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Institut für Kreditwesen
Direktor: Prof. Dr. Andreas Pfingsten

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Banks as Delegated Risk Managers

Dipl.-Math. Hendrik Hakenes

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Banks as Delegated Risk Managers

Hendrik Hakenes*

Abstract: Risk management, although of major importance in the banking industry in practice, plays only a minor role in the theory of banking. We reduce this gap by putting forward a model in which risk managers — specialists that can find out correlations of risky assets — endogenously take over typical functions of banks. They *grant loans*, they consult on financial questions with firms that are threatened by bankruptcy, and they sign tailor-made hedge transactions with these firms. Risk management can thus be seen as a core competence of banks.

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Institut für Kreditwesen (Department of Banking)
University of Münster
Universitätsstraße 14–16
D-48143 Münster
Phone/Fax: + 49 – 251 – 83 22885/22882
E-Mail: `21heha@wiwi.uni-muenster.de`

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1 Introduction

In practice, *risk management* is often seen as a core function of the banking business. Allen and Santomero (1998, p. 1462) suggest that it becomes more and more the essential function of banks (financial intermediaries):

> "[Intermediaries] are facilitators of risk transfer and deal with the increasingly complex maze of financial instruments and markets. Risk management has now become a central activity of many intermediaries. Most current theories of intermediation have little to say about why risk management should play such an important role in the activities of intermediaries."

On p. 1465, Allen and Santomero (1998) confirm that theoretical literature is empty:

> "Little is offered as a cogent argument as to why intermediaries should be the ones offering these services [risk management], and what value they bring to the activity. In short, the intermediation literature is noticeably quiet to why these institutions should be engaged in one of their central areas of activity."

Under *risk management*, we understand two essential aspects, (i) *risk analysis* as the aggregation of information about distribution and correlation of risky assets, and (ii) *risk controlling* as the active structuring and shaping of a risky portfolio. Especially the knowledge of correlation seems to be important for risk management in reality, as it is the basis of portfolio management. A firm can only hedge its risk with financial contracts if it is aware of the correlation between its own business and traded financial contracts (feasible hedging tools).

There is a variety of approaches to explaining the existence of financial intermediaries.[1] All these have one characteristic in common. Risk allocation is often modeled, the acquisition of information about the distribution of random variables is sometimes modeled, whereas the acquisition of information about the correlation of random variables typically remains unmodeled. As a result, an important aspect of risk management is ignored.

One might think that models about risk sharing — such as presented e. g. by Allen and Gale (1994) — would have risk management as an essential component. However, risk analysis rarely plays an explicit role. A reason is often that correlations of risky assets do not appear, e. g. because random variables are independent. Another possible reason is that information about correlation is publicly accessible, so that no meaningful decision about the acquisition of correlation data needs to be made.

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[1] See Baltensperger (1980), Santomero (1984), Bhattacharya and Thakor (1993) or Swank (1996) for an overview.

As an example, take the model of Allen and Gale (1999). There are two parties: The *intermediary* knows the random variable of any asset's outcome. He thus knows their state dependence, distribution and correlations. The *customer* is affected by incomplete information regarding the random variables of traded securities' outcomes. He wants to hedge the risk of his random income, but refrains from engaging on the financial markets because he fears unpleasant surprises. The intermediary possesses the information the customer needs but cannot credibly convey it. In a one-period setting, although intermediary and customer can conclude risk sharing contracts that are favorable for either party, the customer refuses doing so. He fears do be cheated by the intermediary. In a multi-period setting, though, a cheating intermediary would have to expect the termination of the profitable relationship by the customer. Therefore, he refrains from cheating. He offers a risk sharing contract that fits the customer. The anticipating customer dares to conclude the contract, which he needs not necessarily understand. However, the model of Allen and Gale (1999) contains only aspects of *risk controlling* (the customer reduces risk by signing the risk sharing contract) but not of *risk analysis* (the aggregation of information about correlation is not costly for the intermediary). The focus of the model is risk sharing, not risk management.

Furthermore, most of the theoretic approaches view banks as competitors with financial markets. By contrast, empirical evidence suggests that the relationship between banks and financial markets may not be characterized by competition alone. According to Allen and Santomero (2001), the importance of banks has decreased only in relation to the importance of the financial industry. Yet in comparison with GDP, the importance of banks *and* financial firms increases. This hypothesis is supported by empirical research carried out by Scholtens and Wensveen (2000) for US-American markets and by Schmidt et al. (1999) for French, German and British data. Allen and Gale (1999) conclude that a change of paradigm lies ahead. The "traditional paradigm" in which banks and financial markets are seen as competitors needs to be replaced by an "emerging paradigm" in which banks act as a link between customers and financial markets.

In the following, we put forward a model in which risk management is the only competence of banks. In the model, assume that an agent — in the following called the risk analyst — has the ability of *risk analysis*. Here, risk analysis is understood as the costly determination of state dependencies of the random variables of an asset's outcome. The risk analyst is shown below to have typical properties of banks. Another agent — in the following called the entrepreneur — is in need of risk sharing, e. g. because the randomness of his income can lead to bankruptcy. It is shown that the risk analyst then

1. carries out the *risk analysis* on behalf of the entrepreneur by gathering information about the correlation between the entrepreneur's business and states of nature,

2. carries out the *risk controlling* on behalf of the entrepreneur by selling him a hedging tool that reduces the entrepreneur's risk,

3. grants a *loan* to the entrepreneur.

The reason for item 3 is more subtle: By granting a loan, a financial intertwining between risk analyst and entrepreneur is created, which in turn alleviates the problem of delegating the risk analysis. The loan harmonizes the interests of entrepreneur and risk analyst: The entrepreneur is genuinely interested in avoiding his own bankruptcy. After granting the loan, the risk analyst bears a counterparty risk and is therefore (more) willing to help the entrepreneur. Because risk analysis (gathering information about risk), risk controlling (selling tailor-made contracts to customers) and granting loans are typical functions of banks, we may call the previous risk analyst a bank.

In the following, we first present a provisional model where the risk analyst cannot grant loans to customers (Section 2). Properties of the sequential equilibrium (determined in Section 3) are discussed in Section 4. Section 5 extends the provisional model by allowing that the risk analyst may grant loans to the customer. Results are discussed in Section 6. The conclusion of Section 7 completes the paper.

2 The Model

We first present a model — called Game 1 — without allowing for loans. An *entrepreneur* (U) has access to a capital project which requires an investment outlay of I_0 in $t = 0$ and leads to a risky result of $\tilde{Y} : \Omega \to \mathbb{R}_+$ in $t = 1$, Ω being the set of possible states of nature and $\mathbb{R}_+$ being the nonnegative real numbers. The entrepreneur is endowed with exactly the necessary investment outlay I_0; there is thus neither necessity for borrowing, nor can the entrepreneur's liability exceed the project's result. The result has distribution $F_{\tilde{Y}}$ and density $f_{\tilde{Y}}$.[2] The entrepreneur can influence neither the realization of $\tilde{Y}$ nor its distribution $F_{\tilde{Y}}$ — be it by choosing between different levels of effort or different designs of the project. The distribution $F_{\tilde{Y}}$ is public information, whereas the entrepreneur does not know and cannot determine the corresponding random variable $\tilde{Y}$ (i. e. the result's dependence on the states of nature). In $t = 1$, the realization of $\tilde{Y}$ becomes publicly known. If the project fails to yield more than a threshold of K, the entrepreneur is confronted

[2] For the sake of simplicity, let us assume that the project's result is distributed continuously, and $f = F'$ exists. An F without index stands for $F_{\tilde{Y}}$, the project's distribution function. Furthermore, assume that $0 < F(Y) < 1$ for all $Y \in (0, \infty)$, thus arbitrarily small and large results are possible. All results of the model hold under much weaker, very natural assumptions, such as the assumption that the initial probability of U's bankruptcy lies between 0% and 100% ($0 < F(K) < 1$).

with bankruptcy, leading to bankruptcy costs of ϕ_{U}. U is liable with the project's results only.

Because of the bankruptcy risk, the entrepreneur has an incentive to engage in financial markets in order to diminish the probability of bankruptcy. However, as long as he has no information about the state dependency $\tilde{Y}$ of his project, he does not know which financial products to choose.[3] The entrepreneur's ignorance of $\tilde{Y}$ expresses thus the opaqueness of financial markets.

A *risk analyst* (B) can determine the project's state dependency $\tilde{Y}$ at an effort c_{B}. If he does not invest c_{B}, he knows only the distribution function F. There is hence scope for delegation — the entrepreneur may charge the risk analyst with the evaluation of the project's state dependency and then hedge the firm against bankruptcy using the information received. The risk analyst is equipped with a sufficient debt paying capacity. His risk of bankruptcy is neglected. Both entrepreneur and risk analyst are risk neutral and behave rationally. In case of indifference, the players are assumed to decide in favor of the opposite party.

The delegation contract between U and B is called *hedge contract*, because its "aim" is to hedge against U's bankruptcy risk.[4] Of the set of possible hedge contracts, only a subset is considered. We assume:

1. The liabilities that stem from the contract in $t = 1$ depend on the state of nature only. Consequently, the liabilities can be stated as a random variable $\tilde{Z} : \Omega \to \mathbb{R}$. $\tilde{Z}$ can assume positive values — when the entrepreneur is due to pay — and negative values — when the entrepreneur receives money.[5]

2. The functional form of the distribution $F_{\tilde{Z}}$ is such that $\tilde{Z}$ may have the form

$$\tilde{Z}(\omega) = \min\{S, \tilde{Y}(\omega) - K\} \tag{1}$$

[3] Because of the lacking information about state dependency, U cannot calculate correlations between his project's result and the available financial securities. Note that throughout this paper, the notion of correlation contains all information of the dependency on two random variables, not only the correlation coefficient ρ. Formally, it is the copula that contains all information about the correlation of two random variables whose distribution is known. The copula is the two-dimensional distribution modulo the marginals. For a more detailed discussion, cf. Bickel et al. (1993, pp. 155–157).

[4] It is obvious that B must enter into some contract with U. If U would offer B a fixed payment for the risk analysis, without a further contract, B would have no incentive to risk-analyze U's project.

[5] This assumption implies that a hedge contract contains no general clauses such as "If the project fails, B pays a specified sum to U". It may contain only contingencies depending on exogenous variables such as exchange rates, commodity prices, interest rates, other macroeconomic variables or indices. Therefore, the contract may be interpreted as a parcel of derivatives on exogenous variables.

with $\omega \in \Omega$, and $S \in \mathbb{R}$ being a real number. A $\tilde{Z}$ as in (1) leads to liabilities for either party that averts the bankruptcy risk for the entrepreneur, but does not lead to unnecessary payments. Of all hedge contracts that avert the entrepreneur's bankruptcy risk, the payments of a $\tilde{Z}$ as in (1) have the least dispersion. Thus

$$F_{\tilde{Z}} = \begin{cases} F_{\tilde{Y}}(z+K): & z < S \\ 1: & \text{else} \end{cases} \tag{2}$$

3. The entrepreneur pays no fee in advance (in $t = 0$).

Note that the state dependency $\tilde{Z}$ of the hedge contract may (but need not) have the form $\tilde{Z}(\omega) = \min\{S, \tilde{Y}(\omega) - K\}$. We call the hedge contract *fitting* if (1) holds. Even if the contract does not fit, it has the distribution function of a fitting contract because of (2). On this account, U cannot find out whether $\tilde{Z}$ fits or not. If B does not invest c_{B} to find out about $\tilde{Y}$, he generally cannot draw up the fitting contract.

Figure 1 about here.

In Game 1 described above, U is the first mover, his strategy space is $\sigma_{\mathrm{U}} \in \{S, \dagger\}$: $S \in \mathbb{R}$ stands for U's proposal of the hedge contract's terms, † for not offering any contract. B is the second mover, his strategy space is $\sigma_{\mathrm{B}} \in \{\oplus, \ominus, \dagger\}$, where $\oplus$ stands for accepting the proposal S and investing c_{B} (risk-analyzing), $\ominus$ for accepting S but not investing c_{B} (not risk-analyzing), † for not accepting U's proposal.[6]

3 Sequential Equilibrium

We first calculate the expected profits, depending on the moves chosen by the players U and B.[7] That way, the strategic form of Game 1 becomes known, which simplifies the following analysis of equilibrium.

Remark 1 (Hedge Contract after Risk Analysis) *If B risk-analyzes U's project ($\sigma_{\mathrm{U}} = S, \sigma_{\mathrm{B}} = \oplus$), then he draws up a hedge contract, and this contract fits the project.*

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[6] For a precise definition of game theoretic notions, cf. e.g. Fudenberg and Tirole (1991). Note that the chosen way of notation contains already some simplification and is not formally correct: Because B's move is already a response to U's move. Correctly, one would have to write $\sigma_{\mathrm{B}} \in \{(\oplus|S), (\ominus|S), (\dagger|S), (\dagger|\dagger)\}$. In order to avoid clutter, we quote only the moves instead of the whole strategies. Moreover, B's strategy space is actually richer, for he might risk-analyze the project but choose to draw up a contract that does not *fit* the project.

[7] In order to keep track of the calculations, we have to define indices for different moves. In the brackets in the subscript, the first sign denotes U's move, the second denotes B's response.

Proof: If B draws up the fitting contract, he can expect[8]

$$\begin{aligned}
\mathrm{E}[G_{\mathrm{B}}]_{(S,\oplus)} &= \mathrm{E}[\tilde{Z}] - c_{\mathrm{B}} = \int_{\Omega} \tilde{Z}(\omega)\, d\Pr\{\omega\} - c_{\mathrm{B}} && (3)\\
&= \int_0^{\infty} \min\{S, Y-K\}\, f(Y)\, dY - c_{\mathrm{B}}\\
&= \int_0^{S+K} (Y-K)\, f(Y)\, dY + \int_{S+K}^{\infty} S\, f(Y)\, dY - c_{\mathrm{B}}\\
&= S\, F(S+K) - \mathcal{F}(S+K) + (1 - F(S+K))\, S - c_{\mathrm{B}}\\
&= S - \mathcal{F}(S+K) - c_{\mathrm{B}}. && (4)
\end{aligned}$$

These expected profits are an upper bound for the possible expected profits of other contracts with the same distribution,

$$\mathrm{E}[G_{\mathrm{B}}] = \int_{\Omega} \min\{\tilde{Z}(\omega), \tilde{Y}(\omega)\}\, d\Pr\{\omega\} - c_{\mathrm{B}} \le \int_{\Omega} \tilde{Z}(\omega)\, d\Pr\{\omega\} - c_{\mathrm{B}}. \qquad (5)$$

Thus if B draws up a contract at all, he selects the *fitting* contract. However, if B had chosen to enter into no contract at all, then he would have foreseen this before spending c_{B} — hence this cannot occur, for B acts rationally. ■

The entrepreneur's expected profits can be calculated as a residual,

$$\mathrm{E}[G_{\mathrm{U}}]_{(S,\oplus)} = \mathrm{E}[\tilde{Y}]_{(S,\oplus)} - \mathrm{E}[\tilde{Z}]_{(S,\oplus)} = \mathrm{E}[\tilde{Y}] - S + \mathcal{F}(S+K). \qquad (6)$$

If the risk analyst draws up a hedge contract without having risk-analyzed the entrepreneur's project, there are several possibilities. If the risk analyst makes a good guess, the contract written may have a high correlation with the fitting contract, in which case U's bankruptcy risk is nearly eliminated. In the other extreme, if the risk analyst guesses wrong, the contract written may as well correlate negatively with the fitting contract, in which case the hedge contract aggravates the risk of bankruptcy. However, as B knows only the distribution of the fitting contract, one can assume that B's guess — call it $\tilde{X}$ — is i. i. d. with $\tilde{Y}$. Hence the contract written $\tilde{Z}$ is i. i. d. with the fitting contract. Note that we do not claim that a guess $\tilde{X}$ is always independent of the project $\tilde{Y}$. Good guesses are possible as well as bad guesses, while $\tilde{X}$ is a mere representative which leads to correct expectations.

Remark 2 (The Representative Hedge Contract) *Assume that B draws up a hedge contract without having risk-analyzed the project ($\sigma_{\mathrm{U}} = S, \sigma_{\mathrm{B}} = \ominus$). Then the assumption that the contract is based on a random variable $\tilde{X}$ which is i. i. d. with $\tilde{Y}$ leads to the correct expected values.*

It is now possible to calculate the expected profits of a contract which is based on a guess $\tilde{X}$ of B. U's limited liability must be taken into account — B can never obtain

[8] Let F (f) without index always stand for $F_{\tilde{Y}}$ ($f_{\tilde{Y}}$). Be $\mathcal{F}$ the primary function of F.

more than the minimum of the result of U's project Y and the contract's claims, $\min\{X - K, S\}$.

$$\mathrm{E}[G_{\mathrm{B}}]_{(S,\ominus)} = \int_0^\infty \int_0^\infty \min\{Y, X-K, S\}\, f(Y)\, dY\, f(X)\, dX$$
$$= S - \mathcal{F}(S) - \int_0^S F(X)\,(1 - F(X+K))\, dX. \tag{7}$$

Again, the entrepreneur's expected profits can be calculated as a residual, additionally taking into account U's positive probability of bankruptcy.

$$\mathrm{E}[G_{\mathrm{U}}]_{(S,\ominus)} = \mathrm{E}[\tilde{Y}] - \mathrm{E}[G_{\mathrm{B}}]_{(S,\ominus)} - \mathrm{E}[\phi_{\mathrm{U}}]_{(S,\ominus)} \quad \text{with} \tag{8}$$
$$\mathrm{E}[\phi_{\mathrm{U}}]_{(S,\ominus)} = \phi_{\mathrm{U}}\ \Pr\{Y - Z < K\} = \phi_{\mathrm{U}}\ \Pr\{Y < S + K \text{ and } Y < X\}$$
$$= \phi_{\mathrm{U}}\, F(S+K)\, \left(1 - \frac{F(S+K)}{2}\right)$$

and $\mathrm{E}[G_{\mathrm{B}}]_{(S,\ominus)}$ as calculated in (7). Finally, we have to determine the expected profits of both players for the case that no contract is written.

$$\mathrm{E}[G_{\mathrm{U}}]_{(S,\dagger)} = \mathrm{E}[G_{\mathrm{U}}]_{(\dagger,\dagger)} = \mathrm{E}[\tilde{Y}] - \phi_{\mathrm{U}}\ \Pr\{Y < K\} = \mathrm{E}[\tilde{Y}] - \phi_{\mathrm{U}}\, F(K). \tag{9}$$

B does not participate, which implies $\mathrm{E}[G_{\mathrm{B}}]_{(S,\dagger)} = \mathrm{E}[G_{\mathrm{B}}]_{(\dagger,\dagger)} = 0$. Now the expected profits given the moves of the players are known.

Table 1 about here.

When choosing between nonperformance and performance of the risk analysis, B is confronted with a tradeoff. If he risk-analyzes and draws up the fitting contract, he has to spend c_{B}, but faces no counterparty risk — U's bankruptcy is prevented by the contract. If he draws up a contract without risk-analyzing, he saves c_{B}, but runs the risk that U cannot pay although intended by contract. As one can see from Table 1, the expected financial loss due to counterparty risk amounts to $\kappa(S) = \int_0^S F(X)\,(1 - F(X+K))\, dX$. B is indifferent between $\oplus$ and $\ominus$ if $c_{\mathrm{B}} = \kappa(S)$. In order to derive the sequential equilibria, one must now deduce further properties of indifference points.

Proposition 1 (Indifference Points of B) *If U offers a contract to B, he proposes terms S such that U is indifferent between two moves.*

Proof: This follows from the fact that U's expected profits decrease monotonically with S, for $\partial \mathrm{E}[G]_{(S,\oplus)}/\partial S = F(S+K) - 1 < 0$ and $\partial \mathrm{E}[G]_{(S,\ominus)}/\partial S = (F(S) - 1 - \phi_{\mathrm{U}}\, f(S+K))\,(1 - F(S+K)) < 0$. ■

Proposition 1 implies that only a finite set of terms S needs to be considered: $S_{(\oplus \overset{B}{\sim} \ominus)}$ where B is indifferent between a contract with risk analysis and a contract without, $S_{(\oplus \overset{B}{\sim} \dagger)}$ where B is indifferent between a contract with risk analysis and no contract at all, and $S_{(\ominus \overset{B}{\sim} \dagger)}$ where B is indifferent between a contract without risk analysis and no contract.

Remark 3 (Existence and Uniqueness of Indifference Points) *If the point of intersection* $S_{(\oplus \overset{B}{\sim} \ominus)}$ *[*$S_{(\ominus \overset{B}{\sim} \dagger)}$*,* $S_{(\oplus \overset{B}{\sim} \dagger)}$*] exists, it is unique, and for all* $S < S_{(\oplus \overset{B}{\sim} \ominus)}$ *[*$S < S_{(\ominus \overset{B}{\sim} \dagger)}$*,* $S < S_{(\oplus \overset{B}{\sim} \dagger)}$*], B prefers not to carry out the risk analysis [not to draw up any contract, not to draw up any contract].*

If $S_{(\oplus \overset{B}{\sim} \ominus)}$ *[*$S_{(\ominus \overset{B}{\sim} \dagger)}$*,* $S_{(\oplus \overset{B}{\sim} \dagger)}$*] does not exist, B always prefers to carry out the risk analysis [not to draw up any contract, not to draw up any contract].*

Proof: This follows from the fact that $\partial \mathrm{E}[G_{\mathrm{B}}]_{(\dagger,\dagger)}/\partial S < \partial \mathrm{E}[G_{\mathrm{B}}]_{(S,\ominus)}/\partial S < \partial \mathrm{E}[G_{\mathrm{B}}]_{(S,\oplus)}/\partial S$ and in $S = 0$, $\mathrm{E}[G_{\mathrm{B}}]_{(\dagger,\dagger)} > \mathrm{E}[G_{\mathrm{B}}]_{(S,\ominus)} > \mathrm{E}[G_{\mathrm{B}}]_{(S,\oplus)}$. ■

Figure 2 about here.

Proposition 2 (Categorization of Sequential Equilibria) *Depending on the exogenous parameters* F*,* K*,* ϕ_{U} *and* c_{B}*, sequential equilibrium* $\sigma^* = (\sigma^*_{\mathrm{U}}, \sigma^*_{\mathrm{B}})$ *is in one of the four categories*

$$\begin{aligned}
&\sigma^*_0 \text{ with } \sigma^*_{\mathrm{U}} = \dagger, && \sigma^*_{\mathrm{B}} = \dagger \text{ and } \mathrm{E}[G_{\mathrm{B}}] = 0;\\
&\sigma^*_1 \text{ with } \sigma^*_{\mathrm{U}} = S_{(\oplus \overset{B}{\sim} \dagger)}, && \sigma^*_{\mathrm{B}} = \oplus \text{ and } \mathrm{E}[G_{\mathrm{B}}] = 0;\\
&\sigma^*_2 \text{ with } \sigma^*_{\mathrm{U}} = S_{(\oplus \overset{B}{\sim} \ominus)}, && \sigma^*_{\mathrm{B}} = \oplus \text{ and } \mathrm{E}[G_{\mathrm{B}}] > 0; \qquad \text{or}\\
&\sigma^*_3 \text{ with } \sigma^*_{\mathrm{U}} = S_{(\ominus \overset{B}{\sim} \dagger)}, && \sigma^*_{\mathrm{B}} = \ominus \text{ and } \mathrm{E}[G_{\mathrm{B}}] = 0.
\end{aligned}$$

The *proof* draws on Remark 3. If U's global optimum is (as illustrated in Figure 2) reached with $\sigma^*_{\mathrm{B}} = \oplus$, then U implements S as small as possible. That being so, either $\sigma^*_{\mathrm{U}} = S_{(\oplus \overset{B}{\sim} \dagger)}$ with $\mathrm{E}[G_{\mathrm{B}}] = 0$ or $\sigma^*_{\mathrm{U}} = S_{(\oplus \overset{B}{\sim} \ominus)}$ with $\mathrm{E}[G_{\mathrm{B}}] > 0$ holds. If $\sigma^*_{\mathrm{B}} = \oplus$ is optimal for U, then the lower bound of potential S is $S_{(\ominus \overset{B}{\sim} \dagger)}$. If no contract is better than any contract, then U's strategy will be $\sigma^*_{\mathrm{U}} = \dagger$.[9] ■

The most interesting of the four categories of equilibria is surely σ^*_2. There, a delegation of risk analysis is advantageous for U. However, if U offered B a contract setting the terms S sufficiently low that $\mathrm{E}[G_{\mathrm{B}}] = 0$, then B would choose not to risk-analyze U's project. In order to create sufficient incentives, U must raise S and thus B's counterparty risk $\kappa(S)$ until $\kappa(S) = c_{\mathrm{B}}$. Under this condition, B risk-analyzes the project, draws up the fitting contract, and his expected profits $\mathrm{E}[G_{\mathrm{B}}]$ are positive.

By contrast, in σ^*_1 the delegation of risk analysis is favorable for U, but an S with $\mathrm{E}[G_{\mathrm{B}}] = 0$ is sufficient to implement incentives for B to risk-analyze U's project. In σ^*_3, a contract based on B's guess is written — U's project is never risk-analyzed.

[9] There are in fact more equilibria. However, the corresponding payments are equal those of one of the four equilibria listed above. E. g., instead of playing $\sigma^*_{\mathrm{U}} = \dagger$, U can set $S = 0$, which implies $\mathrm{E}[G_{\mathrm{B}}] < 0$. As a result, B cannot possibly agree. The equilibrium $\sigma^* = (\sigma^*_{\mathrm{U}} = 0, \sigma^*_{\mathrm{B}} = \dagger)$ is therefore considered as equivalent to $\sigma^*_0 = (\sigma^*_{\mathrm{U}} = \dagger, \sigma^*_{\mathrm{B}} = \dagger)$.

Because $\mathrm{E}[G_{\mathrm{B}}] = 0$, the contract leads to a mean preserving spread for U, which in this case reduces U's bankruptcy risk. If the mean preserving spread generated by the contract cannot reduce the bankruptcy risk, then no contract is reached (σ_0^*).

Figure 3 about here.

4 Discussion (Part I)

Having categorized the possible sequential equilibria, it is now of interest to deduce how the endogenous variables react to changes of exogenous parameters. In the following Propositions 3 and 4, we concentrate on equilibria of the form σ_2^*. U can solve the delegation problem and make B risk-analyze ($\sigma_{\mathrm{B}} = \oplus$) only by obliging U through raising the terms S to $S_{\oplus \overset{B}{\sim} \ominus}$.

Proposition 3 (Comparative Statics for S^*) *Let F, K, ϕ_{U} and c_{B} be such that equilibrium has the form σ_2^*. Then*

$$\frac{\partial S^*}{\partial c_{\mathrm{B}}} > 0 \quad \textit{and} \quad \frac{\partial S^*}{\partial K} > 0 \tag{10}$$

Proof: In σ_2^*, B is indifferent between $\oplus$ and $\ominus$, so

$$c_{\mathrm{B}} = \int_0^{S^*} F(X)\,(1 - F(X + K))\,dX. \tag{11}$$

Differentiation of (11) subject to c_{B} yields

$$\begin{aligned} 1 &= F(S^*)\,(1 - F(S^* + K))\,\frac{\partial S^*}{\partial c_{\mathrm{B}}} \quad \text{and thus} \\ \frac{\partial S^*}{\partial c_{\mathrm{B}}} &= \frac{1}{F(S^*)\,(1 - F(S^* + K))} > 0. \quad \text{Analogously,} \\ \frac{\partial S^*}{\partial K} &= \frac{\int_0^{S^*} F(X)\,f(X+K)\,dX}{F(S^*)\,(1 - F(S^* + K))} > 0, \end{aligned}$$

which was to be shown. ■

The results of Proposition 3 can be used to analyze the expected profits' dependence on exogenous parameters.

Proposition 4 (Comparative Statics for $\mathrm{E}[G_\mathrm{B}]$ and $\mathrm{E}[G_\mathrm{U}]$) *Let F, K, ϕ_U and c_B be such that equilibrium has the form σ_2^*. Then*[10]

$$\frac{\partial \mathrm{E}[G_\mathrm{B}]}{\partial K} < 0, \qquad \frac{\partial \mathrm{E}[G_\mathrm{U}]}{\partial K} > 0,$$
$$\frac{\partial \mathrm{E}[G_\mathrm{B}]}{\partial c_\mathrm{B}} > 0, \qquad \frac{\partial \mathrm{E}[G_\mathrm{U}]}{\partial c_\mathrm{B}} < 0.$$

Furthermore, $\partial \mathrm{E}[G_\mathrm{B}]/\partial \phi_\mathrm{U} = \partial \mathrm{E}[G_\mathrm{U}]/\partial \phi_\mathrm{U} = 0$.

Proof:

$$\begin{aligned}
\mathrm{E}[G_\mathrm{U}] &= \mathrm{E}[\tilde{Y}] - S^* + \mathcal{F}(S^* + K) \\
\text{s.t.} \quad c_\mathrm{B} &= \int_0^{S^*} F(X)\,(1 - F(X+K))\,dX, \text{ thus} \\
\frac{\partial \mathrm{E}[G_\mathrm{U}]}{\partial K} &= F(S^*+K) - (1 - F(S^*+K))\,\frac{\partial S^*}{\partial K} \\
&= F(S^*+K) - (1 - F(S^*+K))\,\frac{\int_0^{S^*} F(X)\,f(X+K)\,dX}{F(S^*)\,(1 - F(S^*+K))}
\end{aligned}$$

which is negative if and only if

$$\begin{aligned}
F(S^*+K)\,F(S^*) &< \int_0^{S^*} F(X)\,f(X+K)\,dX \\
&= F(S^*)\,F(S^*+K) - \int_0^{S^*} f(X)\,F(X+K)\,dX \qquad \Longleftrightarrow \\
0 &> \int_0^{S^*} f(X)\,F(X+K)\,dX,
\end{aligned}$$

which is a contradiction. Hence $\partial \mathrm{E}[G_\mathrm{U}]/\partial K > 0$ holds. Furthermore,

$$\begin{aligned}
\frac{\partial \mathrm{E}[G_\mathrm{U}]}{\partial c_\mathrm{B}} &= -(1 - F(S^*+K))\,\frac{\partial S^*}{\partial c_\mathrm{B}} < 0, \\
\frac{\partial \mathrm{E}[G_\mathrm{B}]}{\partial K} &= -\frac{\partial \mathrm{E}[G_\mathrm{U}]}{\partial K} < 0 \quad \text{and} \\
\frac{\partial \mathrm{E}[G_\mathrm{B}]}{\partial c_\mathrm{B}} &= -\frac{\partial \mathrm{E}[G_\mathrm{U}]}{\partial c_\mathrm{B}} - 1 \\
&= (1 - F(S^*+K))\,\frac{1}{F(S^*)\,(1 - F(S^*+K))} - 1 = \frac{1}{F(S^*)} - 1 > 0.
\end{aligned}$$

Obviously, $\partial \mathrm{E}[G_\mathrm{B}]/\partial \phi_\mathrm{U} = \partial \mathrm{E}[G_\mathrm{U}]/\partial \phi_\mathrm{U} = 0$. ■

[10] Note that if one of the variables changes too much, equilibrium may switch from σ_2^* to a different form. The derived inequations apply accordingly only locally in the interior of σ_2^*.

Note that nothing can be said about how the expected profits of the players react to changes of F in general.[11] One of the results surprises at first sight: B's expected profits increase with the costs. Intuitively, if the costs of risk analysis rise, U must raise B's counterparty risk $\kappa(S)$ and as a result S in order to keep B indifferent between risk analysis and guessing. Proposition 4 shows that the effect of the risen S outweighs the effect of the higher c_{B}. Consequently, B is overcompensated for his increased costs.

Proposition 5 (Credit Standing of B's Customers) *Given F, c_{B} and ϕ_{U}, an entrepreneur with especially high or low credit standing does not offer B a contract such that B risk-analyzes U's project in equilibrium (hence equilibrium has the form σ_1^* or σ_2^*).*[12]

Proof: First note that because of F's monotonicity, a high (low) K is equivalent to a low (high) credit standing. If K is low ($K \approx 0$), then $\mathrm{E}[G_{\mathrm{U}}]_\dagger = \mathrm{E}[\tilde{Y}] - \phi_{\mathrm{U}}\, F(K) \approx \mathrm{E}[\tilde{Y}]$. If B and U sign a contract and B risk-analyzes, then $\mathrm{E}[G_{\mathrm{B}}] + \mathrm{E}[G_{\mathrm{U}}] = \mathrm{E}[\tilde{Y}] - c_{\mathrm{B}}$, thus $\mathrm{E}[G_{\mathrm{B}}] \geq 0$ implies $\mathrm{E}[G_{\mathrm{U}}] \leq \mathrm{E}[\tilde{Y}] - c_{\mathrm{B}}$. Risk analysis is too expensive for $K \approx 0$. For the case of high K, note that

$$\begin{aligned}\kappa(S) &= \int_0^S F(X)\,(1 - F(X+K))\,dX \\ &\leq S\,F(S)\,(1 - F(K)).\end{aligned}$$

If $\tilde{Y}$ is integrable then $S\,F(S)$ is bounded, and for high K and thus small $1 - F(K)$, there is no S with $\kappa(S) = c_{\mathrm{B}}$. ■

Consequently, the customers of B have a medium credit standing. For entrepreneurs with a high standing, the costs of risk analysis (inclusive of delegation costs) are too high. Entrepreneurs with a low standing cannot afford a hedge contract. The costs of insurance exceed expected profits.

5 Implications of a Loan

In this section, we analyze how a loan — as an example for a deeper financial intertwining between entrepreneur and risk analyst — influences the delegation of risk analysis. First, we take the level of debt as given exogenously.

Assume that U has already received a loan of the amount l with repayment L, which contains redemption and interest. The maturity period is identical with the

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[11] One might assume that $\log \tilde{Y}$ is e.g. normally distributed and then calculate $\partial\mathrm{E}[G_{\mathrm{U}}]/\partial\mu$, $\partial\mathrm{E}[G_{\mathrm{U}}]/\partial\sigma$ and so forth.

[12] Proposition 5 is illustrated by Figure 3: For fixed c_{B}, B risk-analyzes only for medium K.

project's life span. U is liable only with the project's result. If U does not need l to finance the project, one may imagine that he consumes l directly.[13] Because of the additional liabilities of U due to the loan, the bankruptcy threshold rises from K to $K + L$. That being so, a hedge contract between U and B must be based on $K + L$.

Assuming that B has risk-analyzed U's project and written the fitting contract ($\sigma_{\mathrm{B}} = \oplus$), B receives S (plus L because of U's liabilities towards B) only if the project delivers more than $S + K + L$. Otherwise B receives the whole output Y less K which he transfers to U in order to avoid bankruptcy. Thus[14]

$$\begin{aligned}
\mathrm{E}[G_{\mathrm{B}}]_{(S,\oplus),(L,l)} &= \int_0^{S+K+L} (Y-K)\, f(Y)\, dY + \int_{S+K+L}^{\infty} (S+L)\, f(Y)\, dY - c_{\mathrm{B}} - l \\
&= S + L - \mathcal{F}(S+K+L) - c_{\mathrm{B}} - l, \text{ and} \qquad (12) \\
\mathrm{E}[G_{\mathrm{U}}]_{(S,\oplus),(L,l)} &= \mathrm{E}[\tilde{Y}] - S - L + \mathcal{F}(S+K+L) + l. \qquad (13)
\end{aligned}$$

The same procedure can be applied to calculate the expected profits for the other strategies,

$$\begin{aligned}
\mathrm{E}[G_{\mathrm{B}}]_{(S,\ominus),(L,l)} &= \int_0^{\infty}\int_0^{\infty} \min\{Y, X-K, S+L\}\, f(Y)\, dY\, f(X)\, dX - l \\
&= S + L - \mathcal{F}(S+L) - \int_0^{S+L} F(X)\,(1 - F(X+K))\, dX - l. \qquad (14) \\
\mathrm{E}[G_{\mathrm{U}}]_{(S,\ominus),(L,l)} &= \mathrm{E}[\tilde{Y}] - \mathrm{E}[G_{\mathrm{B}}]_{(S,\ominus),(L,l)} - \mathrm{E}[\phi_{\mathrm{U}}]_{(S,\ominus),(L,l)} \quad \text{with} \qquad (15) \\
\mathrm{E}[\phi_{\mathrm{U}}]_{(S,\ominus),(L,l)} &= \phi_{\mathrm{U}} \Pr\{Y - Z < K + L\} = \phi_{\mathrm{U}} \Pr\{Y < S + L + K \text{ and } Y < X\} \\
&= \phi_{\mathrm{U}}\, F(S+L+K)\left(1 - \frac{F(S+L+K)}{2}\right)
\end{aligned}$$

and $\mathrm{E}[G_{\mathrm{B}}]_{S,\ominus}$ as calculated in (14). Furthermore,

$$\begin{aligned}
\mathrm{E}[G_{\mathrm{B}}]_{(\dagger,\dagger),(L,l)} &= L - \mathcal{F}(L) - l, \\
\mathrm{E}[G_{\mathrm{U}}]_{(\dagger,\dagger),(L,l)} &= \mathrm{E}[\tilde{Y}] + l - L + \mathcal{F}(L) - \phi_{\mathrm{U}}\, F(K+L).
\end{aligned}$$

One can now determine how the loan parameters l and L influence the expected profits of each strategy.

Remark 4 (Relocation of the Indifference Point)

$$\begin{aligned}
\mathrm{E}[G_{\mathrm{B}}]_{(S,\oplus),(L,l)} &= \mathrm{E}[G_{\mathrm{B}}]_{(S+L,\oplus),(0,l)} \ \textit{and} \\
\mathrm{E}[G_{\mathrm{B}}]_{(S,\ominus),(L,l)} &= \mathrm{E}[G_{\mathrm{B}}]_{(S+L,\ominus),(0,l)}, \ \textit{thus} \\
S_{(\oplus \overset{B}{\sim} \ominus),(L,l)} &= S_{(\oplus \overset{B}{\sim} \ominus)} - L. \qquad (16)
\end{aligned}$$

[13] Note that the reason for U to borrow may, but need not be the need of funding the project.

[14] Unfortunately, the expected profits must now be indexed with two more variables, L and l. Note that $\mathrm{E}[G_{\mathrm{B}}]_{(S,\oplus),(L=0,l=0)} = \mathrm{E}[G_{\mathrm{B}}]_{(S,\oplus)}$ and $\mathrm{E}[G_{\mathrm{U}}]_{(S,\oplus),(L=0,l=0)} = \mathrm{E}[G_{\mathrm{U}}]_{(S,\oplus)}$, for $L = l = 0$ means that no loan exits. The same applies for $\mathrm{E}[G_{\mathrm{B}}]_{(S,\ominus)}, \mathrm{E}[G_{\mathrm{U}}]_{(S,\ominus)}$, and so forth.

Comparing (4) with (12) and (7) with (14) yields the *proof.* ■

Remark 4 has far-reaching consequences on equilibrium. As (16) states, the higher U's indebtedness to B, the less further incentives (by raising S) U must offer to make B indifferent between risk-analysis and no risk-analysis. As a result, one can summarize that debt mitigates the delegation problem of risk analysis between U and B.

Let us now extend Game 1 by assuming that L and l can be chosen endogenously. Before entering the subgame of risk analysis delegation (as described in Figure 1), U can propose to take up a loan (determined by L and l) at B. B can accept reject U. Furthermore, assume that U disposes over all market power: If U rejects him, he can apply for a loan at another risk analyst.[15] Because U can now choose L and l, his space of moves is enlarged: $\sigma_{\mathrm{U}} \in \{L, l\} \times \{S, \dagger\}$ with $L, l, S \in \mathbb{R}$. B can reject the loan *and/or* the hedge contract, $\sigma_{\mathrm{B}} \in \{\checkmark, \dagger\} \times \{\oplus, \ominus, \dagger\}$, where ✓ stands for accepting the loan. In the following, the game described above is called Game 2.

Figure 4 about here.

Proposition 6 (Relocation of Expected Profits) *Assume that F, c_{B}, ϕ_{U} and K are such that in Game 1 (without debt),* $\mathrm{E}[G_{\mathrm{B}}]_{l=0,L=0} > 0$ *(thus equilibrium has the form σ_2^*). Then with the same exogenous parameters in Game 2 (with debt), if U chooses $\bar{l} \in [0, \mathrm{E}[G_{\mathrm{B}}]_{l=0,L=0}]$ and S and $\bar{L}$ such that $\bar{l} = \bar{L} - \mathcal{F}(\bar{L})$, $S = S_{(\oplus \overset{B}{\sim} \ominus),(\bar{l},\bar{L})} - \bar{L}$, then*

$$\mathrm{E}[G_{\mathrm{U}}]_{l=\bar{l}} = \mathrm{E}[G_{\mathrm{U}}]_{l=0} + \bar{l} \quad \textit{and} \quad \mathrm{E}[G_{\mathrm{B}}]_{l=\bar{l}} = \mathrm{E}[G_{\mathrm{B}}]_{l=0} - \bar{l}.$$

Proof: (16) means that with raising L, S is lowered by the same amount. The sum $S + L$ remains constant. Only the change of l affects expected profits, and as one can see from (12) and (13), $\partial \mathrm{E}[G_{\mathrm{B}}]/\partial l = -1$ and $\partial \mathrm{E}[G_{\mathrm{U}}]/\partial l = 1$ if $\sigma_{\mathrm{U}} = \oplus$. The condition $\bar{l} = \bar{L} - \mathcal{F}(\bar{L})$ guarantees that after granting the loan, B and U are still interested in achieving a hedge contract. ■

According to Proposition 6, if B's expected profits are positive, a higher level of debt l lets them shrink, whereas those of U grow. This is economically intuitive: Without the loan, U would have to oblige B by increasing S, otherwise B would not choose to risk-analyze ($\sigma_{\mathrm{U}} = \oplus$). Now if U is indebted at U, U already has a genuine incentive to help U to avoid his bankruptcy. This is due to the interest-harmonizing impact of debt. Because U understands this impact, he applies for a loan at B. In other words, he holds out the prospect of the positive expected profits of the hedge contract for B, but only after accepting the loan. If B rejects the loan, U turns to another risk analyst.

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[15] Note that the reason for rejection cannot be a low credit standing, as the probability of U's default is public information. If U's credit standing is low, he knows that no risk analyst would grant the loan, so he does not need to apply.

Proposition 7 (Categorization of Sequential Equilibria) *Depending on the exogenous parameters F, K, ϕ_{U} and c_{B}, sequential equilibrium $\sigma_l^* = (\sigma_{l,\mathrm{U}}^*, \sigma_{l,\mathrm{B}}^*)$ is in one of the four categories*

$$\begin{array}{llll}
\sigma_{l,0}^* \textit{ with } \sigma_{l,\mathrm{U}}^* = (l=0, \dagger), & \sigma_{l,\mathrm{B}}^* = \dagger \textit{ and } \mathrm{E}[G_{\mathrm{B}}] = 0; & \\
\sigma_{l,1}^* \textit{ with } \sigma_{l,\mathrm{U}}^* = (l=0, S_{(\oplus \overset{B}{\sim} \dagger)}), & \sigma_{l,\mathrm{B}}^* = \oplus \textit{ and } \mathrm{E}[G_{\mathrm{B}}] = 0; & \\
\sigma_{l,2}^* \textit{ with } \sigma_{l,\mathrm{U}}^* = (l>0, S_{(\oplus \overset{B}{\sim} \ominus)}), & \sigma_{l,\mathrm{B}}^* = \oplus \textit{ and } \mathrm{E}[G_{\mathrm{B}}] = 0; & \textit{or} \\
\sigma_{l,3}^* \textit{ with } \sigma_{l,\mathrm{U}}^* = (l=0, S_{(\ominus \overset{B}{\sim} \dagger)}), & \sigma_{l,\mathrm{B}}^* = \ominus \textit{ and } \mathrm{E}[G_{\mathrm{B}}] = 0. &
\end{array}$$

The *proof* can be led analogously to the one of Proposition 2. U chooses $S + L$ just large enough to guarantee the desired response of B. A positive $\mathrm{E}[G_{\mathrm{B}}]$ is impossible, because of Proposition 6, U would rather raise l and pocket the expected profits himself.[16] ■

Again, not all four equilibria are of major interest. In $\sigma_{l,0}^*$ (as in σ_0^*), no contract between U and B is reached, so there is no need for harmonization of interests. In $\sigma_{l,3}^*$ (as in σ_3^*), U chooses not to raise $S + L$ until incentive compatibility is given, hence again nothing speaks for $l > 0$. In $\sigma_{l,1}^*$ (as in σ_1^*), incentive compatibility is reached already with $l = L = 0$. Only in $\sigma_{l,3}^*$, U applies for a loan in order to lower delegation costs by harmonizing interests with B.

Because loans are advantageous in only one of the four types of equilibria, a remark on the relevance of the results may be appropriate. From an economic point of view, it is clear that there is no scope for loans in all equilibria. The reason is that only one aspect of loans is considered — the harmonization of diverging interests. Other important aspects, such as the raising of funds for investment, are not taken into account.[17] Of course, parameter constellations under which there is no scope for the delegation of risk analysis are conceivable, such as high analysis costs ($c_{\mathrm{B}} \to \infty$) or a high credit standing of U ($K \to 0$). In the model, the missing desirability of risk analysis ruins possible advantages of a loan. However, which type of equilibrium is most realistic depends on the parameter assignment (of F, K, ϕ_{U} and c_{B}) that comes closest to reality.

Figure 5 about here.

Remark 5 (Comparison of Equilibria) *By allowing for the possibility of the loan, the delegation of risk analysis gains importance.*

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[16] Analogously to Proposition 2 (as stated in Footnote 9), there are additional equilibria which are equivalent to the ones listed above with reference to the payments.

[17] The lack of disadvantages of debt is also the reason that the l of Proposition 7 is only a lower bound for an optimal loan amount. A further increase of l leads neither to advantages nor to disadvantages, neither for U nor for B.

In other words, parameter constellations under which σ_0^* of σ_3^* are equilibria in Game 1 (with $\sigma_{\mathrm{B}}^* = \ominus$) may lead to equilibria of the type $\sigma_{l,2}^*$ in Game 2 (with $l > 0$ and $\sigma_{\mathrm{B}}^* = \oplus$). Delegation of risk analysis becomes more likely. The reason is that delegation costs that may be prohibitively high in Game 1 are lowered in Game 2. The opposite direction of change of equilibrium is not possible. The parameter constellations under which $\sigma_{l,1}^*$ and σ_1^* form the equilibrium are identical. Therefore, the "area" in which risk analysis takes place expands.[18]

6 Discussion (Part II)

Now that a loan between entrepreneur and risk manager is considered, a further discussion of results is appropriate.

Remark 6 (Comparative Statics for l) *Let F, K, ϕ_{U} and c_{B} be such that equilibrium has the form $\sigma_{l,2}^*$. Then*

$$\frac{\partial l}{\partial K} < 0 \quad \text{and} \quad \frac{\partial l}{\partial c_{\mathrm{B}}} > 0.$$

Furthermore, $\partial l / \partial \phi_{\mathrm{U}} = 0$.

Proof: By Proposition 6, each additional monetary unit of l increases $\mathrm{E}[G_{\mathrm{U}}]$ and decreases $\mathrm{E}[G_{\mathrm{B}}]$ by one unit. The rest can be derived from Proposition 4. ∎

Note that B grants loans only to entrepreneurs with medium credit standing. This can be derived directly from Proposition 5. Beyond the comparative statics, further aspects may be worth discussing. The following aspects are unrelated, so the order is arbitrary.

(i) In the model, it is the counterparty risk created by the loan that harmonizes interests. Because collateral curtails the potential losses for the risk analyst, it has a worsening effect on the delegation problem. For this reason, the costs of collateral for the entrepreneur may be higher and the optimal degree of collateral lower than without consideration of the delegation problem.

(ii) In Remark 6 as well as Proposition 4, the size of the entrepreneur is disregarded. Are firms that delegate risk analysis rather small or large? However, assuming that there is a size variable ς and that all monetary variables concerning the enterprise (thus $\tilde{Y}$, K and ϕ_{U}) are proportional to ς, the answer can be derived easily. The only monetary variable not concerning U are the costs of risk analysis c_{B}. Choosing ς as numéraire, c_{B} becomes inversely proportional to ς. A doubling of the firm's

[18] For an illustration, compare Figure 3 with Figure 5.

size ς has the same effect on the model as a halving of the costs of risk analysis c_B. Now assuming that c_B is low ($c_B \approx 0$), the terms S^* that make B indifferent between $\oplus$ and $\ominus$ are small ($S^* \approx 0$, cf. (11)). Therefore, U does not need to oblige B by raising S^* above the "fair" level in order to make B risk-analyze. There are no delegation costs for U, and the first best solution can be reached. Hence large firms do delegate risk analysis, and they do not need to take up a loan in order to reduce delegation costs. By contrast, $\varsigma \approx 0$ is equivalent to $c_B \approx \infty$, implying that the entrepreneur does not delegate risk analysis. However, no monotonic relation between e. g. expected profits of the risk analyst and size of the firm exist.[19]

(iii) We must discuss the question whether the risk analyst B can be interpreted as a bank. The answer to this question naturally depends on how narrow one chooses the definition of the word "bank". If one (like Allen and Gale (2000)) distinguishes between banks (that refinance by illiquid deposits and are subject to bank runs) and intermediaries (that refinance by means of bonds), the question must remain unanswered. The risk analyst's refinancing is not considered in the model. If one defines banks as firms that grant loans, the risk analyst surely is a bank. At least, the entrepreneur prefers to borrow from his risk analyst. Because in the described model risk analysts have no disadvantage in lending compared to other organizations, risk analysts are lenders and thus banks.

(iv) Note that the preceding results can be reinterpreted in a different manner. In Game 1 (without loan), the constellations of parameters are such that in equilibrium, the expected profits of B are positive. In Game 2 (with the loan), U has to cede part of his expected profits to B in order to create incentive compatibility. Now Remark 3 states that it is irrelevant whether incentive compatibility is achieved by higher terms S or a loan repayment L — which is coupled with the loan amount l. One can define a different game which leads to the same results: U auctions off the prospect of the contract. The bidders are risk analysts. A risk analyst then bids as high as the future expected payoffs of the hedge contract.[20] In equilibrium, all expected cash flows are identical with the ones of Game 2.

7 Conclusion

As the last sections have shown, an agent that is able to obtain information about the correlations of risky assets endogenously carries out risk analysis, risk controlling and grants loans, all on behalf of a principal, an entrepreneur. The model suggests

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[19] The reason is that an increase of the numéraire ς directly increases expected profits, but via decreasing c_B decreases $E[G_B]$. There are therefore two opposite effects of which none strictly dominates the other.

[20] The auction's mechanism (English, Dutch, Vickrey, First Price) is irrelevant if risk analysts have the same information about the value of the future contract.

thus that there are economies of scope between lending and risk management. It can therefore be seen as one possible answer to the complaint of Allen and Santomero (1998) that there is no cogent theory as to why banks should be the ones offering the service of risk management.

Several simplifying assumptions allowed us to derive our results analytically. The following extensions seem desirable from an empirical point of view, but may render the explicit analysis of the model impossible.

(i) Unlike the entrepreneur, the risk analyst is not threatened by bankruptcy in the model. This has two implications. First, the entrepreneur does not need to take the risk analyst's possible insolvency into consideration. As a result, the risk analyst can easily insure any conceivable risk. Second, unlike a real bank, the risk analyst has no intrinsic incentive to manage (own) risk. The incentive emerges extrinsically from the entrepreneur's delegation. As a possible extension of the above model, one can impose financial restrictions on the risk analyst. The risk analyst then weighs up his own against his customers' interests risk sharing. In case of doubt, the customers' interests may be neglected.

(ii) The entrepreneur's initial probability of bankruptcy is given exogenously. In reality, it seems obvious that this probability can be influenced, e. g. by the way the entrepreneur operates his business. As an example, the entrepreneur may be able to agree upon different terms with his customers. The terms that his business partners grant may in turn depend on the entrepreneurs decision about whether to hedge against bankruptcy risk.[21] A possible extension of the above model might consider the reaction of the entrepreneur's business partners on his hedging decision.

(iii) In the model, the risk analyst can choose between carrying out the risk analysis or refrain from it.[22] In reality, one should think that the risk analyst's set of options is larger and smoother. He may e. g. choose between different levels of effort when carrying out the risk analysis.

[21] The price a customer is willing to pay for a good may e. g. depend on the producers probability of bankruptcy, especially if the good need future servicing. Even multiple equilibria might exist: If an entrepreneur's business partners — especially his lenders — believe that his credit worthiness is high, they will demand only a low risk premium, which reduces the entrepreneur's cost of funds and thus his probability of bankruptcy. When they believe that the credit worthiness is low, one can argue the other way round.

[22] This causes the jumps in the expected profits functions of Figure 2.

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A Figures and Tables

Figure 1: Time Structure of Game 1 (without Loan)

- $t = 0$: U "finds" a project and decides on its realization.
- U decides whether to offer B a hedge contract. If he offers a contract, he chooses its terms S.
- B decides whether to accept the contract. If he accepts, he further decides whether to risk-analyze U's project, i. e. to find out the project's state dependency $\tilde{Y}$.
- B decides how to draw up the state dependencies $\tilde{Z}$ of the contract.[23]
- U signs the contract and carries out the project.
- $t = 1$: Nature "chooses" a state ω and with it a project realization $\tilde{Y}(\omega)$ and a contract liability $\tilde{Z}(\omega)$.
- If possible, payments flow. If $\tilde{Y}(\omega) - \tilde{Z}(\omega) < K$, the entrepreneur files for bankruptcy which leads to costs of ϕ_{U}.

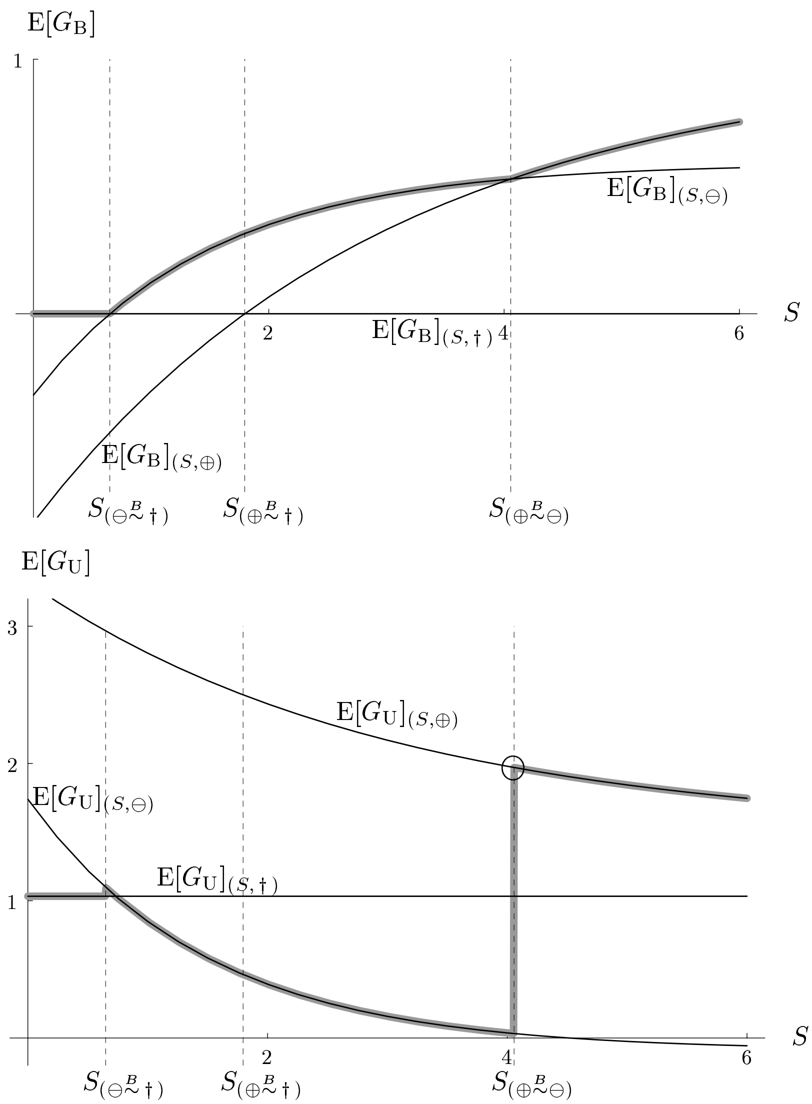
Table 1: Survey of Expected Profits in Dependence on Selected Moves

	Risk Analysis $\sigma = (S, \oplus)$	No Risk Analysis $\sigma = (S, \ominus)$	No Contract $\sigma = (\dagger, \dagger)$
$E[G_{\mathrm{B}}]_\sigma$	$S - \mathcal{F}(S+K) - c_{\mathrm{B}}$	$S - \mathcal{F}(S+K)$ $- \int_0^S F(X)\,(1 - F(X+K))\,dX$	0
$E[G_{\mathrm{U}}]_\sigma$	$\mathrm{E}[\tilde{Y}] - S + \mathcal{F}(S+K)$	$\mathrm{E}[\tilde{Y}] - S + \mathcal{F}(S+K)$ $+ \int_0^S F(X)\,(1 - F(X+K))\,dX$ $- \phi_{\mathrm{U}}\,F(S+K)\,\frac{2 - F(S+K)}{2}$	$\mathrm{E}[\tilde{Y}] - \phi_{\mathrm{U}}\,F(K)$
$\sum$	$\mathrm{E}[\tilde{Y}] - c_{\mathrm{B}}$	$\mathrm{E}[\tilde{Y}]$ $- \phi_{\mathrm{U}}\,F(S+K)\,\frac{2 - F(S+K)}{2}$	$\mathrm{E}[\tilde{Y}] - \phi_{\mathrm{U}}\,F(K)$

Note: The table contains the strategic form of Game 1: Expected profits are listed for all possible moves.

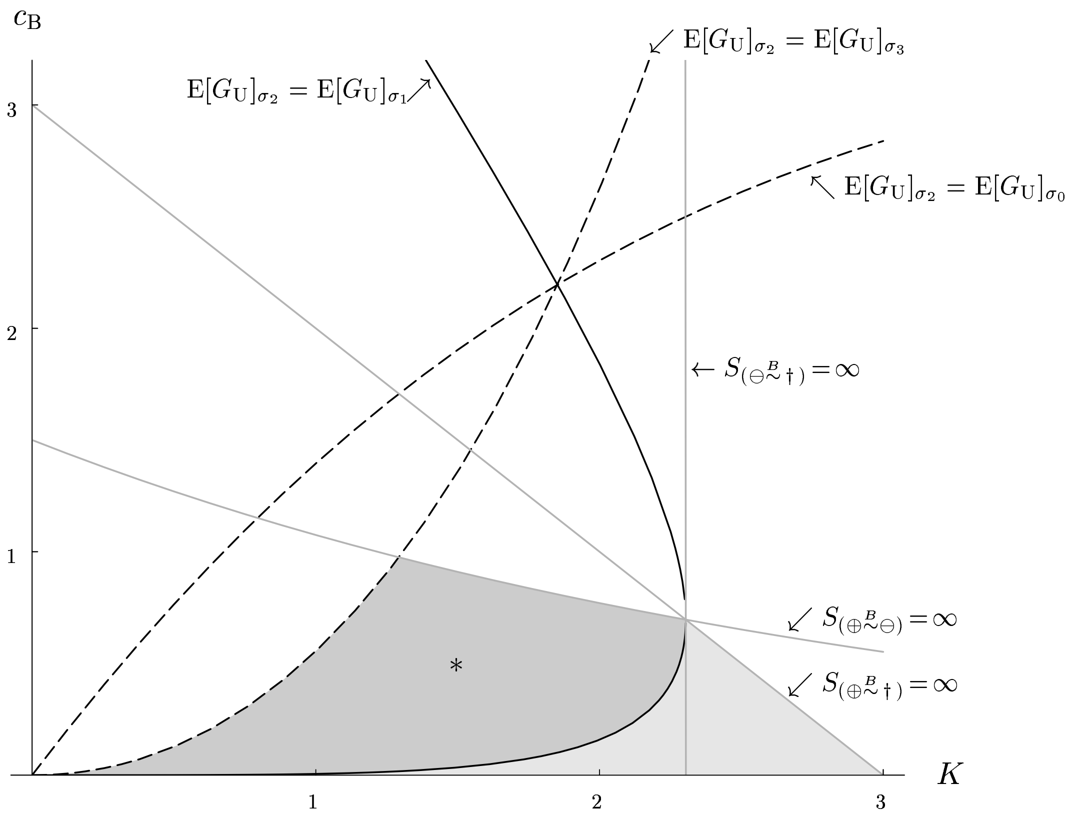
[23] B cannot decide on the terms S which has already been chosen by U. However, B can select the state dependencies $\tilde{Z}$ for U is unable to assess them.

Figure 2: Expected Profits of Risk Analyst and Entrepreneur as a Function of S



Note: The figure is based on exponential distribution of $\tilde{Y}$, thus $F(Y) = 1 - e^{-Y/\mu}$ with $\mu = 3$, furthermore $K = 1.5$, $c_\mathrm{B} = 0.5$ and $\phi_\mathrm{U} = 5$. U's expected profits are maximized when B is indifferent between risk-analysis and no risk-analysis ('$\bigcirc$') — the equilibrium has the form σ_2^*. Here, B's expected profits are positive.

Figure 3: Sequential Equilibria of Game 1 in Dependence on K and c_{B}

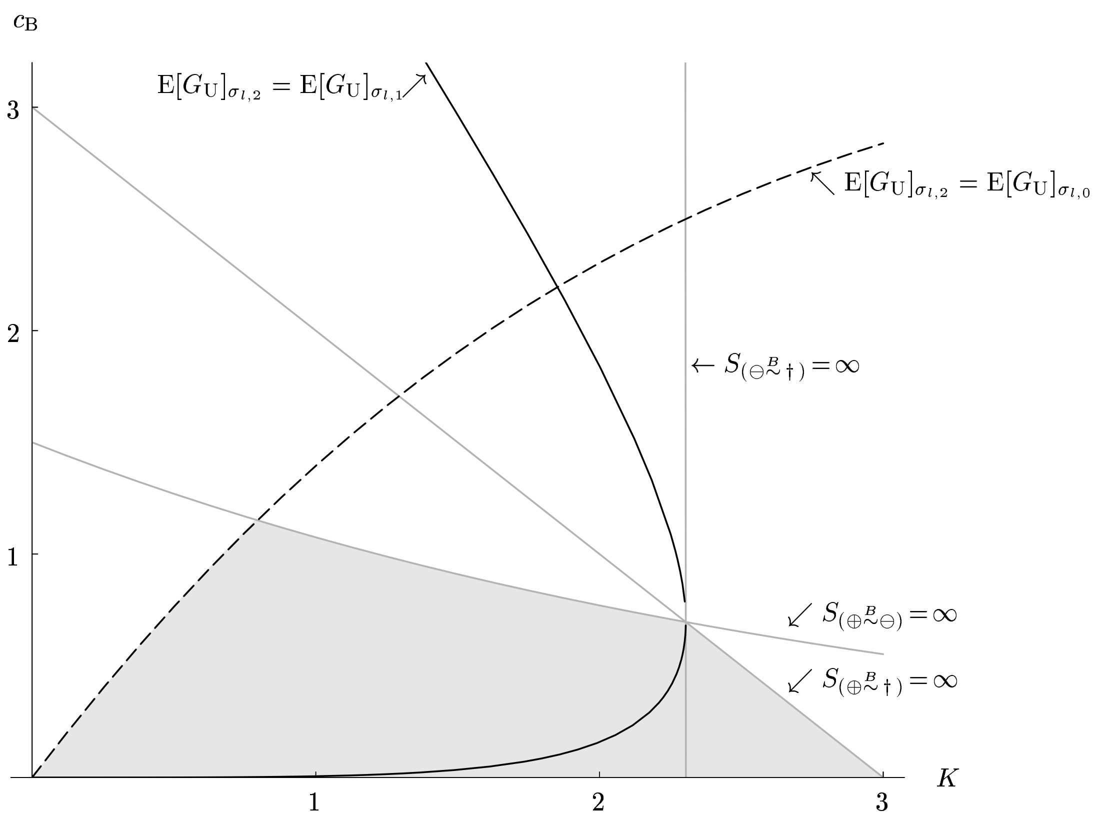


Note: Like in Figure 2, $\tilde{Y}$ is exponentially distributed with $\mu = 3$, and $\phi_{\mathrm{U}} = 5$. Depending on c_{B} and K, different sequential equilibria are played. Points where σ_2^* (with $\sigma_{\mathrm{B}}^* = \oplus$ and $\mathrm{E}[G_{\mathrm{B}}] > 0$) is played are backed dark gray, σ_1^* (with $\sigma_{\mathrm{B}}^* = \oplus$ and $\mathrm{E}[G_{\mathrm{B}}] = 0$) is backed light gray and σ_3^* is backed white. Under exponential distribution, σ_0^* is never played. Therefore, demarcation lines between σ_0^* and other equilibria are not noted. As demonstrated in Figure 2, σ_2^* is the equilibrium when $c_{\mathrm{B}} = 0.5$ and $K = 1.5$ ('$*$').

Figure 4: Time Structure of the Game 2 (with Loan)

- $t = 0$: U "finds" a project and decides on its realization.
- U applies for a loan at B, he proposes the loan amount l and the repayment L.
- B decides whether to grant the loan.
- U decides whether to offer B a hedge contract. If he offers a contract, he chooses its terms S.
- B decides whether to accept the contract. If he accepts, he further decides whether to risk-analyze U's project, i. e. to find out the project's state dependency $\tilde{Y}$.
- B decides how to draw up the state dependencies $\tilde{Z}$ of the contract.
- U signs the contract and carries out the project.
- $t = 1$: Nature "chooses" a state ω and with it a project realization $\tilde{Y}(\omega)$ and a contract liability $\tilde{Z}(\omega)$.
- If possible, payments flow. If $\tilde{Y}(\omega) - \tilde{Z}(\omega) < K$, the entrepreneur files for bankruptcy which leads to costs of ϕ_{U}.

Figure 5: Sequential Equilibria of Game 2 in Dependence on K and c_{B}



Note: Like in Figure 3, $\tilde{Y}$ is exponentially distributed with $\mu = 3$, and $\phi_{\mathrm{U}} = 5$. The area where U risk-analyzes ($\sigma^*_{\mathrm{U}} = \oplus$) is backed gray. l is positive in the gray area between the two curved black lines. In comparison with Figure 3, one can see that the gray area has expanded: The possibility of a loan makes the performance of risk analysis more likely.