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Abstract

For a small open economy with decreasing returns to scale within industries, increasing any factor supply may increase all sector outputs. A necessary and sufficient condition for these seemingly perverse Rybczynski adjustments may even be satisfied for stable equilibria.

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1 Motivation [†]

The famous theorem by Rybczynski (1955) is one of the single most significant insights of modern pure trade theory. Given the standard Heckscher-Ohlin framework of two commodities and factors, the theorem states that an extra supply of some input will induce the production sector where this input is used more intensively to expand more than proportionally. At the same time, the other sector will contract. Proofs of this theorem usually rely on the convenient simplification that scale effects can be assumed away due to constant returns in production.

Not surprising, the theorem does not in general carry over to variations of the standard model. While Blackorby/Schworm/Venables (1993, p. 426) have shown that under a regime of joint production deviations from the usual Rybczynski adjustments may even occur for constant returns to scale, other authors have introduced technologies with scale economies or diseconomies. So-called comparative-statics paradoxes may emerge and may sometimes persist despite supplementary assumptions. However, the previous literature indicates that these paradoxes do not occur when the commodity markets are Marshallian stable (cf. Jones (1968), Mayer (1974), Panagariya (1980), Ide/Takayama (1988, 1990), Ingene/Yu (1991), Ishikawa (1994)).

While other authors have modeled non-market externalities (e.g. economies of scope) which are *external* to single industries, in our study output scale effects that are *internal* to an industry matter. We feel that this represents a natural first arena for extended research on the comparative statics of international trade. Furthermore, while much of the contemporary economic literature seems to be concerned with *economies* of scale, we focus on scale *diseconomies* in production. This does not only refer to a classical economic theme in the tradition, e.g., of D. Ricardo and J.H. von Thünen, but also reflects our assessment of empirical issues. Among others, the aggregate nature of real-world inputs (implying that in fact more quantity may mean less average quality) as well as the role of increases internal complexity (rendering the replication argument for constant returns to scale less convincing) motivate the analysis of decreasing returns to scale.

Except for decreasing returns to scale, our model with two or more goods and factors is fairly standard. Following Woodland (1982), the analysis is carried out entirely in dual space. In particular, we are starting from (minimum) cost functions and, hence, exploit

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the structure and regularity imposed by duality theory irrespective of functional form. At the same time, we need no longer refer to traditional, however less convenient, concepts such as the 'production possibility locus' of an economy (cf. Herberg/Kemp (1969)).

We present a seemingly perverse Rybczynski adjustment in the sense that *all* outputs tend to *benefit* from larger endowments. Contrary to what was suggested by earlier writers, such comparative-statics responses do *not* exclude Walrasian and Marshallian stability of an equilibrium. Apart from proving this by a selection of examples, *necessary and sufficient* conditions for the advent of non-standard adjustments are provided. It is shown that these outcomes can occur in a reasonable number of relevant cases. In particular, inferior inputs (not possible under constant returns) and substitution as well as non-substitution technologies are included in our analysis.[1] It also turns out that, unlike what is true in the 2×2 model with constant returns to scale, *all* factor prices may fall when any single endowment is increased.

The paper is organized as follows: In Section 2 we briefly outline our general model and its comparative-statics results. Various parameterizations of an example and their non-standard adjustments to endowment changes are presented in Section 3. In Section 4 stability is shown, and Section 5 concludes.

2 General Model

We consider $n \geq 2$ profit maximizing sectors or firms each producing $x_i > 0$ $(i = 1, \ldots, n)$ units of a single net output from at least one net input. The economy's total number of inputs is $m \geq 2$. There are fixed factor endowments $\mathbf{v}' = (v_1, \ldots, v_m)$ where $v_j > 0$ for all $j = 1, \ldots, m$.[2] The small, open economy takes world output prices $p_i > 0$ $(i = 1, \ldots, n)$ as exogenous whereas domestic factor prices $w_j > 0$ $(j = 1, \ldots, m)$ are determined endogenously due to factor immobility. We also assume that the minimum cost in each sector i of producing x_i output units at factor prices $\mathbf{w}' = (w_1, \ldots, w_m)$ can be computed from a twice-differentiable cost function $C^i : \mathbb{R}^m_{++} \times \mathbb{R}_{++} \to \mathbb{R}_{++}$ with the following properties satisfied for all $(\mathbf{w}, x_i) \in \mathbb{R}^m_{++} \times \mathbb{R}_{++}$: Cost is a nondecreasing first-order homogeneous and (weakly) concave function of factor prices $\mathbf{w}$. Marginal cost is positive and increases as output increases (decreasing returns to scale in production).

Our analysis relies on the postulate that the economy attains an equilibrium state in which all factors are at full employment. Hence, $\sum_{i=1}^{n} C^i_{w_j}(\mathbf{w}, x_i) = v_j$ for all j, where $C^i_{w_j}$ is short-hand notation for $\partial C^i / \partial w_j$ and thus stands for the quantity demanded of factor j

[1] Our results also hold in the presence of factor complements when more than two factors are considered.

[2] The prime $'$ denotes transposes.

in sector i according to Shephard's Lemma. At the same time, marginal costs shall equal (finite) output prices in all sectors i: $C^i_{x_i}(\mathbf{w}, x_i) = p_i$. Firms will thereby make positive profits. Yet, these profits shall be sufficiently small such that no further firm will want to enter the markets. We are thus given $m + n$ model equations. Total differentiation yields:

$$\begin{pmatrix} \mathbf{C_{ww}} & \mathbf{C_{xw}} \\ \mathbf{C'_{xw}} & \mathbf{D} \end{pmatrix} \begin{pmatrix} d\mathbf{w} \\ d\mathbf{x} \end{pmatrix} = \begin{pmatrix} d\mathbf{v} \\ d\mathbf{p} \end{pmatrix}, \qquad (1)$$

introducing as $\mathbf{x}$ and $\mathbf{p}$ the vectors of sector outputs and output prices, respectively, and with matrices $\mathbf{C_{ww}} := \sum_{k=1}^n \mathbf{C}^k_{\mathbf{ww}} = \sum_{k=1}^n (C^k_{w_i w_j})$, $\mathbf{C_{xw}} := (C^1_{x_1\mathbf{w}}, \ldots, C^n_{x_n\mathbf{w}})$ and $\mathbf{D} := \operatorname{diag}(C^i_{x_i x_i})$.[3]

If we had assumed constant returns to scale, $\mathbf{D}$ would possess zero elements throughout. Then (1) would correspond to (5.1) in Diewert/Woodland (1977) and (A9) in Jones/Scheinkman (1977) as well as to (6) in Chang (1979) and (48)-(49) in Woodland (1982, Section 4.7.). In the present context, however, all entries along the diagonal of $\mathbf{D}$ are positive. Therefore, firstly, $\mathbf{D}$ is positive definite and, secondly, the diagonal inverse $\mathbf{D}^{-1}$ exists and is likewise positive definite.

We define $\mathbf{M} := \mathbf{C_{ww}} - \mathbf{C_{xw}}\mathbf{D}^{-1}\mathbf{C'_{xw}}$ (cf. Gantmacher (1990, pp. 45-46)) and require that $\det(\mathbf{M}) \neq 0$. This not only eliminates borderline singular cases of little economic significance but, given existence of $\mathbf{D}^{-1}$, is also necessary and sufficient for a unique solution of (1) in terms of changes in factor rents $\mathbf{w}$ and sector outputs $\mathbf{x}$. The following comparative-statics responses to changes in the world market prices $\mathbf{p}$ and national endowments $\mathbf{v}$ result (cf. Hadley (1974, pp. 107-109)):

$$\begin{pmatrix} d\mathbf{w} \\ d\mathbf{x} \end{pmatrix} = \begin{pmatrix} \mathbf{M}^{-1} & -\mathbf{M}^{-1}\mathbf{C_{xw}}\mathbf{D}^{-1} \\ -\mathbf{D}^{-1}\mathbf{C'_{xw}}\mathbf{M}^{-1} & \mathbf{D}^{-1} + \mathbf{D}^{-1}\mathbf{C'_{xw}}\mathbf{M}^{-1}\mathbf{C_{xw}}\mathbf{D}^{-1} \end{pmatrix} \begin{pmatrix} d\mathbf{v} \\ d\mathbf{p} \end{pmatrix}. \qquad (2)$$

Consequently:

$$\frac{\partial \mathbf{x}}{\partial \mathbf{v}} = -\mathbf{D}^{-1}\mathbf{C'_{xw}}\mathbf{M}^{-1}. \qquad (3)$$

Recall that $\mathbf{D}^{-1}$ has a positive diagonal (with elements $1/C^i_{x_i x_i}$). Therefore, the respective Rybczynski effects $\partial\mathbf{x}/\partial\mathbf{v}$ will always possess the same sign as the corresponding entries to $-\mathbf{C'_{xw}}\mathbf{M}^{-1}$. Formally, introducing as $\mathbf{O}$ the $m \times n$ zero matrix:

Proposition 1. *All Rybczynski derivatives $\partial\mathbf{x}/\partial\mathbf{v}$ will come out positive if, and only if,* $-\mathbf{M}^{-1}\mathbf{C_{xw}} > \mathbf{O}$.

This proposition provides a necessary and sufficient condition for an increase in all sector outputs as a response to an increase in any single factor supply, i.e. a seemingly

[3] Subscripts attached to C again represent partial derivatives.

perverse Rybczynski adjustment. Examples highlighting the range of this result are provided in the next section. Note from the following proposition that any such example must allow for input substitution in at least one production sector:

Proposition 2. *Suppose that no sector's technology permits input substitution. Then the matrix inequality* $-\mathbf{M}^{-1}\mathbf{C_{xw}} > \mathbf{O}$ *cannot be satisfied.*

Proof. If all inputs are non-substitutes, then $\mathbf{C_{ww}} = \mathbf{O}$. For the same reason, $\mathbf{C_{xw}} \geq \mathbf{O}$. Hence, $\mathbf{M} = -\mathbf{C_{xw}}\mathbf{D}^{-1}\mathbf{C}'_{\mathbf{xw}}$. Now let $\mathbf{I}$ denote the $m \times m$ identity matrix such that $\mathbf{I} = \mathbf{M}^{-1}\mathbf{M} = (-\mathbf{M}^{-1}\mathbf{C_{xw}})\mathbf{D}^{-1}\mathbf{C}'_{\mathbf{xw}}$. Therefore, if all entries to the matrix product in brackets are positive, $e_{ij} = 0$ $(i \neq j)$ requires that the entire column j of $\mathbf{C}'_{\mathbf{xw}}$ vanishes, contradicting $e_{jj} = 1$. Consequently, not all elements of $-\mathbf{M}^{-1}\mathbf{C_{xw}}$ can be positive at the same time. □

Basically, this observation says: whenever no input substitution is possible, diseconomies of scale do not matter, i.e. the standard Rybczynski outcome persists. In this case, increasing the output in all sectors would require more of *every*, and not just one, input.

Let us briefly address the adjustments in factor prices. In the standard Heckscher-Ohlin framework of two commodities and factors with constant returns to scale, the price of the factor of which an extra supply has become available will decrease whereas the other factor price will increase (cf. Woodland (1982, p. 79)). In our model economy, factor price responses to endowment changes are given by $\mathbf{M}^{-1}$ (cf. equation (2)). By definition, $\mathbf{M}$ (and hence $\mathbf{M}^{-1}$) is negative definite.[4] Together with the examples in the following section this proves:

Proposition 3. *The own-price effects of arbitrary supply shocks are normal, whereas for* $i \neq j$ *the adjustments* $\partial w_i / \partial v_j$ *may be of either sign.*

In particular, different from the standard adjustments, all factor prices may decrease when any single endowment increases.

3 Examples with Seemingly Perverse Rybczynski Adjustments

Let $n = m = 2$ and consider the cost functions $C^1(w_1, w_2, x_1) = (0.1w_1 + 0.05w_2)\, x_1^{1.26}$, which comes from a production process with a fixed factor intensity, and $C^2(w_1, w_2, x_2) = w_1 x_2^{1.2} + 2\sqrt{w_1 w_2}\, x_2^{1.1} - w_2 x_2^{1.3}$ over some domain in the neighborhood of the economy's

[4] Note that $\mathbf{r}'\mathbf{M}\mathbf{r} = \mathbf{r}'\mathbf{C_{ww}}\mathbf{r} - \mathbf{r}'\mathbf{C_{xw}}\mathbf{D}^{-1}\mathbf{C}'_{\mathbf{xw}}\mathbf{r} \leq 0$ for all $\mathbf{r} \neq \mathbf{o}$ of length m, since $\mathbf{C_{ww}}$ is negative semi-definite (cf. Diewert (1982, p. 567)) and $\mathbf{D}^{-1}$ is positive definite (with $\mathbf{C}'_{\mathbf{xw}}\mathbf{r}$ possibly equal to the null vector). Hence, $\mathbf{r}'\mathbf{M}\mathbf{r} < 0$ by the assumed regularity of $\mathbf{M}$. (Cf. Diewert/Woodland (1977, p. 392) for a similar argument in the context of constant returns to scale.)

assumed equilibrium state $w_1 = 1.5$, $w_2 = 1$ and $x_1 = 1$, $x_2 = \eta$. Some lengthy calculations lead essentially to the following results, depending on the level of η:

1. Let $\eta = 1.0$. We find:

$$\mathbf{C_{ww}} = \begin{pmatrix} -0.272 & 0.408 \\ 0.408 & -0.612 \end{pmatrix}, \quad \mathbf{C_{xw}} = \begin{pmatrix} 0.126 & 2.098 \\ 0.063 & 0.047 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 0.066 & 0 \\ 0 & 0.239 \end{pmatrix},$$

$$\mathbf{M} = \begin{pmatrix} -18.900 & -0.127 \\ -0.127 & -0.682 \end{pmatrix}, \quad \mathbf{M}^{-1} = \begin{pmatrix} -0.053 & 0.010 \\ 0.010 & -1.468 \end{pmatrix}, \quad -\mathbf{C}'_{\mathbf{xw}}\mathbf{M}^{-1} = \begin{pmatrix} 0.006 & 0.091 \\ 0.111 & 0.049 \end{pmatrix}.$$

2. Let $\eta = 1.15$. Then:

$$\mathbf{C_{ww}} = \begin{pmatrix} -0.317 & 0.476 \\ 0.476 & -0.714 \end{pmatrix}, \quad \mathbf{C_{xw}} = \begin{pmatrix} 0.126 & 2.145 \\ 0.063 & 0.011 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 0.066 & 0 \\ 0 & 0.206 \end{pmatrix},$$

$$\mathbf{M} = \begin{pmatrix} -22.906 & 0.245 \\ 0.245 & -0.775 \end{pmatrix}, \quad \mathbf{M}^{-1} = \begin{pmatrix} -0.044 & -0.014 \\ -0.014 & -1.294 \end{pmatrix}, \quad -\mathbf{C}'_{\mathbf{xw}}\mathbf{M}^{-1} = \begin{pmatrix} 0.006 & 0.083 \\ 0.094 & 0.043 \end{pmatrix}.$$

3. Let $\eta = 1.5$. This gives:

$$\mathbf{C_{ww}} = \begin{pmatrix} -0.425 & 0.638 \\ 0.638 & -0.957 \end{pmatrix}, \quad \mathbf{C_{xw}} = \begin{pmatrix} 0.126 & 2.237 \\ 0.063 & -0.065 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 0.066 & 0 \\ 0 & 0.154 \end{pmatrix},$$

$$\mathbf{M} = \begin{pmatrix} -33.215 & 1.465 \\ 1.465 & -1.045 \end{pmatrix}, \quad \mathbf{M}^{-1} = \begin{pmatrix} -0.032 & -0.045 \\ -0.045 & -1.020 \end{pmatrix}, \quad -\mathbf{C}'_{\mathbf{xw}}\mathbf{M}^{-1} = \begin{pmatrix} 0.007 & 0.070 \\ 0.069 & 0.034 \end{pmatrix}.$$

In all three cases, all entries of $-\mathbf{C}'_{\mathbf{xw}}\mathbf{M}^{-1}$ are positive. Therefore, $\partial x_i/\partial v_j$ is positive for all $i,j = 1,2$. This appears to be a perverse Rybczynski adjustment, because an increase in any single factor endowment implies an increase in *all* sector outputs, i.e. *no* sector will shrink.

The differences between the three parameterizations are as follows: In the first case, both factors are superior and the matrix $\mathbf{M}^{-1}$ exhibits the standard factor price responses. In the second case, each input is still superior. However, all factor prices will now decrease because all entries of $\mathbf{M}^{-1}$ are negative (cf. Proposition 3). This also holds in the third case, where factor 2 is inferior in the second industry.[5]

4 Stability

The above cases, simple as they are, demonstrate clearly that non-Rybczynski outcomes may emerge in a standard 2×2-setting if decreasing returns to scale are assumed. Ac-

[5]It can be shown for the 2×2 case that an inferior factor and standard factor price responses are incompatible with seemingly perverse Rybczynski adjustments.

cording to Samuelson's correspondence principle (cf. Samuelson (1947)), however, such comparative-statics results are meaningful only if stability of the equilibrium is guaranteed. The argument, so-called comparative-statics paradoxes were usually associated with unstable equilibria and would otherwise vanish, has been put forward repeatedly in the literature on international trade (e.g. Mayer (1974), Ide/Takayama (1988, 1990)). For this reason, it is important to note that in our model equilibrium factor prices and outputs are *always* (locally) stable once the market-clearing process operates according to some simple, yet intuitively appealing, principles.

We suggest two standard types of adjustment processes in the factor and commodity markets, respectively (cf. Mayer (1974)). To begin with, we propose that rent variations are proportional by scalars k_j (> 0) to the amount of excess input demand. This Walrasian price-tâtonnement shall be complemented in the goods markets by a Marshallian adjustment mechanism: outputs evolve in the direction of the difference between world demand prices and domestic supply prices (in terms of marginal cost of production), again by some factors of proportionality l_i (> 0). Assuming for simplicity that $k_j = l_i = 1$ for all j and i, we obtain:[6]

$$\dot{w}_j = \sum_{i=1}^{n} C^i_{w_j}(\mathbf{w}, x_i) - v_j \quad \text{for all } j\,, \tag{4}$$

$$\dot{x}_i = p_i - C^i_{x_i}(\mathbf{w}, x_i) \quad \text{for all } i\,, \tag{5}$$

where Shephard's Lemma has been used in (4).

Associated with the adjustment model (4)-(5) is the following linear approximation system (cf. Takayama (1974, p. 311)) which has been evaluated around an equilibrium state $(\mathbf{w}^*, \mathbf{x}^*)$:

$$\begin{pmatrix} \dot{\mathbf{w}} \\ \dot{\mathbf{x}} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \mathbf{w} - \mathbf{w}^* \\ \mathbf{x} - \mathbf{x}^* \end{pmatrix} \tag{6}$$

with the Jacobian matrix $\mathbf{A}$ defined as:

$$\mathbf{A} := \begin{pmatrix} \mathbf{C_{ww}} & \mathbf{C_{xw}} \\ -\mathbf{C}'_{\mathbf{xw}} & -\mathbf{D} \end{pmatrix}. \tag{7}$$

We are now prepared to introduce our final proposition:

Proposition 4. *All eigenvalues of* $\mathbf{A}$ *are negative real numbers. The equilibrium state* $(\mathbf{w}^*, \mathbf{x}^*)$ *is stable.*

Proof. Let λ and $\mathbf{z}$ denote an eigenvalue and related eigenvector, respectively, of $\mathbf{A}$. Hence,

[6] The dot '·' indicates total differentiation with respect to time.

$\mathbf{Az} = \lambda\,\mathbf{z}$. Furthermore, partition $\mathbf{z}$ into two vectors $\mathbf{z}_1$ and $\mathbf{z}_2$ of suitable length such that

$$\begin{aligned}\mathbf{z}'\mathbf{Az} &= (\mathbf{z}_1'\mathbf{z}_2')\begin{pmatrix}\mathbf{C_{ww}} & \mathbf{C_{xw}}\\ -\mathbf{C}'_{\mathbf{xw}} & -\mathbf{D}\end{pmatrix}\begin{pmatrix}\mathbf{z}_1\\ \mathbf{z}_2\end{pmatrix}\\ &= \mathbf{z}_1'\mathbf{C_{ww}}\mathbf{z}_1 - \mathbf{z}_2'\mathbf{D}\mathbf{z}_2\\ &= \lambda\,\mathbf{z}'\mathbf{z}\,. \end{aligned} \tag{8}$$

Note that $\mathbf{C_{ww}}$ is a negative semi-definite matrix (cf. Diewert (1982, p. 567)). Also recall that $\mathbf{D}$ is positive definite. Therefore, $-\mathbf{D}$ will be negative definite, and we may conclude from (8) that all eigenvalues of $\mathbf{A}$ must be real and non-positive. Regularity of $\mathbf{D}$ implies that $\det(-\mathbf{D}) \neq 0$ while $\det(\mathbf{M}) \neq 0$ by assumption (cf. Section 2). Therefore

$$\begin{aligned}\det(\mathbf{A}) &= \det(-\mathbf{D})\,\det(\mathbf{C_{ww}} - \mathbf{C_{xw}}\,\mathbf{D}^{-1}\,\mathbf{C}'_{\mathbf{xw}})\\ &= \det(-\mathbf{D})\,\det(\mathbf{M})\end{aligned} \tag{9}$$

(cf. Gantmacher (1990, pp. 45-46)) requires $\det(\mathbf{A}) \neq 0$. Consequently, zero can not be an eigenvalue of $\mathbf{A}$. Thus all eigenvalues of $\mathbf{A}$ must be negative reals which is sufficient for $(\mathbf{x}^*, \mathbf{w}^*)$ to be stable (see Takayama (1974, p. 310)). □

We thus find, unlike Mayer (1974) and Ide/Takayama (1988, 1990), that (seemingly) paradoxical comparative-statics effects may be observed even in well-behaved settings with locally stable equilibria. The following differences in the approach taken by these authors on the one hand and our model on the other hand appear to be crucial. Firstly, we assume that prices equal *marginal* rather than *average* costs. Therefore, firms will make positive profits due to decreasing returns if there are no fixed costs. Secondly, Mayer and Ide/Takayama impose several restrictions upon the equilibrium outcomes of their models, following Jones (1968). They thereby neglect inferior inputs and also require that an increase in any factor price must raise the average *equilibrium* production cost of each commodity. Therefore, certain non-obvious cases of technologies and endowment changes are excluded from their approach. According to our results, the cases allowing for non-Rybczynski responses of industry outputs in otherwise stable equilibria must be among them.

5 Conclusion

The purpose of this paper was to examine the comparative statics of endowment changes in a small open and competitive economy with a finite number of goods and factors. We assumed that this economy is exposed to diseconomies of scale within each industry. Common intuition suggests that all production sectors should then be able to benefit from an extra supply of any scarce resource. We proved as our major result that this conjecture, although in conflict with Rybczynski's theorem, can hold true, indeed, if the production technologies permit input substitution. In contrast to the literature, we showed that such seemingly perverse Rybczynski adjustments are fully compatible with the Walrasian and Marshallian stability of the underlying equilibrium.

In view of the standard reciprocity relation (cf. Samuelson (1953/54)), our model could also be evaluated with respect to the comparative-statics effects of changes in the world market prices upon domestic outputs and factor rents. The interpretation of deviations from standard Stolper-Samuelson effects (cf. Stolper/Samuelson (1941)), however, is less straightforward and beyond the scope of this paper. For instance, residual profits under decreasing returns could also be seen as rents of specific factors in a constant-returns context.

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