Westfälische Wilhelms-Universität Münster

Institut für Kreditwesen
Direktor: Prof. Dr. Andreas Pfingsten

Institut für Kreditwesen

Diskussionsbeitrag 02-01

An Empirical Investigation of the Rank Correlation between different Risk Measures

Dipl.-Kfm. Carsten Hahn
Prof. Dr. Andreas Pfingsten
Dipl.-Kfm. Peter Wagner

ISSN 0949-6610

Februar 2002

Abstract

The Value at Risk (VaR) is widely used to measure the risk of banks' trading books, but it also has been criticized in the previous academic literature due to some odd properties. Other risk measures have been suggested as potentially being superior from a theoretical perspective. This gives rise to the question whether or not it makes much difference in practical applications which index is being used. Certainly, the numbers indicating risk will be different. Whenever risk measures are ordinal, however, it only matters whether or not the ranking of risky alternatives (portfolios, say) coincides. Therefore, taking actual data from two trading books, we analyze the rank correlation of different measures. In most cases we observe high values of Spearman's rank correlation coefficient, indicating that the index choice might not be too important. But it also becomes apparent that the VaR could as well be replaced by indices with a more convincing theoretical basis which do not generate a very different ranking of risks and in fact seem to compromise better between competing risk judgements. On the basis of theoretical arguments, we modify the data in various ways and repeat the analysis. It turns out that the results do not vary much, i.e., they seem to be quite robust.

Key Words: risk measures, downside risk, poverty measurement, shortfall risk measurement, trading book, rank correlation, Value at Risk, Lower Partial Moments.

JEL classification: D81, G20, I32

We are indebted to Paul Hagen, Robert Demon, and Jürgen W. Bauer (HSBC Trinkaus & Burkhardt, Düsseldorf) for the data and a stimulating discussion. Frank Altrock, Dan Gordon, Jose A. Lopez, Ulrich Müller-Funk, and Dirk Thiel, as well as referees, added further helpful suggestions. We also thank participants of the 56th European Meeting of the Econometric Society (Lausanne, Switzerland, August 2001), of the wi/if 2001 (Augsburg, Germany, September 2001), and of the Annual Conference of the German Society of Operations Research (GOR, Duisburg, Germany, September 2001) for discussions and for helpful comments on an earlier version of the paper. The responsibility for all shortcomings rests with us.

Westfälische Wilhelms-Universität Münster
Institut für Kreditwesen
Universitätsstraße 14-16
D-48143 Münster
Telefon: +49 251 83-22881
Telefax: +49 251 83-22882
E-Mail: `21anpf@wiwi.uni-muenster.de`

1 Introduction

One of the main problems in risk management is the correct quantification of risk. For this purpose several different risk measures have been developed. In financial analysis, for example the Value at Risk (*VaR*), which measures the loss that is only exceeded with a given probability (1 %, say) over a certain time period (e.g., one day), plays a key role in practical applications (cf. Jorion (2001)). Alas, the index did not find a likewise unanimous backing in academia. Artzner et al. (1997), Artzner et al. (1999), Guthoff et al. (1997a) and Johanning (1998) are among those who have contributed to a critical appraisal of *VaR*. These and other papers have also discussed alternative traditional risk measures which were thought to be superior for one reason or the other. Pfingsten et al. (2000) consider downside-risk measures derived from poverty measures as further candidates for a "good" risk measure.

It certainly is an intellectually demanding and important issue to narrow down the set of reasonable risk measures by convincing theoretical arguments. But this is not what we have in mind here. We are basically starting from the practitioner's question, although not leaving theory behind: Does it really matter which index I choose?

Different measures produce different numbers. For ordinal indices, however, it only matters whether risky alternatives are ordered in the same or in different ways by various indices. For cardinal indices identical rankings do not necessarily limit the importance of the choice of a particular index.

Apart from theoretical arguments, this calls for an empirical analysis, and that is exactly what this paper is all about. The key idea, see a more detailed exposition in subsection 3.3, is to compare the risk ranking different measures generate for a sample of profit and loss distributions by Spearman's rank correlation coefficient. If all measures are highly correlated then the choice of a particular one seems not to be very important (for qualifications see below), whereas low values of the coefficient indicate that the index choice should not be taken lightly.

We examine different groups of measures, namely traditional measures (Standard Deviation, Lower Partial Moments, and Value at Risk) and poverty-based downside risk measures (including the Sen-index). The risk rankings generated by these measures are compared within each of the two groups and between the groups.

The analysis rests on the complete 1999 data for two proprietary trading books. Therefore we are justified to claim that we are not performing an artificial analysis with self-generated data, but obviously have to admit the limited generality of our findings to-date, since we are only covering one year and the books contain only interest rate related instruments.

Apart from this introduction, the paper is divided into three sections. In section 2 traditional risk measures and downside-risk measures based on poverty measures are presented. In section 3 we describe the data, present two normality tests and a rank correlation analysis and investigate the robustness of our key findings. Section 4 summarizes the main results of the paper and concludes.

2 Risk Measures

In the following subsections we present different types of risk-measures. In subsection 2.1 we describe what we call traditional risk measures. In subsection 2.2 we consider measures originally used for poverty measurement, but which are currently discussed in the economics literature as being applicable in downside-risk management.

Throughout the paper we will examine situations of the following type: An actual trading book will usually change its value from one day to the other. There are n possible future states of nature, each of them yielding a change in value $x_i (i = 1, ..., n)$. For simplicity, we assume that these changes are increasingly ordered $x_1 \leq x_2 \leq ... \leq x_n$. All states of nature have the same probability $1/n$, but the same value change may occur in more than one state. The mean change of value is defined as

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i. \tag{1}$$

We will analyze the data of one year, but will not distinguish the days by superscripts in order to keep the notation as simple as possible. Whenever necessary we will denote the absolute minimum by x_a which is the biggest integer smaller than the minimum value reached during the year. Therefore, x_a is just a little bit smaller than the smallest of the values x_1 of all days.

2.1 Traditional Risk Measures

Measure 1 *Standard Deviation*

As known from the Portfolio Selection Theory of Markowitz (1952) the Standard Deviation can be used to measure the riskiness of assets. The Standard Deviation as a risk measure has been widely used, especially in modern capital market theory, e.g. the CAPM developed by Sharpe (1964), Lintner (1965) and Mossin (1966), and in the option pricing theory by Black and Scholes (1973) and Merton (1976). Given our notation the Standard Deviation is defined as follows:[1]

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}. \tag{2}$$

The measurement of the shortfall and excess returns by the Standard Deviation is very important in case of a probability distribution function which is normally distributed.

[1] The careful reader will observe that we are treating the distributions as discrete distributions with n events. Using the formula for the unbiased estimator of the variance from a sample, we would have to replace n by $(n - 1)$. Given that n ranges between 1,000 and 2,500, this would most likely make not much difference. Also, we are sticking to our interpretation of a discrete distribution throughout rather than deriving unbiased estimators for all other indices.

Then the distribution can be fully described by the mean and the Standard Deviation. If the probability distribution function is not normal, also skewness and kurtosis are often needed in order to describe the distribution. In the economics literature it is criticized that the Standard Deviation evaluates the shortfalls (often characterized as risk in a narrow sense) as well as the excess returns (normally interpreted as chance) in the same way, i.e. the Standard Deviation evaluates deviations from the mean symmetrically. Decision makers in the investment context, however, often associate risk with the failure to reach a target return. From this point of view the use of the Standard Deviation as a risk measure must be questioned.

Measure 2 *Value at Risk*

To a large extent driven by banking regulation, the Value at Risk (*VaR*) is becoming more and more important. The *VaR* is the concept suggested by the Basle Accord for banks' internal market risk models (Basle Committee on Banking Supervision (1996)). These internal models are often used instead of standard methods to calculate the amount of capital that a bank is required to hold as a cushion against the risk contained in its trading book. It measures that shortfall from the target z that is not exceeded with a given probability over a certain time period.[2] To define it formally, let α be a fixed number called confidence level with $0 < \alpha < 1$ and z a real number (e.g., zero) as a reference point for measuring losses. The Value at Risk relating to α is then defined for our purposes as[3]

$$\mathit{VaR}\ 1\text{-}\alpha = \max\{0, z - x_j\}, \tag{3}$$

where j is the smallest integer greater than $(1 - \alpha) \cdot n$. As it is expressed in money units, the *VaR* is easy to interpret and to communicate, in particular to the senior management level, and it can be added across business units. From this point of view it should not be surprising, how popular this measure is in practice. Simple summation, of course, ignores any correlation. From the theoretical point of view one should also note that the *VaR* only takes one single point of the distribution function into account. This feature is very unsatisfactory and has been criticized in previous papers.[4]

Measure 3 *Lower Partial Moments*

Lower Partial Moments by contrast are risk measures that only take the lower part of a probability distribution into account.[5] For this reason Lower Partial Moments belong to the class of downside-risk measures. The downside part of a distribution does not have to be identical to all outcomes below the mean. Instead, some target

[2] Cf. Jorion (2001), pp. 108.

[3] The measures are functions. However, to simplify notation we refrain from repeating their arguments throughout, assuming that this will not cause ambiguities.

[4] Artzner et al. (1997), Artzner et al. (1999), Guthoff et al. (1997b), and Johanning (1998).

[5] Lower Partial Moments were discussed in Fishburn (1977).

z has to be defined which separates shortfalls from the rest. To simplify notation we introduce q for the number of shortfall events, i.e. $q = \max\{i \mid x_i < z\}$. We will always assume $x_1 < z$. When analyzing a distribution of value changes, z need not be zero (although this seems to be a rather natural choice). Alternatively, some $z > 0$ representing a required minimal return or some $z < 0$ indicating losses that cause considerable financial distress could be used as well. In our investigation we always assume $z = 0$. Given our notation and assumptions, the Lower Partial Moment Zero (LPM_0) is defined as follows:

$$LPM_0 = \frac{1}{n}\sum_{i=1}^{q}(z - x_i)^0 = \frac{q}{n}. \tag{4}$$

As is apparent from the second equality, LPM_0 measures the percentage of events in which a shortfall with respect to the target occurs. But LPM_0 does not measure the amount of the shortfall from the target. In contrast to LPM_0 higher Lower Partial Moments consider the amount of the shortfall from the given target. For all payments above the target, the target is reached and therefore the shortfall is zero: payoffs that are higher than the target cannot compensate payoffs below the target. LPM_1 gives the expected amount by which the target is missed and is defined as follows:

$$LPM_1 = \frac{1}{n}\sum_{i=1}^{q}(z - x_i)^1. \tag{5}$$

LPM_1 is also the average shortfall, where outcomes above the target are assigned a shortfall of zero.

$$LPM_2 = \frac{1}{n}\sum_{i=1}^{q}(z - x_i)^2, \tag{6}$$

by squaring the shortfalls from the target, high shortfalls are weighted higher than shortfalls scarcely below the target. Given the same normalization as above, LPM_2 is the mean squared shortfall. For $z = \bar{x}$, it is equal to the semi-variance.

2.2 Poverty-Based Downside-Risk Measures

The application of poverty measures to quantify downside-risk is not a trivial issue (cf. Pfingsten et al. (2000)). People are usually assumed to have non-negative incomes and hence poverty measures are usually defined on the basis of this assumption. Since in risk management negative changes in values of trading books are relevant, the measures have to be adapted.

Without deeply examining the axiomatic implications, we perform a transformation, wherever possible, in our present study for each of the poverty measures analyzed: We shift all distributions by $-x_a$, i.e. by the worst possible outcome of the whole year. This modification is somewhat ad-hoc and not innocuous.

The shift by $-x_a$ will by definition change the risk values for all measures that are not translation-invariant.[6] Our shift therefore may cause a re-ranking of the riskiness of different distributions. Notice that measures which are not translation invariant will in general assign different values to the wealth distribution and the profit and loss distribution.

Zheng (1997) investigates poverty measures and their characteristics. In his article he analyzes axioms which a poverty measure should fulfill and classifies the measures in accordance with these axioms in four different categories.[7] We have chosen at least one measure from every class Zheng (1997) develops.

Measure 4 *Watts measure*

A poverty measure which has often been neglected in the literature is the Watts measure. Harold Watts introduced this first distribution-sensitive poverty measure in Watts (1968). As defined in Zheng (1997), p. 151, the Watts measure can be written as

$$W = \frac{1}{n} \sum_{i=1}^{q} \left[\ln z - \ln x_i \right]. \tag{7}$$

As every poverty measure the Watts measure is defined for non-negative incomes x_i and a given poverty line z, which separates the population into the poor and the non-poor.

Because of the negative changes of the values of a trading book we shift the distribution by $-x_a$. According to our notation the modified Watts measure is defined as

$$W^a = \frac{1}{n} \sum_{i=1}^{q} \left[\ln(z - x_a) - \ln(x_i - x_a) \right]. \tag{8}$$

Measure 5 *Sen-index and modifications*

A poverty index based on a suggestion by Sen, comprises three measures which describe distinct characteristics of poverty that can be transformed into characteristics of a downside-risk measure (cf. Eggers et al. (1999)). The incidence is defined as the ratio between the number of the poor and the population (Headcount-Ratio):

6 This property is explained in Breitmeyer et al. (1999). Intuitively, translation-invariance requires that adding the same number to all x_i and to z does not change the value of the index.

7 Zheng (1997), p. 143, gives an overview about the axioms, classes, and the poverty measures.

$$H = \frac{q}{n} = LPM_0. \tag{9}$$

The intensity is measured by the Income Gap Ratio, which can be defined as the percentage of the average income shortfall of the poor to the poverty line and is denoted in Zheng (1997), p. 142, as

$$I = 1 - \frac{\mu_p}{z}, \tag{10}$$

where μ_p is the mean income of the poor:

$$\mu_p = \frac{1}{q} \sum_{i=1}^{q} x_i. \tag{11}$$

Rearranging terms yields the following expression for the Income Gap Ratio:

$$I = \frac{\sum\limits_{i=1}^{q} (z - x_i)}{qz}. \tag{12}$$

The reader may notice that

$$I = \frac{n}{q} \cdot \frac{LPM_1}{z} = \frac{1}{H} \cdot \frac{LPM_1}{z}. \tag{13}$$

The Gini-index can be used as a measure of the inequality of the income distribution among the poor, the third characteristic of poverty. The Gini-index in this case is defined as follows:

$$G = \frac{1}{2q^2 \mu_p} \sum_{i=1}^{q} \sum_{j=1}^{q} |x_i - x_j|. \tag{14}$$

Given the indices H, I, and G, the original Sen-index can be closely approximated (Zheng (1997), p. 145) by

$$S = H \cdot (I + (1 - I) \cdot G). \tag{15}$$

H is well defined for negative or zero values z or x_i, and it is also invariant with respect to shifts. I requires some transformation to become meaningful for $z \leq 0$; it is scale-invariant, but not translation-invariant. The same basically holds for G with $\mu_p \leq 0$ being the critical case. In light of these observations, we simply use the shift by $-x_a$ and thus the Sen-index is defined as follows:

$$S^a = \frac{q}{n} \cdot \left[\frac{\sum\limits_{i=1}^{q} (z - x_i)}{q(z - x_a)} + \frac{\sum\limits_{i=1}^{q} (x_i - x_a)}{q(z - x_a)} \cdot \frac{1}{2q^2(\mu_p - x_a)} \cdot \sum_{i=1}^{q} \sum_{j=1}^{q} |x_i - x_j| \right]. \tag{16}$$

This formula can be approximated by

$$S^a = \frac{2}{qn(z - x_a)} \sum_{i=1}^{q} \left[(z - x_i)(q + 0,5 - i)\right].^8 \tag{17}$$

Measure 6 *Clark, Hemming, Ulph measure*

Clark et al. (1981) also noticed the close relationship between the Sen-index and Headcount-Ratio, Income-Gap-Ratio and Gini-index and proposed another poverty measure. They replaced the Gini-index with an alternative inequality measure, the Atkinson measure.[9] Again according to Zheng (1997), p. 148, the original Clark, Hemming, Ulph measure can be written as

$$CHU = \frac{q}{nz} \left[\frac{1}{q} \sum_{i=1}^{q} (z - x_i)^{\epsilon}\right]^{\frac{1}{\epsilon}}, \ \epsilon > 1. \tag{18}$$

Modified by a shift in the distribution of $-x_a$, we denote the modified Clark, Hemming, Ulph measure as

$$CHU^a = \frac{q}{n(z - x_a)} \left[\frac{1}{q} \sum_{i=1}^{q} (z - x_i)^{\epsilon}\right]^{\frac{1}{\epsilon}}, \ \epsilon > 1. \tag{19}$$

The parameter ϵ determines how large (absolute) deviations are weighted in comparison to small (absolute) deviations. The careful reader may notice a resemblance between the Clark, Hemming, Ulph measure with $\epsilon = 2$ and the LPM_2 introduced in section 2.1. Therefore we calculate the modified Clark, Hemming, Ulph measure with $\epsilon = 4$ "punishing" more heavily larger shortfalls.

Measure 7 *Chakravarty measure*

To complete the list of downside-risk measures based on poverty measures we have chosen the Chakravarty measure. Although it belongs to the same class of poverty measures as the Clark, Hemming, Ulph measure, the Chakravarty measure fulfills nearly all axioms Zheng (1997) investigates whereas the Clark, Hemming, Ulph measure does not. According to Zheng (1997), p. 150, the original Chakravarty measure is denoted as

$$C = \frac{1}{n} \sum_{i=1}^{q} \left[1 - \left(\frac{x_i}{z}\right)^{e}\right], \ 0 < e < 1. \tag{20}$$

[8] Cf. Zheng (1997) p. 145, for details.

[9] For this inequality measure cf. Atkinson (1970).

Here, e is weight for relative deviations. We shift the distribution by $-x_a$ and denote this modified Chakravarty measure as

$$C^a = \frac{1}{n}\sum_{i=1}^{q}\left[1-\left(\frac{x_i - x_a}{z - x_a}\right)^e\right],\ 0 < e < 1. \tag{21}$$

For expository purposes, we calculate the modified Chakravarty measure with $e = 0.5$ to represent some sort of an intermediate weight.

3 Empirical Analysis

3.1 Data Set

Our data comes from a bank which is an active participant in international investment banking. It covers for two different trading books, the so-called "government bond options book" and the so-called "interest rate derivatives book", every business day of the year 1999. The data we analyze are profit and loss distributions for both trading books on a daily basis, generated by an internal model of the bank which is used in risk management as well as for regulatory purposes.[10] The basis for this model is a Monte Carlo simulation.[11] The internal model starts with 1000 possible changes of the trading book from one business day to the next one. The number of simulations was extended during the year via 1250, 1500, and 2000 up to 2500 (cf. Table 1) to improve the results from the internal model.

Table 1: Simulations included

Period	Business days	Days included for *interest rate derivatives book*	Days included for *government bond options book*	Number of simulations per day
Jan 1^{st} - May 9^{th}	**88**	85	81	1,000
May 10^{th} - Jun 15^{th}	**24**	24	24	1,250
Jun 16^{th} - Sep 14^{th}	**65**	65	65	1,500
Sep 15^{th} - Nov 14^{th}	**43**	42	43	2,000
Nov 15^{th} - Dec 31^{st}	**33**	33	33	2,500
Sum	**253**	249	246	./.

[10] Our objective is to compare the risk measures for the distributions which we assume as given. We are not concerned with the performance of the profit and loss distribution forecasts of the banks internal model. Cf. Diebold and Lopez (1996) and Diebold et al. (1997) for details on this topic.

[11] It is not really a random experiment, because the scenarios on which the simulations are based do not change (except for actual parameter realizations) as long as the number of draws does not change. The reason for this approach is that changes in risk figures then cannot be the result of an "extraordinary" draw.

To explain the difference between the number of business days and the number actually included for each trading book (cf. Table 1) it is important to know that the 22^{nd}, 23^{rd}, and 25^{th} of February and the 19^{th} of October for the interest rate derivatives book, the same days in February, the 23^{rd} and 26^{th} of March, and the 14^{th} and 20^{th} of April for the government bond options book are excluded from further analysis, because simulated profits and losses were zero.

Our data set is limited. It only contains the changes of the values of the trading books but not the values as such. Therefore only absolute and not relative changes can be analyzed. Table 2 gives an idea of the (non-ordered) profit and loss distributions on which our analysis is based.

It is necessary to identify the minima of both trading books to shift the distributions and to use the poverty measures as downside-risk measures (cf. subsection 2.2). Rounded to the next lowest integer the minimum of the interest rate derivatives book (government bond options book) is $x_a^i = -1,855,364$ Euro ($x_a^g = -415,231$ Euro).

A few statistical characteristics are provided to gain a better understanding of our data. We reckon mean (cf. section 2 equation (1)), median, skewness and kurtosis (cf. Table 3). The median can be regarded as the center of a distribution. This is the point which divides the probability distribution into two equal parts and can be written as

$$x_{Me} = x_{\frac{n+1}{2}} \; (n \, odd) \; or \; x_{Me} = \frac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2} \; (n \, even) \tag{22}$$

if the values of a distribution x_i $(i = 1, 2, ..., n)$ are increasingly ordered. Skewness and kurtosis are often used when studying the shape of a probability distribution. Skewness (Sk) describes the lack of symmetry of a distribution and the kurtosis (K) gives information about its tallness or flatness.[12] The skewness can be measured as:

$$Sk = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{\sqrt{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^3}}, \tag{23}$$

and the kurtosis is defined as:

$$K = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^2}. \tag{24}$$

[12] Cf., e.g., Hartung (1998) for further information about these statistical parameters.

Table 2: Extract of data (interest rate derivatives book)

Simulation	Date													
	Jan 4^{th}	...	May 7^{th}	May 10^{th}	...	Jun 15^{th}	Jun 16^{th}	...	Sep 14^{th}.	Sep 15^{th}	...	Nov 12^{th}	Nov 15^{th}	...	Dec 30^{th}
1	-369,928.97	...	-109,412.62	16,896.09	...	96,052.73	30,701.88	...	150,844.63	111,519.16	...	179,994.79	88,025.91	...	-333,187.41
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
999	-100,678.09	...	13,590.47	-10,008.46	...	67,216.98	561.98	...	166,619.27	99,633.85	...	63,668.98	172,976.81	...	-122,813.83
1,000	-569,667.61	...	480,208.77	447,808.57	...	27,555.18	82,820.27	...	382,699.41	386,311,64	...	165,446.29	179,520.50	...	-464,588.92
1,001				-124,640.52	...	330,545.27	220,086.09	...	10,002,27	15,555.02	...	-69,120.11	-177,853.41	...	298,780.64
...				...	...	...	...	...	...	...	...	...	...	...	...
1,250				20,587.20	...	-3,100.31	5,002.30	...	40,616.26	77,443.58	...	-160,114.48	-126,879.86	...	221,951.45
1,251							22.810.64	...	-7,111.12	-63,886.32	...	180,366.88	130,143.75	...	-299,026.55
...							...	...	...	...	...	...	...	...	...
1,500							10,694.09	...	-5,717.62	-59,444.11	...	67,172.66	37,333.28	...	-203,958.40
1,501										106,567.95	...	-59,074.73	91,768.75	...	358,770.36
...										...	...	...	...	...	...
2,000										-193,264.07	...	-16,251.40	-32,370.50	...	95,860.93
2,001													35.653,44	...	36.472,67
...													...	...	...
2,499													-323,726.19	...	355,680.45
2,500													10,324.06	...	6,942.54

Table 3: Statistical characteristics of both trading books

	interest rate derivatives book					*government bond options book*				
	< 0	$= 0$	> 0	Minimum	Maximum	< 0	$= 0$	> 0	Minimum	Maximum
x_1	249	0	0	-1,855,363.20	-51,447.15	246	0	0	-415,230.73	-7,759.09
x_n	0	0	249	143,386.06	2,905,459.57	0	0	246	60,630.88	2,013,522.65
$\bar{x}$	47	0	202	-30,524.58	185,792.03	0	0	246	3,333.73	177,267.24
x_{Me}	45	4	200	-29,042.03	65,885.11	2	0	244	-147.95	70,095.06
σ	./.	./.	./.	48,046.43	505,443.97	./.	./.	./.	7,251.79	277,857.37
Sk	66	0	183	-0.7658	2.1578	0	0	246	0.1012	6.9805
$K-3$	18	0	231	-0.4380	10.1794	2	0	244	-0.8687	75.2332

We are not aiming at a complete discussion of these figures, but a few remarks seem appropriate. It is an interesting feature that the mean was positive for the government bond options book for every business day, whereas this was not true for the interest rate derivatives book. There are certainly a number of possible explanations, e.g. that traders (but why only those responsible for the interest rate derivatives book?) do not care for the average outcome of the Monte Carlo simulations. A more interesting, if true, economic reason might be that the set of instruments available for the management of the interest rate derivatives book prevented traders once in a while to obtain a positive expected value.

The values for Sk and the ranges for K give a first vague indication that the daily distributions are not normal, because this would require $Sk = 0$ and $K = 3$ (cf. Hartung (1998), pp. 49, or Spremann (2000), pp. 93). In particular the values for K hint on the commonly claimed fat tails of these distributions.

3.2 Testing for Normality

An important question concerning the profit and loss distributions is whether they are normally distributed or not. If in particular the data follows a standard normal distribution then the traditional risk measures especially the *VaR* 1% and *VaR* 5% order the days according to their risk in the same way and also in the same way as the Standard Deviation.[13] We apply two statistical tests for normality: the Kolmogorov-Smirnov (KS) test and the Jarque-Bera (JB) test.

The Kolmogorov-Smirnov test is used to decide whether a sample comes from a specific distribution.[14] It is based on the empirical distribution function. An attractive feature of this test is that the distribution of the Kolmogorov-Smirnov test statistic itself does not depend on the underlying cumulative distribution function being tested. Another advantage is that it is an exact test, while the common Chi-square goodness of fit test depends on an adequate sample size for the approximations to be valid (which, however,

[13] The order would be the same, because e.g. *VaR* $1\% = 2.33 \cdot \sigma$ and *VaR* $5\% = 1.645 \cdot \sigma$. Cf., e.g., Jorion (2001), pp. 112.

[14] Cf., e.g., DeGroot (1975), pp. 463, or Lindgren (1975), pp. 221.

would not cause problems here). Despite these advantages the Kolmogorov-Smirnov test has several important limitations. It only applies to continuous distributions and tends to be more sensitive near the center of the distribution than it is at the tails. The most serious limitation is that the distribution must be fully specified. That is, if location, scale, and shape parameters are estimated from the data, the critical region of the Kolmogorov-Smirnov test is no longer valid. The Kolmogorov-Smirnov test for normality is defined by: H_0 = The data follows a normal distribution, H_a = The data does not follow a normal distribution. The Kolmogorov-Smirnov test statistic is defined as

$$d_n = \max\left\{d_n^+, d_n^-\right\}; \tag{25}$$

with

$$d_n^+ = \max\left\{\max_{1\leq i\leq n}\{F_n(x_i^*) - F_0(x_i^*)\}; 0\right\} \tag{26}$$

and

$$d_n^- = \max\left\{\max_{1\leq i\leq n}\{F_0(x_i^*) - F_n(x_{i-1}^*)\}; 0\right\}, \tag{27}$$

where $F_0(x_i^*)$ is the theoretical cumulative normal distribution with the observed Mean and Standard Deviation and $F_n(x_i^*)$ is the true cumulative distribution of the daily value changes of the trading books. Given a confidence level (α) we approximate

$$d_{n;1-\alpha} \approx \sqrt{-\frac{1}{2n}\ln\frac{\alpha}{2}}. \tag{28}$$

If the calculated $d_n > d_{n;1-\alpha}$ the null hypothesis (H_0) has to be rejected.

To verify the results from of the Kolmogorov-Smirnov test, we choose the Jarque-Bera (JB) test for normality. This is a large sample, or asymptotic, test and the following test statistic is used:

$$JB = n \cdot \left[\frac{Sk^2}{6} + \frac{(K-3)^2}{24}\right]. \tag{29}$$

Under the null hypothesis that the residuals are normally distributed, Jarque and Bera (1987) show that in large samples the statistic presented in equation (29) follows the Chi-square distribution with 2 degrees of freedom (*df*). Gujarati (1995) describes this test detailed on pp. 141. For both tests we use a confidence level of $\alpha = 0.01$. Cf. Table 4 for the results of both tests.

Since the Kolmogorov-Smirnov test is often characterized as a conservative test, it is not surprising that it recognizes less days where the null hypothesis has to be rejected than the Jarque-Bera test. It is obvious from Table 4 that in both tests more days of the interest rate derivatives book seem to have normally distributed profits and losses than of the government bond options book. A tentative explanation is that the government bond options book indeed contains more options. These have values that are nonlinear in the value of the underlying and therefore, provided they do not compensate each other, tend to yield asymmetric profit and loss distributions (cf. Table 3).

Table 4: Results of normality tests

	Number of days where the assumption of a *normal distribution* had to be rejected			
	in both tests	only in KS test	only in JB test	in none of the tests
interest rate derivatives book	92	0	94	63
government bond options book	240	0	5	1

We will now check whether this result has an impact on the rankings of days generated by different risk measures.

3.3 Rank Correlation Analysis

3.3.1 Results

We calculate the values of each of six traditional risk measures (Standard Deviation, LPM_0, LPM_1, LPM_2, VaR 1%, and VaR 5%) and four poverty measures (C^a, W^a, CHU^a, and S^a) for both trading books on a day by day basis. Tables 5 and 6 show an extract from our calculations for the time period September 28th to October 6th, both for the interest rate derivatives book. See the appendix for the corresponding Tables 11 and 12 for the government bond options book.

Table 5: Selected values of the traditional risk measures (interest rate derivatives book)

Date	σ	LPM_0	LPM_1	LPM_2	VaR 1%	VaR 5%
...	...	...	...	...	...	...
Sep 28th	104,802.29	0.3975	29,563.48	3,574,327,671	218,717.85	143,668.29
Sep 29th	113,537.31	0.4225	32,979.71	3,959,595,533	215,909.61	155,646.33
Sep 30th	113,118.51	0.4140	32,881.07	4,068,197,471	218,289.21	156,034.31
Oct 1st	138,303.01	0.4425	43,448.59	6,809,134,039	293,058.76	199,222.53
Oct 4th	168,038.29	0.4540	55,366.94	10,946,546,316	353,465.07	251,633.46
Oct 5th	171,373.38	0.4460	56,132.62	11,420,207,698	377,180.54	248,722.32
Oct 6th	175,096.32	0.4410	58,171.47	12,423,483,713	389,146.02	263,758.87
...	...	...	...	...	...	...

Table 5 shows an often criticized feature of the Value at Risk. A reranking of days can be found in this table. According to VaR 1%, September 29th is less risky than September 30th and this day is less risky than September 28th. The VaR 5% shows a

Table 6: Selected values of the poverty-based downside risk measures (interest rate derivatives book)

Date	C^a	W^a	CHU^a	S^a
...	...	...	...	...
Sep 28^{th}	0.00810	0.01648	0.02713	0.02279
Sep 29^{th}	0.00904	0.01838	0.02850	0.02497
Sep 30^{th}	0.00902	0.01835	0.02871	0.02504
Oct 1^{st}	0.01197	0.02449	0.03909	0.03328
Oct 4^{th}	0.01535	0.03160	0.05067	0.04266
Oct 5^{th}	0.01558	0.03210	0.05201	0.04308
Oct 6^{th}	0.01617	0.03336	0.05314	0.04480
...	...	...	...	...

different picture: September 28^{th} is the least risky day of all three and September 29^{th} is less risky than September 30^{th}.[15] Note that the risk difference between September 28^{th} and 30^{th} is close to zero for VaR 1%, but more than 12,000 Euro for VaR 5%. Note also that October 4^{th} and 5^{th} exhibit another reranking. This shows clearly that the choice of the confidence level has some influence on the risk ranking.

Table 6 proves that rerankings also occur within the group of poverty-based downside risk measures: C^a and W^a indicate a decrease, but CHU^a and S^a an increase in risk comparing comparing September 30^{th} to September 29^{th}.

On account of this interesting feature we investigate and compare the intensity of the differences in the rankings generated by different risk measures in the next step. We therefore sort, for each trading book, the days of the year 1999 according to their risk corresponding to each risk measure. As an outcome we receive, for each trading book, a ranking of days for every risk measure, i.e. the business days of 1999 are sorted according to their risk by every risk measure. We observe that every risk measure delivers a different ranking of the days according to their riskiness. Table 7 shows an example for the different rankings of three risk measures for the interest rate derivatives book. The most risky day is assigned rank 1 and the day with the least risk rank 249.

Apparently the three risk measures deliver different rankings of the days and assess the risk of a trading book on a single day in a different way although exhibiting some similarity with respect to bad (Jul 20^{th} and May 25^{th}) and good (Jun 15^{th} and Jun 16^{th}) days.[16]

[15] The same period of days shows a reranking of days (September 29^{th} and 30^{th}) by VaR 1% and VaR 5% also for the government bond options book. Cf. Table 11 in the appendix.

[16] A more detailed overview is given in Table 13 for the interest rate derivatives book and in Table 14 for the government bond options book in the appendix.

Table 7: Selected ranks of the risk measures (interest rate derivatives book)

Rank	σ	LPM_2	C^a
1	Jul 20^{th}	Jul 20^{th}	Jul 20^{th}
2	Mar 23^{rd}	May 25^{th}	May 25^{th}
3	May 25^{th}	May 20^{th}	May 20^{th}
...	...	...	...
100	Feb 15^{th}	Apr 23^{rd}	Apr 28^{rd}
...	...	...	...
200	Sep 1^{st}	Jun 28^{th}	Sep 16^{th}
...	...	...	...
247	Jul 13^{th}	Apr 14^{th}	Jun 15^{th}
248	Jul 14^{th}	Jun 15^{th}	Apr 14^{th}
249	Aug 26^{th}	Jun 16^{th}	Jun 16^{th}

To evaluate the similarity in the risk rankings generated by the indices, we apply Spearman's rank correlation coefficient[17]. Unlike the traditional correlation, it is invariant with respect to all strictly increasing transformations of the measures. The coefficient of rank correlation is a statistic that describes the correlation between the ranks of two distributions. As long as our m daily values of one index V, named $v_1, ..., v_m$, and the values $u_1, ..., u_m$ of another index U for the same trading book are at least ordinally scaled, we can form ranks for one index ($r_1, ...r_m$ for V, say) and for the other index ($s_1, ...s_m$ for U, say), and use the rank correlation coefficient R_s as a measure for the correlation of the ranks of the two indices. In order to determine the similarity of the information by different risk measures, it might seem natural to calculate their correlation, in particular since the index values are all real numbers. The poverty-based measures, however, are only ordinal indices. This seems to us to exclude the use of traditional correlation measures. Spearmans's rank correlation coefficient can be written as:

$$R_s = \frac{\sum_{i=1}^{m} (r_i - \overline{r})(s_i - \overline{s})}{\sqrt{\sum_{i=1}^{m} (r_i - \overline{r})^2 \sum_{i=1}^{m} (s_i - \overline{s})^2}} \tag{30}$$

where $\overline{r}$ and $\overline{s}$ represent the means of the ranks. Spearman's coefficient of rank correlation is bounded between -1 and $+1$ and can be interpreted as follows: If R_s is close to $+1$ this represents a strong positive rank correlation between the variables V and U. If v_i has a high (a low) rank r_i then u_i has a high (a low) rank s_i and vice versa. If R_s is close to -1 this reveals a strong negative rank correlation between the indices V and U. If v_i has a high (a low) rank r_i then u_i has a low (a high) rank s_i and vice

[17] For further details see Hartung (1998), pp. 553.

versa. If R_s is approximately zero then there is no rank correlation between the indices V and U, i.e. r_i does not convey any information about s_i.

Table 8 presents the rank correlation coefficients of the risk measures for the interest rate derivatives book (above the diagonal) and for the government bond options book (below the diagonal).

Table 8: Rank correlation coefficients (interest rate derivatives book and government bond options book)

	σ	LPM_0	LPM_1	LPM_2	VaR 1%	VaR 5%	C^a	W^a	CHU^a	S^a
σ		0.21499	0.79090	0.80662	0.77094	0.81262	0.79186	0.79236	0.74888	0.78121
LPM_0	0.03017		0.65318	0.59509	0.55562	0.56339	0.65020	0.64804	0.66614	0.65439
LPM_1	0.24861	0.86867		0.99462	0.97421	0.98789	0.99995	0.99987	0.99287	0.99928
LPM_2	0.27821	0.79473	0.98777		0.98849	0.99758	0.99517	0.99561	0.99207	0.99562
VaR 1%	0.28086	0.68040	0.93743	0.97732		0.99024	0.97523	0.97612	0.98490	0.97860
VaR 5%	0.29063	0.76151	0.97741	0.99632	0.98426		0.98864	0.98931	0.98657	0.98969
C^a	0.24853	0.86688	0.99995	0.98849	0.93914	0.97839		0.99996	0.99308	0.99936
W^a	0.24924	0.86443	0.99986	0.98931	0.94109	0.97944	0.99993		0.99330	0.99944
CHU^a	0.21485	0.85038	0.99070	0.99037	0.95647	0.98007	0.99110	0.99152		0.99573
S^a	0.24317	0.86123	0.99938	0.99097	0.94563	0.98154	0.99951	0.99962	0.99371	

Since all rank correlation coefficients in Table 8 are positive, we use a one-sided test to test the significance of the relationship between two indices. We state the null hypothesis and the alternative hypothesis as follows: $H_0 : R_s \leq 0$, there is no positive relationship between two indices, $H_a : R_s > 0$, there is a significant positive rank correlation between two indices. The significance of the rank correlation coefficient can be tested by calculating

$$t = R_s \sqrt{\frac{n-2}{1-R_s^2}} \tag{31}$$

and then referring to a Student's t distribution with $n-2$ degrees of freedom (df).[18] We use a significance level of 0.01. In this case and with 247 degrees of freedom for the interest rate derivatives book (244 for the government bond options book) the critical value of t is 2.326. Any value of t higher than this will fall within the critical region and permit us to reject the null hypothesis.

As a short cut we can also calculate the critical rank correlation coefficient $\overline{R}_s$ for each trading book as follows

$$\overline{R}_s = \frac{t}{\sqrt{t^2+n-2}}. \tag{32}$$

[18] Cf. Toutenburg (2000), pp. 184.

At the 1 per cent level, i.e. for $t = 2.326$, the critical rank correlation coefficient is 0.1464 for the interest rate derivatives book and 0.1482 for the government bond options book.

Using the above mentioned test statistic we can reject the null hypothesis of no positive rank correlation for the interest rate derivatives book completely. There is a significant positive rank correlation between all indices for this trading book, because every rank correlation coefficient exceeds the critical value. For the government bond options book, the same is true with the exception of the rank correlation between the Standard Deviation and the LPM_0. The high values of basically all pairwise rank correlation coefficients suggest to test for perfect positive rank correlation. Such a test, unfortunately, would be degenerate because $R_s < 1$ implies that rank correlation cannot be perfect.

3.3.2 Interpretation

The results above and below the diagonal in Table 8 seem to be fairly similar. Obviously the difference between the distributions for the two trading books pointed out in section 3.2 seems to have only a little impact on the rank correlation coefficients. For some qualifications to this statement and a more detailed interpretation of our results see the following remarks.

To start with, we have to underline that the Standard Deviation and the LPM_0 take a special position in our analysis. The rank correlation coefficients between the Standard Deviation and all remaining risk measures are relatively small for both trading books (but with a clear difference; see next paragraph). This is probably due to the conceptual difference between the Standard Deviation and the remaining risk measures. As pointed out in subsection 2.1, the Standard Deviation evaluates the shortfalls as well as the excess returns symmetrically, while the remaining risk measures evaluate only a special (lower) part of the distribution.

The rank correlation coefficients of the Standard Deviation with all other risk measures (except the LPM_0) are between 0.74888 and 0.81262 for the interest rate derivatives book and between 0.21485 and 0.29063 for the government bond options book. This difference may result from the different distributions observed in subsection 3.2. For normal distributions (which are less often rejected for the interest rate derivatives book), the *VaR* 1% and the Standard Deviation would rank identical.[19] This is in line with the rank correlation coefficient between the Standard Deviation and the *VaR* 1% which is only 0.28086 for the government bond options book and 0.77094 for the interest rate derivatives book. If distributions are symmetrical (and a fortiori if they are normal) then the Standard Deviation and Lower Partial Moment 2 should deliver similar results, too, at least if the latter uses a target level close to the mean.

As introduced in subsection 2.1, the LPM_0 measures the percentage of events in which a shortfall with respect to the target occurs. The LPM_0 is hence not distribution-sensitive, i.e. it does not take the amount of the shortfall from the given target into

[19] Note that for the standard normal distribution, e.g., the 1% quantile is at roughly 2.33σ.

account, unlike most of the remaining risk measures. It is therefore hardly surprising that it differs from all other risk measures when analyzing the rank correlation coefficients of both trading books. The rank correlation between the LPM_0 and the remaining risk measures is relatively small for both trading books, too. Other than for the Standard Deviation, however, the rank correlations of the LPM_0 (with all measures except the Standard Deviation) are higher for the government bond options book than for the interest rate derivatives book. This can be exemplified by the rank correlation between the LPM_0 and the *VaR* 1%. The coefficient is 0.68040 for the government bond options book and only 0.55562 for the interest rate derivatives book. It also turns out that the rank correlation of the LPM_0 with any other measure is higher (lower) than the rank correlation of the Standard Deviation with this measure for the government bond options book (interest rate derivatives book).

Next, we examine whether the remaining traditional risk measures, LPM_1, LPM_2, *VaR* 1%, and *VaR* 5%, assess the risk in a different or in the same way. Guthoff et al. (1997b) suggested the LPM_1 as an alternative superior to the *VaR*. But the rank correlation coefficients between LPM_1, LPM_2, *VaR* 1%, and *VaR* 5% are all very high for both the interest rate derivatives book and the government bond options book, ranging from 0.93743 to 0.99758 (cf. Table 8). This is not an evidence of a superiority of either the Value at Risk or the Lower Partial Moments, but means that the choice of one of these traditional risk measures does not matter too much (rank-wise), when assessing the risk of both trading books. All of these four traditional measures rank the days of 1999 in nearly the same way, i.e., LPM_1, LPM_2, *VaR* 1%, and *VaR* 5% deliver nearly the same ranking of days according to their riskiness.

Another interesting feature is that for both trading books the *VaR* 5% has higher rank correlation coefficients with all other risk measures than the *VaR* 1% has.[20] This indicates a closer relationship between the other risk measures and *VaR* 5% than with *VaR* 1%. One could argue that this is a hint to greater reliability of the *VaR* 5% values. This is in line with anecdotal evidence provided by practitioners who state that their results at the 5% level are more robust against outliers than at the 1% level.

We cannot observe much difference between the rank correlation coefficients of the traditional risk measures between the interest rate derivatives book and the government bond options book. Although they are a little higher for the interest rate derivatives book, the differences of the distributions seem to have no or just a marginal effect on the coefficients.

Breitmeyer et al. (1999) and Pfingsten et al. (2000) first transferred the ideas of poverty measurement systematically to downside-risk measurement. Eggers et al. (1999) explained particular problems of the transfer of poverty-based measures to downside-risk measurement, especially the treatment of negative changes in values of trading books. In our current study we evaluate the suitability of some poverty-based measures for downside-risk measurement for the first time on an empirical basis. A close look at

[20] It is also worth noting that something similar holds for the LPM_1. It has higher rank correlation coefficients (although not too visible) with the poverty-based risk measures than LPM_2.

the rank correlation coefficients between the four poverty-based measures shows that the rank correlation coefficients are very high, all being above 0.99. Maybe because of their conceptual similarities, the four poverty-based measures deliver similar rankings of days. This result, however, might alternatively be a consequence of the shifts in the distributions we performed in order to make the indices applicable for negative values. This issue deserves further attention (cf. subsection 3.3.3).

Furthermore, we cannot notice much difference between the poverty-based risk measures' rank correlation coefficients for the interest rate derivatives book and for the government bond options book. Therefore the differences of the distributions pointed out in subsection 3.2 seem to have no or just a marginal influence on the coefficients for these measures as well.

Finally, we examine the rankings of the days when using the traditional measures on the one hand and the poverty-based measures on the other hand. The rank correlation coefficients between these risk measures are very high for both trading books, ranging between 0.93914 and 0.99995 (cf. Table 8). For example the rank correlation coefficient between LPM_2 and CHU^a is 0.99207 for the interest rate derivatives book and 0.99037 for the government bond options book. Obviously the traditional risk measures and the poverty-based measures assess the risk in nearly the same way and deliver nearly identical rankings. This means that both the poverty-based measures and the traditional measures evaluate the risk of the trading books similar. So it does not really matter which risk measure, traditional or from poverty measurement, a bank uses in risk measurement. Thus, the poverty-based measures are as well suited to evaluate the risk of a trading book as the traditional risk measures.

Once more we cannot discover a large difference between the rank correlation coefficients for the two trading books. In most cases the coefficients are slightly higher for the interest rate derivatives book. Consequently the difference of the distributions, once again, seems to have no or just a marginal influence on the rank correlation coefficients.

3.3.3 Robustness

Our results are derived, and hence valid, only for two specific trading books and a given time period. However, we will not extend the analysis in the direction of applying it to other real world data sets. Instead we check the robustness of our results with respect to three modifications of the data which are important also from a theoretical perspective:

- shifting the distributions,
- eliminating the riskiest days (with and without changes in the shifts),
- scaling the distributions.

These modifications are examined in turn.

Firstly, to make the poverty-based measures of downside risk applicable for our analysis, we had shifted all profits and losses by $-x_a$ (cf. subsection 2.2). This shift possibly distorted the ranking of days because the daily distributions may be affected in a different way and to a different extent. Therefore we want to know whether the high rank correlation coefficients observed are merely an artifact of an inappropriate change of data.

From the definitions in section 2 we can distinguish three cases:

- The traditional risk measures (Standard Deviation, LPM_0, LPM_1, LPM_2, $VaR\,1\%$, and $VaR\,5\%$) do not depend on x_a and therefore do not change along with it.
- The poverty-based measures S^a and CHU^a are such that a change in x_a only affects the factor $\frac{1}{(z-x_a)}$. Hence the index values change when x_a is varied, but the order of days does not change because all daily index values vary by the same factor.
- The poverty-based measures W^a and C^a are such that a change in x_a affects each term, i.e. the contribution of each below target outcome to the index value, differently. Thus, not only the daily index values change, but their ranking may change as well.

The empirical examination confirms these theoretical observations. First of all we addressed this problem in shifting the profit and loss distributions of both portfolios by varying amounts: the government bond options book by $x_a^i = -1,855,364$ Euro, the absolute minimum of the interest rate derivatives book, and both trading books by $2,000,000$, $3,000,000$ and $10,000,000$ Euro, keeping q fixed. The rank correlation coefficients between the traditional risk measures (Standard Deviation, LPM_0, LPM_1, LPM_2, $VaR\,1\%$, and $VaR\,5\%$) did not change, because these measures were not affected by the shift. We also observed no changes of the rank correlation coefficients between the traditional risk measures and CHU^a and S^a, and between CHU^a and S^a themselves. This holds true for both trading books.

Also for both trading books, we noticed fairly small changes of the rank correlation coefficients between C^a, W^a and all remaining risk measures. For example the rank correlation coefficient between the modified Watts measure (W^a) and the $VaR\,1\%$ decreased from 0.97612 to 0.97454 for the interest rate derivatives book when we shifted the profit and loss distribution by $10,000,000$ Euro. The impact of our initial shift of the profit and loss distributions by the worst possible outcome of the year is still unknown, but our additional shifts of the distributions had either no or only a very moderate influence on our results.

The previous experiment enlightens the effect of an increase in the shift $-x_a$. A decrease is not feasible because this renders the poverty-based measures ill-defined for some days. Therefore secondly, to obtain at least a little indication what might happen, we have decided to eliminate some "bad" days from the sample. Basically there are two ways to do that. Method *a*: eliminate the days which determine the absolute minimum across all outcomes for a trading book. Method *b*: eliminate days which are detected as very risky by most risk measures (cf. Table 13 in the appendix). For the interest rate derivatives book, both methods yield the same first three days to be eliminated: July 20^{th}, May 25^{th}, and May 20^{th}. The next two days depend on the method chosen: December 30^{th} and 27^{th} vs. March 4^{th} and May 19^{th} (cf. Table 9).

For the government bond options book the two methods yield eight different days to be eliminated (cf. Tables 15 and 16 in the appendix).

Eliminating the "most risky" days from the sample has two effects:

- On average, and in particular when method b is applied, days will be eliminated where the risk measures agree on the ranking (more or less). This should tend to decrease the rank correlation coefficients.
- The shift by the now smaller absolute minimum has an effect which seems to be unpredictable in general (cf. our earlier discussion for some qualifications).

In order to get a better understanding of both effects, we proceed in steps. The analysis is carried out for both elimination methods.[21] Recall in passing that for 10 risk measures there exist 45 different pairwise rank correlation coefficients.

Eliminating the days mentioned while keeping the shift parameter at its initial level reveals the first effect. Denoting the initial rank correlation coefficients by R_{old} and the ones after elimination of days by R_{eli}, we obtain the following results. For both methods, for all 5 days, and in all 45 cases the difference $\delta_1 = R_{eli} - R_{old}$ is negative. The reduction in the rank correlation increased with each day eliminated, ranging between -0.004180 and -0.000001 after eliminating July 20^{th} and between -0.023948 (respectively -0.024090) and -0.000003 after eliminating five days (cf. Table 9). This clear-cut empirical result confirms our intuition for the first effect remarkably well.

Also in line with what we anticipated, the second effect is ambiguous. Denoting the rank correlation coefficients after reducing the shift to the new minima by R_{new}, the second effect is calculated as $\delta_2 = R_{new} - R_{eli}$. For both elimination methods, there exist rank correlation coefficients which increase and others which decrease when the shifts are reduced (cf. elimination of day 1 in Table 9). Note that the numbers provided for the changes δ_2 in coefficients are cumulated, i.e. refer to R_{eli} before any change in shifts. The number 28 occurs because of the eight measures invariant with respect to changes in $-x_a$.

The ambiguity of the second effect extends to the total effect $\delta = \delta_1 + \delta_2 = R_{new} - R_{old}$. Positive changes in R as well as negative changes can be observed, but with a clear dominance in number and size[22] for the latter due to the unambiguous first-effect. Closer inspection reveals that in particular Chakravarty measure, Watts measure, or

[21] We frankly admit that eliminating 5 days, i.e. 2% of the sample, is somewhat arbitrary, but in accordance with usual treatment of outliers (cf. Barnett and Lewis (1994), pp. 27-54).

[22] Remember that the values of the coefficients were quite high initially, leaving only little room for further increases.

LPM_0 were involved when the rank correlation coefficient increased, which did hardly happen for Standard Deviation or LPM_1. Again the picture is qualitatively similar for both elimination methods although the numerical results are, of course, different.

Table 9: Differences in rank correlation coefficients (interest rate derivatives book)*

	1	2	3	4a	5a	Sum a	4b	5b	Sum b
(additional) day eliminated	Jul 20^{th}	May 25^{th}	May 20^{th}	Dec 30^{th}	Dec 27^{th}		Mar 4^{th}	May 19^{th}	
shift by	-1,855,364						-1,855,364		
biggest δ_1	-0.000001	-0.000001	-0.000002	-0.000003	-0.000003		-0.000003	-0.000003	
smallest δ_1	-0.004180	-0.010467	-0.015136	-0.020861	-0.023948		-0.019356	-0.024090	
$\delta_1 > 0$	0	0	0	0	0	0	0	0	0
$\delta_1 = 0$	0	0	0	0	0	0	0	0	0
$\delta_1 < 0$	45	45	45	45	45	225	45	45	225
shift by	-1,260,354	-1,168,473	-1,139,051	-1,025,208	-969,705		-1,139,051		
biggest δ_2	0.001069	0.001082	0.001510	0.001925	0.002149		0.001529	0.001548	
smallest δ_2	-0.003070	-0.003097	-0.0004298	-0.005369	-0.006395		-0.004329	-0.004367	
$\delta_2 > 0$	11	11	11	11	11	55	11	11	55
$\delta_2 = 0$	28	28	28	28	28	140	28	28	140
$\delta_2 < 0$	6	6	6	6	6	30	6	6	30
biggest δ	0.000779	0.000498	0.000627	0.000741	0.000746		0.000467	0.000402	
smallest δ	-0.004180	-0.010467	-0.015136	-0.020861	-0.023948		-0.019356	-0.024090	
$\delta > 0$	9	8	7	7	7	38	6	5	35
$\delta = 0$	0	0	0	0	0	0	0	0	0
$\delta < 0$	36	37	38	38	38	187	39	40	190

*Examples for the calculation (referring to the column headed 3):

-0.015136: Calculate each of the 45 coefficients for the data set after July 20th, May 25th, and May 20th have been eliminated; subtract the respective coefficients calculated from the whole data set; enter the smallest of these 45 values.

-0.004298: Calculate, for the data set after the same 3 days have been eliminated, each of the 45 coefficients once with the (new) shift of -1,139,051 and once with the (old) shift of -1,855,364; calculate the 45 differences; enter the smallest of these 45 values.

-0.015136: This is the smallest of the 45 differences between a coefficient calculated from the data set after 3 days have been eliminated and the new shift is applied and the same coefficient calculated from the whole data set. (In this case it is not the sum of the above numbers, because they come from different pairs of measures.)

It is suspicious in Table 9 that the values for smallest δ_1 and smallest δ coincide in all cases. This is neither an error nor a general result. The differences all come from the rank correlation coefficients between LPM_0 and σ. Both of these measures are invariant with respect to changes in $-x_a$ which implies $\delta_2 = 0$ and hence $\delta = \delta_1$. Apparently, for the six cases with $\delta_2 < 0$ the corresponding δ_1 is so large that in total δ is greater than the values provided as minima.[23]

[23] A similar observation holds for Table 15 in the appendix. There, in the first three cases LPM_0 and VaR determine the minima.

Altogether, the changes induced by the elimination of a few days are visible, but hardly dramatic. We have performed this exercise for the government bond options book as well. The results can be found in Tables 15 and 16 in the appendix. The outcome is comparable. Interesting differences are as follows: Method a now gives rise to an ambiguous first effect because the days determining the absolute minima are not always ranked similarly. Since x_g^a is not determined by the five most riskiest days, method b generates no possibility for different shifts, i.e. $\delta_2 = 0$ in all cases.

Thirdly, let us turn to the issue of scaling. We do not have any information about the daily sizes of the trading books.[24] But these sizes might matter for the rank correlation as becomes obvious from the following consideration:

Suppose, an investor doubles his portfolio (by exact replication) from day to day. Also, none of two risk measures be scale-invariant, i.e. doubling all profits and losses in general changes (increases, say) the value of these risk measures.[25] In this case we would observe an identical risk ranking of both measures, irrespective of their further special properties, solely driven by portfolio size. It would require (major) changes in the portfolio composition to generate different rankings.

With this hypothetical and clearly very stylized example in mind, we ask whether the daily sizes of the trading books could have predetermined the high rank correlation in our case. To answer this question, we changed the scales of the original profit and loss distributions of both trading books in the following way: For each business day t we chose a number ϵ_t from a uniform distribution on $[0; 1]$. By means of

$$\gamma_t = 9.9 \cdot \epsilon_t + 0.1 \tag{33}$$

this was transformed into a scaling factor $\gamma_t \in [0.1; 10]$. Each entry of the profit and loss distribution of day t was then multiplied by γ_t, where stretches of days t's profit and loss distribution ($\gamma_t > 1$) are ten times as likely as compressions ($\gamma_t < 1$) because $\epsilon_t = \frac{1}{11}$ implies $\gamma_t = 1$. With the modified profit and loss distributions we repeated our rank correlation analysis described in subsection 3.3.1. We did the random selection for each trading book five times, clearly not aiming at a more complete simulation study. A typical example of the impact of these random changes of our initial data set on the rank correlation coefficients of both trading books is shown in Table 10. Again, the values of the interest rate derivatives book are presented above the diagonal and the values of the government bond options book below it.

The following results for the interest rate derivatives book emerge: As shown in Table 10 the changes of the rank correlation coefficients between LPM_1, LPM_2, $VaR\,1\%$, $VaR\,5\%$, C^a, W^a, CHU^a and S^a are all positive but fairly small.[26] The changes vary from 0.00002 (between C^a and W^a) and 0.01483 (between C^a and $VaR\,1\%$). Table

[24] Actually, the "size" of a portfolio of derivatives is a difficult term anyway because traditional concepts like market value may be misleading when long and short positions are included.

[25] Of the measures discussed, only the LPM_0 is scale invariant.

[26] Notice that the high initial values of the coefficients do not leave much room for increases anyway.

Table 10: Changes of the rank correlation coefficients of both trading books (randomly modified according to equation (33))

	σ	LPM_0	LPM_1	LPM_2	VaR 1%	VaR 5%	C^a	W^a	CHU^a	S^a
σ		-0.05953	0.12462	0.12167	0.14829	0.11908	0.12392	0.1238	0.15069	0.12991
LPM_0	-0.07242		-0.21379	-0.20219	-0.18886	-0.19068	-0.21206	-0.21135	-0.21635	-0.21199
LPM_1	0.29744	-0.17813		0.00261	0.01552	0.00701	0.00003	0.00008	0.00427	0.00042
LPM_2	0.33923	-0.20911	-0.00023		0.00738	0.00128	0.00222	0.00194	0.00401	0.00181
VaR 1%	0.37585	-0.21428	0.01061	0.00701		0.00590	0.01483	0.01417	0.00810	0.01248
VaR 5%	0.33676	-0.20696	0.00215	0.00074	0.00489		0.00647	0.00599	0.00701	0.00546
C^a	0.29837	-0.17816	0.00001	-0.00042	0.01005	0.00185		0.00002	0.00415	0.00037
W^a	0.29864	-0.17708	0.00007	-0.00089	0.00885	0.00126	0.00004		0.00396	0.00030
CHU^a	0.32469	-0.18260	0.00266	0.00071	0.00725	0.00341	0.00253	0.00224		0.00259
S^a	0.30375	-0.18050	-0.00007	-0.00038	0.00974	0.00177	-0.00009	-0.00014	0.00251	

10 shows strong negative deviations (order of magnitude: -0.2) of the rank correlation coefficients between LPM_0 and all remaining risk measures. This is extra evidence that the LPM_0 takes a special position in our analysis as a distribution-insensitive measure. Except for LPM_0, the changes of the rank correlation coefficients between Standard Deviation and all remaining risk measures are positive (around 0.13). Apparently, the type of scaling lets the profit and loss distributions of the interest rate derivatives book behave more like a normal distribution. Altogether the changes of the rank correlation coefficients are quite similar for all our five random selections.

We derive the following results for the government bond options book: First of all we notice that the changes of the rank correlation coefficients between all risk measures are quite similar for the five random selections for this trading book, too. The changes of the rank correlation coefficients between LPM_1, LPM_2, VaR 1%, VaR 5%, C^a, W^a, CHU^a and S^a are again quite small, but this time often negative. The changes of the rank correlation coefficients between this risk measures vary from -0.00089 (between W^a and LPM_2) and 0.01061 (between VaR 1% and LPM_1). Again except for LPM_0 the changes of the rank correlation coefficients between Standard Deviation and all remaining risk measures are positive (around 0.33). The results of the interest rate derivatives book for the LPM_0 are confirmed by this trading book.

To sum up, one may say that these random changes of profit and loss distributions had only little impact on the rank correlation coefficients (except for LPM_0 and Standard Deviation, which are conceptually different anyway).

One may criticize our approach, because the probability to stretch the initial profit and loss distribution was ten times the probability for a compression of the distribution. Therefore we changed the profit and loss distributions in another way:

The draws ϵ_t from a uniform distribution on $[0;1]$ were transferred into scaling factors according to

$$\eta_t = \begin{cases} 1.8 \cdot \epsilon_t + 0.1 & 0 \leq \epsilon_t \leq 0.5 \\ 18 \cdot \epsilon_t + -8 & 0.5 < \epsilon_t \leq 1 \end{cases} . \quad (34)$$

With this scaling, stretching ($\eta_t > 1$) and compressing ($\eta_t < 1$) of the probability distribution are equally likely. The interval for the scaling factor coincides with the first case, i.e. $\eta_t \in [0.1; 10]$. The resulting changes of the rank correlation coefficients are shown in Table 17 in the appendix and we refrain from a detailed interpretation because the outcome is very similar, qualitatively, to the previous case.

Scaling the daily profit and loss distributions according to equation (34) implies equal probabilities for stretching and compressing but is asymmetric in the absolute size of changes. One of our earlier intentions was to analyze smaller shifts. To guarantee a smaller shift we added one more way of scaling

$$\theta_t = \epsilon_t. \tag{35}$$

With this scaling the profit and loss distributions are automatically compressed. We obtained changes of the rank correlation coefficients shown in Table 18 in the appendix for one example. Again we refrain from a detailed interpretation because the outcome is very similar, qualitatively, to the previous cases. In all our simulations according to equation (35) the minima have changed. For example, Table 18 is based on $x_a^i = -1,044,110.11$ Euro and $x_a^g = -300,710.04$ Euro as compared to the values $-1,855,363.20$ and $-415,230.73$, respectively, of the true data. The shifts to generate positive numbers therefore were decreased by more than 40%, respectively almost 30%. Quite surprisingly many changes in the rank correlation coefficients are still positive, although one might have though that random compression of daily distributions and smaller shifts could potentially reduce the similarity of the risk rankings.

Summarizing subsection 3.3.3, we are justified to say that it does not look as if the fairly high rank correlation coefficients were derived artificially. Neither increasing nor decreasing (after eliminating some days) the shift, needed to include negative data in poverty-based risk measures, nor any scaling we tried affects the coefficients to a large extent. This need not be the final word, but for the moment we feel comfortable with our findings.

4 Summary and Outlook

We have started this study by presenting some traditional risk measures, including the Standard Deviation which evaluates the shortfalls as well as the excess returns from the mean symmetrically. Alternatives as pure downside risk measures are the Lower Partial Moments and the Value at Risk. Next, we introduced some downside-risk measures borrowed from poverty measurement: the Chakravarty measure, the Watts measure, the Clark, Hemming, Ulph measure, and the Sen-index. The original poverty measures are defined for non-negative incomes only, whereas in downside-risk measurement in general and in the data we received from an investment bank in particular, negative changes in the values of trading books are relevant, too. Therefore we had to modify the poverty measures to be applicable for our data. We decided to shift the daily distributions by the minimum of each trading book's profits and losses.

After having introduced all these risk measures, we outlined our empirical study. Our first test was, whether the profit and loss distributions are normally distributed or not. Both the Kolmogorov-Smirnov test and the Jarque-Bera test of normality showed that the profit and loss distributions were generally not normally distributed. For the government bond options book normality had to be rejected almost always whereas it could not be rejected for the interest rate derivatives book in a fair number of cases (depending on the test chosen).

Then, we calculated the values of every risk measure for the interest rate derivatives book and the government bond options book on a day by day basis and ranked the days according to their risk by every risk measure. Every risk measure delivered a different ranking of the business days of the year 1999. To compare the intensity of these differences we calculated, for each of the trading books separately, the rank correlation coefficients between each of the risk measures. These are our main results, which basically hold for both trading books (with some differences mentioned in the preceding section): The rank correlation coefficients between the Standard Deviation and the LPM_0 on the one hand and all remaining risk measures on the other hand are relatively small when compared to the other coefficients, i.e., Standard Deviation and LPM_0 rank the business days quite differently from the remaining risk measures. The conceptual differences between the Standard Deviation (a symmetrical risk measure), the LPM_0 (a distribution-insensitive measure), and the remaining risk measures may be reasons for that.

The other traditional risk measures, i.e. LPM_1, LPM_2, $VaR\,1\%$, and $VaR\,5\%$, assess the risk in nearly the same way, i.e., they deliver similar risk rankings of the business days.[27] The same holds within the group of poverty-based measures. They also assess the risk of both trading books in nearly the same way and deliver similar risk rankings of the business days.[28] Comparing traditional measures and poverty-based measures, it turns out that, again, both deliver similar risk rankings of the business days.

Finally, the high rank correlations observed seem to be robust against a number of modifications: different shifts of the distribution, elimination of riskiest days (with and without changes in shifts), different scalings of the distributions.

Subject to all necessary qualifications, the results indicate that the choice of a particular one of the risk measures, with the exception of both the Standard Deviation and the LPM_0, does not seem to matter very much when assessing the risk of the bank's trading books. All these risk measures lead to similar risk rankings, i.e. for our data traditional risk measures and poverty-based measures evaluate the risk in nearly the same way.

[27] Although conceptually the Value at Risk is most closely related to the LPM_0, for both trading books the LPM_2 has the highest rank order correlation of all three Lower Partial Moments with both Value at Risk measures.

[28] The poverty-based measures show, again for both trading books, a partially circular relationship, i.e., the Clark, Hemming, Ulph measure is most closely resembled by the Sen-index, which itself is best mirrored in the Watts measure, this being then particularly close to the Chakravarty measure, which finally is most closely related to the Watts measure.

Are the similarities in the risk rankings, good news or bad news? A practitioner may feel happy because sticking to the beloved Value at Risk does not seem to leave him or her with an apparently odd risk ranking. But an academic may feel happy, too. He or she can now advocate superior measures not only with the argument that they are well-founded theoretically, but can also validly say that they do not lead to "strange" risk rankings if applied to real world data. Thus, more research is needed in order to determine whether or not a replacement of the Value at Risk by some other downside-risk measure, including those not discussed here, should be seriously pushed. For example, an economic model might be appropriate to determine losses incurred if sticking to a specific one of the risk measures used. Basically, this involves a deeper normative justification of the indices applied.

Risk can be viewed very differently; and decisions on risk-taking are often made by groups, e.g. boards of directors. Innocent as these two observations are, they hint at another line for further research. In some sense the risk rankings generated by different indices are driven by the different (often implicit) value judgements embodied in these measures. Since there is no way of discriminating between right or wrong personal value judgements, indices might be sought that are good compromises between competing views. Therefore, a downside-risk measure with a high rank correlation with other measures might for this reason be preferred.[29] For example, given the Value at Risk is to be used, one might want to choose *VaR* 5% instead of *VaR* 1%, because the *VaR* 5% exhibits, for both trading books, a higher rank correlation with every single one of the other indices than the *VaR* 1%. This observation is strikingly clear from our data, and it is in line with what we have heard from a number of practitioners, namely that *VaR* 5% were more stable, in particular with regard to "fat tails" of profit and loss distributions. How general this last observed feature is, will be yet another matter for future research.

[29] The meaning of "high" must be specified, and it also has to be determined whether maybe the average correlation matters instead, to make this an operational statement.

A Appendix

Table 11: Selected values of the traditional risk measures (government bond options book)

Date	σ	LPM_0	LPM_1	LPM_2	VaR 1%	VaR 5%
...	...	...	...	...	...	...
Sep 28^{th}	42,705.43	0.4120	11,315.01	449,231,876.31	68,599.73	50,274.59
Sep 29^{th}	53,950.33	0.2285	2,494.99	46,429,155.10	29,512.70	17,070.56
Sep 30^{th}	52,817.92	0.3255	3,610.92	54,016,711.93	24,380.46	17,535.23
Oct 1^{st}	65,301.87	0.3130	11,285.16	633,116,349.41	94,886.74	62,142.92
Oct 4^{th}	64,613.67	0.2720	9,171.90	510,164,060.06	94,309.46	56,792.92
Oct 5^{th}	71,985.55	0.1965	4,932.14	200,733,884.31	62,763.17	35,930.60
Oct 6^{th}	73,055.82	0.2065	4,824.32	175,745,858.17	56,351.85	34,480.58
...	...	...	...	...	...	...

Table 12: Selected values of the poverty-based downside risk measures (government bond options book)

Date	C^a	W^a	CHU^a	S^a
...	...	...	...	...
Sep 28^{th}	0.01397	0.02866	0.04095	0.03745
Sep 29^{th}	0.00304	0.00615	0.01065	0.00873
Sep 30^{th}	0.00439	0.00886	0.01218	0.01159
Oct 1^{st}	0.01409	0.02926	0.04431	0.03834
Oct 4^{th}	0.01145	0.02377	0.03788	0.03180
Oct 5^{th}	0.00609	0.01252	0.02031	0.01695
Oct 6^{th}	0.00594	0.01217	0.01893	0.01640
...	...	...	...	...

Table 13: Selected ranks of the risk measures (interest rate derivatives book)

Rank	σ	LPM_0	LPM_1	LPM_2	VaR 1%	VaR 5%	C^a	W^a	CHU^a	S^a
1	Jul 20^{th}	Dec 16^{th}	Jul 20^{th}	Jul 20^{th}	Jul 20^{th}	Jul 20^{th}	Jul 20^{th}	Jul 20^{th}	Jul 20^{th}	Jul 20^{th}
2	Mar 23^{rd}	Dec 29^{th}	May 25^{th}	May 25^{th}	May 25^{th}	May 25^{th}	May 25^{th}	May 25^{th}	May 25^{th}	May 25^{th}
3	May 25^{th}	Oct 25^{th}	May 20^{th}	May 20^{th}	Mar 4^{th}	May 20^{th}	May 20^{th}	May 20^{th}	May 20^{th}	May 20^{th}
4	May 26^{th}	Dec 20^{th}	Mar 4^{th}	Mar 4^{th}	May 20^{th}	Mar 4^{th}	Mar 4^{th}	Mar 4^{th}	Mar 4^{th}	Mar 4^{th}
5	Mar 4^{th}	Jul 13^{th}	May 19^{th}	May 19^{th}	May 19^{th}	May 19^{th}	May 19^{th}	May 19^{th}	May 19^{th}	May 19^{th}
...	...	...	...	...	...	...	...	...	...	...
49	Aug 8^{th}	Jan $4^{th\star}$ Jan 12^{th}	Jan 21^{st}	Dec 17^{th}	Dec 20^{th}	Jan 6^{th}	May 12^{th}	May 12^{th}	Jan 21^{st}	Sep 13^{th}
50	Oct 15^{th}	May 26^{th}	Sep 13^{th}	Dec 20^{th}	Feb 16^{th}	Mar 24^{th}	Sep 13^{th}	Sep 13^{th}	Jan 7^{th}	May 12^{th}
51	Jan 8^{th}	May 3^{rd}	Jun 9^{th}	Jan 7^{th}	Aug 12^{th}	May 31^{st}	Jan 7^{th}	Jan 7^{th}	Jan 18^{th}	Jan 7^{th}
...	...	...	...	...	...	...	...	...	...	...
99	Dec 20^{th}	Apr 23^{rd}	Oct 8^{th}	Jun 29^{th}	Apr 28^{rd}	Jun 29^{th}	Oct 8^{th}	Mar 3^{rd}	Feb 2^{nd}	Mar 3^{rd}
100	Feb 17^{th}	./.	Apr 28^{rd}	Apr 23^{rd}	Feb 9^{th}	Feb 12^{th}	Apr 28^{rd}	Apr 28^{rd}	Apr 23^{rd}	Oct 26^{th}
101	Jun 7^{th}	Jan $07^{th\circ}$ Jan 14^{th} Feb 24^{th}	Nov 12^{th}	Nov 15^{th}	Mar 2^{nd}	Mar 3^{rd}	Nov 12^{th}	Nov 12^{th}	Jul 26^{th}	Nov 12^{th}
...	...	...	...	...	...	...	...	...	...	...
199	Nov 24^{th}	Feb $17^{th\bullet}$ Oct 13^{th} Nov 13^{th}	Sep 17^{th}	Apr 29^{th}	Jun 10^{th}	Mar 30^{th}	Sep 17^{th}	Sep 16^{th}	Apr 7^{th}	Apr 7^{th}
200	Sep 1^{st}	Jul 15^{th}	Mar 16^{th}	Jun 28^{th}.	Sep 8^{th}	Apr 7^{th}	Sep 16^{th}	Sep 17^{th}	Sep 17^{th}	Sep 17^{th}
201	Nov 11^{th}	Sep 23^{rd}	Aug 3^{rd}	Sep 22^{nd}	Apr 22^{nd}	Apr 20^{th}	Aug 3^{rd}	Aug 3^{rd}	Aug 3^{rd}	Aug 3^{rd}
...	...	...	...	...	...	...	...	...	...	...
247	Jul 13^{th}	Jun 18^{th}	Jun 15^{th}	Apr 14^{th}	Apr 13^{th}	Apr 13^{th}	Jun 15^{th}	Jun 15^{th}	Apr 14^{th}	Jun 15^{th}
248	Jul 14^{th}	Apr 14^{th}	Apr 14^{th}	Jun 15^{th}	Jun 15^{th}	Jun 15^{th}	Apr 14^{th}	Apr 14^{th}	Jun 15^{th}	Apr 14^{th}
249	Aug 26^{th}	Jun 16^{th}	Jun 16^{th}	Jun 16^{th}	Jun 16^{th}	Jun 16^{th}	Jun 16^{th}	Jun 16^{th}	Jun 16^{th}	Jun 16^{th}

The * marked box is occupied twice, so the rank of both days is 48.5. The $^\circ$ marked box is occupied three times, so the rank of all three days is 101 (the ranks 100 and 102 are vacant). The $^\bullet$ marked box is also occupied three times, so the rank of all three days is 198. Ranks 197 and 199 are vacant.

Table 14: Selected ranks of the risk measures (government bond options book)

Rank	σ	LPM_0	LPM_1	LPM_2	VaR 1%	VaR 5%	C^a	W^a	CHU^a	S^a
1	May 14th	Feb 24th	May 14th	May14th	May 14th	May 14th	May 14th	May14th	May 14th	May 14th
2	Jun 23th	May 14th	Jun 11th	Sep 7th	Jun 18th	Sep 7th	Jun 11th	Sep 7th	Sep 7th	Sep 7th
3	Jun 22nd	Jul 20th	Sep 7th	Jun 11th	Sep 17th	Jun 11th	Sep 7th	Jun 11th	Jun 11th	Jun 11th
4	Jun 11th	Jun 11^{th*} Apr 7th	Apr 12th	Jun 18th	Jun 11th	Apr 12th	Apr 12th	Jun 18th	Jun 18th	Apr 12th
5	Sep 7th	./.	Sep 23rd	Apr 12th	Apr 7th	Jun 18th	Sep 23rd	Apr 12th	Apr 12th	Jun 18th
...	...	...	...	...	...	...	...	...	...	...
49	May 10th	Feb 26$^{th\otimes}$ Dec 1st	Aug 8th	Feb 26th	Jan 29th	Feb 26th	Mar 1st	Mar 1st	May 4th	May 4th
50	Jul 14th	./.	Feb 12th	Mar 1st	Jun 1st	Jun 22nd	Feb 12th	Feb 12th	Feb 12th	Feb 12th
51	Oct 7th	Feb 15$^{th\circ}$ Feb 19th	May 11th	Jan 11th	Feb 26th	Jan 11th	May 11th	May 11th	Oct 1st	Oct 1st
...	...	...	...	...	...	...	...	...	...	...
99	Sep 2nd	./.	Jan 14th	Dec 1st	Apr 15th	Feb 16th	Jan 14th	Jan 25th	Apr 15th	Jan 22nd
100	Mar 24th	Jan 18^{th*} May 10th Oct 8th	Jan 25th	Aug 3rd	May 5th	Jan 26th	Jan 25th	Sep 6th	Jan 27th	Dec 30th
101	Jun 16th	./.	Jan 22nd	Jan 20th	Jan 26th	Jan 20th	Jan 22nd	Jan 22nd	Dec 30th	Jan 27th
...	...	...	...	...	...	...	...	...	...	...
199	Feb 16th	Jun 29$^{th\bullet}$ Sep 16th Nov 11th	Nov 23rd	Aug 27th	Apr 21st	Aug 20th	Nov 23rd	Nov 23rd	Nov 11th	Nov 23rd
200	Jan 18th	./.	Jun 4th	Jun 2nd	Jun 4th	Jul 28th	Jun 4th	Jun 4th	Nov 23rd	Jun 4th
201	Jan 19th	Jun 4th	May 6th	Jul 28th	Apr 27th	Dec 28th	May 6th	May 6th	Jun 4th	May 6th
...	...	...	...	...	...	...	...	...	...	...
244	Aug 26th	Jun 16th	Jun 16th	Aug 30th	Dec 21st	Dec 20th	Jun 16th	Jun 16th	Jun 16th	Jun 16th
245	Dec 22nd	Dec 21st	Dec 20th	Dec 20th	Dec 20th	Jun 16th	Dec 20th	Dec 20th	Dec 21st	Dec 21st
246	Dec 23rd	Dec 20th	Dec 21st	Dec 21st	Aug 30th	Dec 21st	Dec 21st	Dec 21st	Dec 20th	Dec 20th

The * marked box is occupied twice, so the rank of both days is 4.5. The $^{\otimes}$ and the $^{\circ}$ marked boxes are also occupied twice, so the rank of both days is 49.5 or 51.5. The * marked box is occupied three times, so the rank of all three days is 100. Ranks 99 and 101 are vacant. The $^{\bullet}$ marked box is also occupied three times, so the rank of all three days is 199. Ranks 198 and 200 are vacant.

Table 15: Differences in rank correlation coefficients method a, (government bond options book)

	1a	2a	3a	4a	5a	Sum a
(additional) day eliminated	Sep 17^{th}	Jul 20^{th}	Jun 18^{th}	May 14^{th}	Apr 7^{th}	
shift by	-415,231					
biggest δ_1	0.000804	0.001251	0.000021	0.000020	0.000019	
smallest δ_1	-0.003738	-0.007486	-0.011254	-0.020361	-0.026588	
$\delta_1 > 0$	10	8	3	3	2	26
$\delta_1 = 0$	0	0	0	0	0	0
$\delta_1 < 0$	35	37	42	42	43	199
shift by	-343.806	-307.831	-268.378	-238.761	-214.747	
biggest δ_2	0.000489	0.000427	0.001466	0.002191	0.002911	
smallest δ_2	-0.000525	-0.000531	-0.001985	-0.002857	-0.003835	
$\delta_2 > 0$	12	12	12	12	12	60
$\delta_2 = 0$	28	28	28	28	28	140
$\delta_2 < 0$	5	5	5	5	5	25
biggest δ	0.000804	0.001251	0.000242	0.000389	0.000542	
smallest δ	-0.003738	-0.007486	-0.011254	-0.020361	-0.026588	
$\delta > 0$	13	11	7	7	5	43
$\delta = 0$	0	0	0	0	0	0
$\delta < 0$	32	34	38	38	40	182

Table 16: Differences in rank correlation coefficients method b, (government bond options book)

	1b	2b	3b	4b	5b	Sum b
(additional) day eliminated	May 14^{th}	Sep 7^{th}	Jun 11^{th}	Jun 18^{th}	Apr 12^{th}	
shift by	-415,231					
biggest δ_1	-0.000001	-0.000001	-0.000002	-0.000002	-0.000003	
smallest δ_1	-0.011821	-0.023645	-0.035567	-0.041917	-0.051609	
$\delta_1 > 0$	0	0	0	0	0	0
$\delta_1 = 0$	0	0	0	0	0	0
$\delta_1 < 0$	45	45	45	45	45	225
shift by	-415,231					
biggest δ_2	0	0	0	0	0	0
smallest δ_2	0	0	0	0	0	0
$\delta_2 > 0$	0	0	0	0	0	0
$\delta_2 = 0$	45	45	45	45	45	225
$\delta_2 < 0$	0	0	0	0	0	0
biggest δ	-0.000001	-0.000001	-0.000002	-0.000002	-0.000003	
smallest δ	-0.011821	-0.023645	-0.035567	-0.041917	-0.051609	
$\delta > 0$	0	0	0	0	0	0
$\delta = 0$	0	0	0	0	0	0
$\delta < 0$	45	45	45	45	45	225

Table 17: Changes of the rank correlation coefficients of both trading books (randomly modified according to equation (34))

	σ	LPM_0	LPM_1	LPM_2	VaR 1%	VaR 5%	C^a	W^a	CHU^a	S^a
σ		-0.26637	0.16139	0.15919	0.19708	0.15500	0.16082	0.16064	0.20004	0.17005
LPM_0	0.01252		-0.48062	-0.47219	-0.46851	-0.45696	-0.47938	-0.47858	-0.49119	-0.48092
LPM_1	0.54439	-0.31938		0.00258	0.01658	0.00810	0.00003	0.00007	0.00542	0.00056
LPM_2	0.56525	-0.34793	0.00290		0.00888	0.00159	0.00217	0.00183	0.00526	0.00180
VaR 1%	0.58269	-0.32124	0.03168	0.01489		0.00679	0.01583	0.01513	0.00862	0.01292
VaR 5%	0.55480	-0.33193	0.01022	0.00196	0.00891		0.00752	0.00700	0.00900	0.00650
C^a	0.54501	-0.31868	0.00004	0.00236	0.03032	0.00945		0.00002	0.00527	0.00050
W^a	0.54481	-0.31709	0.00012	0.00167	0.02862	0.00856	0.00006		0.00507	0.00041
CHU^a	0.58000	-0.32739	0.00456	0.00292	0.02214	0.00900	0.00425	0.00387		0.00313
S^a	0.55279	-0.32276	0.00024	0.00170	0.02768	0.00827	0.00013	0.00004	0.00309	

Table 18: Changes of the rank correlation coefficients of both trading books (randomly modified according to equation (35))

	σ	LPM_0	LPM_1	LPM_2	VaR 1%	VaR 5%	C^a	W^a	CHU^a	S^a
σ		-0.18323	0.11747	0.11877	0.15214	0.11562	0.11694	0.11639	0.14785	0.12463
LPM_0	-0.04631		-0.32820	-0.32477	-0.32135	-0.31151	-0.32710	-0.32573	-0.33586	-0.32980
LPM_1	0.34094	-0.19755		0.00190	0.01263	0.00629	0.00003	0.00008	0.00420	0.00038
LPM_2	0.38032	-0.23063	-0.00005		0.00725	0.00143	0.00153	0.00116	0.00416	0.00150
VaR 1%	0.41181	-0.23291	0.01411	0.00849		0.00603	0.01195	0.01122	0.00657	0.01027
VaR 5%	0.37443	-0.22600	0.00336	0.00039	0.00449		0.00576	0.00520	0.00745	0.00528
C^a	0.34235	-0.19801	0.00001	-0.00020	0.01356	0.00306		0.00003	0.00405	0.00032
W^a	0.34342	-0.19793	0.00004	-0.00044	0.01280	0.00271	0.00002		0.00390	0.00027
CHU^a	0.36769	-0.20056	0.00337	0.00053	0.00845	0.00305	0.00323	0.00302		0.00264
S^a	0.34657	-0.19949	0.00006	-0.00063	0.01220	0.00232	0.00001	-0.00003	0.00269	

References

Artzner, Philippe, Freddy Delbaen, Jean-Marc Eber and David Heath (1997), Thinking Coherently, *Risk*, vol. 10, pp. 68–71.

Artzner, Philippe, Freddy Delbaen, Jean-Marc Eber and David Heath (1999), Coherent Measures of Risk, *Mathematical Finance*, vol. 9, pp. 203–228.

Atkinson, Anthony B. (1970), On the Measurement of Inequality, *Journal of Economic Theory*, vol. 2, pp. 244–263.

Barnett, Vic and Toby Lewis (1994), Outliers in Statistical Data, 3rd ed., Wiley, Chichester et al.

Basle Committee on Banking Supervision (1996), Amendment to the Capital Accord to Incorporate Market Risks, Technical report, Basle, January.

Black, Fischer and Myron Scholes (1973), Pricing of Options and Corporate Liabilities, *Journal of Political Economy*, vol. 81, pp. 637–654.

Breitmeyer, Carsten, Hendrik Hakenes, Andreas Pfingsten and Christoph Rechtien (1999), Learning From Poverty Measurement: An Axiomatic Approach to Measure Downside Risk, Technical Report 99-03, Institut für Kreditwesen, Westfälische Wilhelms-Universität Münster.

Clark, Stephen, Richard Hemming and David Ulph (1981), On Indices for the Measurement of Poverty, *Economic Journal*, vol. 91, pp. 515–526.

DeGroot, Morris H. (1975), Probability and Statistics, Addison-Wesley Publishing Company, Reading, Mass.

Diebold, Francis X., Todd A. Gunther and Anthony S. Tay (1997), Evaluating Density Forecast, Technical Report 215, National Bureau of Economic Research, Cambridge, Mass.

Diebold, Francis X. and Jose A. Lopez (1996), Forecast Evaluation and Combination, in: G. S. Maddala and C. R. Rao (eds.), *Handbook of Statistics*, North-Holland, Amsterdam, pp. 241–268.

Eggers, Frank, Andreas Pfingsten and Sven Rieso (1999), Three Dimensions of Shortfall Risk: Transformation and Extension of Sen's Poverty Index, Technical Report 99-04, Institut für Kreditwesen, Westfälische Wilhelms-Universität Münster.

Fishburn, Peter C. (1977), Mean-Risk Analysis with Risk Associated with Below-Target Returns, *The American Economic Review*, vol. 67, No. 2, pp. 116–126.

Gujarati, Damodar N. (1995), Basic Econometrics, 3rd ed., McGraw-Hill International editions, New York et al.

GUTHOFF, Anja, Andreas PFINGSTEN and Juliane WOLF (1997a), Effects on Risk Taking Resulting from Limiting the Value at Risk or the Lower Partial Moment One, in: INSTITUTE OF ACTUARIES OF AUSTRALIA (ed.), *7th International AFIR Colloquium Proceedings*, Southwood Press, Sydney, pp. 355–378.

GUTHOFF, Anja, Andreas PFINGSTEN and Juliane WOLF (1997b), On the Compability of Value at Risk, Other Risk Concepts, and Expeted Utility Maximation, in: Christian Hipp ET. AL. (ed.), *Geld, Finanzwirtschaft, Banken und Versicherungen: 1996; Beiträge zum 7. Symposium Geld, Finanzwirtschaft, Banken und Versicherungen an der Universität Karlsruhe vom 11.-13. Dezember 1996*, Verlag Versicherungswirtschaft e.V., Karlsruhe, pp. 591–614.

HARTUNG, Joachim (1998), Statistik: Lehr- und Handbuch der angewandten Statistik, 11th ed., Oldenbourg, München.

JARQUE, Carlos M. and Anil K. BERA (1987), A Test for Normality of Observations and Regression Residuals, *Internationl Statistical Review*, vol. 55, pp. 163–172.

JOHANNING, Lutz (1998), Value-at-Risk zur Marktrisikosteuerung und Eigenkapitalallokation, Uhlenbruch, München.

JORION, Philippe (2001), Value at Risk: The New Benchmark for Managing Financial Risk, 2nd ed., McGraw-Hill, New York et al.

LINDGREN, Bernard W. (1975), Basic Ideas of Statistic, Macmillan Publishing, New York.

LINTNER, John (1965), The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, *The Review of Economics and Statistics*, vol. 47, pp. 13–37.

MARKOWITZ, Harry M. (1952), Portfolio Selection, *Journal of Finance*, vol. 7, pp. 77–91.

MERTON, Robert C. (1976), Option Pricing When Underlying Stock Returns are Discontinuous, *Journal of Financial Economics*, vol. 3, pp. 125–144.

MOSSIN, Jan (1966), Equilibrium in a Capital Asset Market, *Econometrica*, vol. 34, pp. 768–783.

PFINGSTEN, Andreas, Carsten BREITMEYER, Frank EGGERS, Hendrik HAKENES, Christoph RECHTIEN and Sven RIESO (2000), Armutsmaße als Downside-Risikomaße: Ein Weg zu Risikomaßen, die dem Value-at-Risk überlegen sind, in: Lutz JOHANNING and Bernd RUDOLPH (eds.), *Handbuch Risikomanagement*, Uhlenbruch Verlag, Bad Soden, pp. 85–107.

SHARPE, William F. (1964), Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk, *Journal of Finance*, vol. 19, pp. 425–442.

SPREMANN, Klaus (2000), Portfoliomanagement, Oldenbourg, München et al.

TOUTENBURG, Helge (2000), Induktive Statistik, 2nd ed., Springer Verlag, Berlin et al.

WATTS, Harold W. (1968), An Economic Definition of Poverty, in: Daniel P. MOYNIHAN (ed.), *On Understanding Poverty*, Basic Books Inc., New York and London, pp. 316–329.

ZHENG, Buhong (1997), Aggregate Poverty Measures, *Journal of Economic Surveys*, vol. 11, pp. 123–162.