testing techniques can be adjusted to allow for stress testing applications for specific characteristics of such products.

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