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Bank Capital Regulation, Asset Risk, and Subordinated Uninsured Debt

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Abstract

Bank capital regulation is intended to limit the insolvency risk of banks. Whether more stringent capital requirements lead to reduced or to increased bank risk-taking has been discussed controversially in the academic literature. In this paper we drop the common assumptions that banks only issue deposits whose returns are guaranteed by a subsidized deposit insurance and that deposit insurance is free. If a value-maximizing bank additionally issues subordinated uninsured debt and has to pay a flat-rate deposit insurance premium, its reaction to a higher capital requirement may change substantially. We identify situations in which banks increase asset risk due to the enforcement of a more stringent capital requirement.

JEL classification: G21, G28
Key words: capital regulation, bank risk-taking, deposit insurance, subordinated uninsured debt

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1 Introduction

The risk-taking of banks is limited by regulatory capital requirements in an attempt to prevent bank insolvency. In the eighties the US capital regulation was modified several times accompanied by a controversial discussion of the effect of a rise of capital requirements on banks' risk-choice. Portfolio models as well as state-preference models and models based on option pricing theory were used to analyse the question whether a more stringent capital requirement induces higher bank risk-taking. The answers vary due to the different analytical frameworks. For example Kahane (1977), Koehn and Santomero (1980), and Kim and Santomero (1988) apply portfolio theory to show that higher capital requirements can give a utility-maximizing bank the incentive to increase the risk of its portfolio. The increase in asset risk may offset the desired effect of reduced leverage. Thus the regulatory authority cannot be sure to achieve a reduction of insolvency risk. Furlong and Keeley (1989) point out the relevance of the deposit insurance guarantee. In models based on state-preference theory and option pricing theory, respectively, they prove that it is not optimal for value-maximizing banks to raise asset risk as a reaction to a higher capital requirement. The discussion has been continued until today (e.g., Lam and Chen, 1985, Flannery, 1989, Zarruk, 1989, Gennotte and Pyle, 1991, Kendall, 1991, Rochet, 1992, Gjerde and Semmen, 1995, Besanko and Kanatas, 1996, Blum, 1999, Santos, 1999) accompanied by numerous empirical investigations (e.g., Furlong, 1988, Karels et al., 1989, Avery and Berger, 1991, Bock, 1995, Grenadier and Hall, 1995, the survey article of Wall and Peterson, 1996, and the references cited therein, Ediz et al., 1998).

Last year, the Basle Committee on Banking Supervision (1999) issued a proposal for a new capital adequacy framework replacing the 1988 Basle capital accord. Hence the question about the effect of changing capital requirements seems to be even more relevant than before. We are once again going to analyse the impact of tightening the capital requirement, for "the theoretical issue of how higher required equity ratios affect bank risk-taking is [still] unresolved" (Berger et al., 1995). In this paper we concentrate on the asset-substitution effect: Does the enforcement of a more stringent capital adequacy rule lead a bank to increase asset risk?

In our opinion the asset-substitution effect has not been examined sufficiently yet. Some important aspects, which are interesting from the theoretical point of view as well as from a practical perspective, are missing in the literature mentioned above. Firstly, the capital structure of the bank is often assumed to be quite simple, consisting only of equity and insured deposits. Knowing that in reality banks do not hold only insured deposits, this assumption does not seem to be very reasonable. For this reason, in our paper banks are allowed to issue deposits whose repayments are guaranteed by a deposit insurance but also subordinated uninsured debt. The issue of subordinated uninsured debt is an important aspect in the discussion about the new capital adequacy framework mentioned above. The Basle Committee on Banking Supervision (1999) promotes enhanced transparency, e.g., of the level of seniority, to support market discipline as one of the three pillars of its framework. The Shadow Financial Regulatory Committees (1999) even recommend "a minimum subordinated debt requirement as a means to bring market discipline to bear on bank risk and capital management." However, the issue of subordinated debt may also induce some negative effects. We will demonstrate that the inclusion of subordinated uninsured debt changes the answer to the question how the incentive for a bank to expand the risk of its asset portfolio varies with a rise in capital requirements. In some cases the bank may *increase* asset risk.[1]

Secondly, the role of imperfections on the markets for insured and uninsured liabilities is analysed in our paper. We are going to show in which market scenario one must drop the assumption of banks with only insured deposits. Thirdly, Furlong and Keeley (1989) say that their conclusions are the same whether deposit insurance is free or the bank has to pay a positive, fixed-rate premium. The question whether the statement is still valid in a modified analytical framework is answered in this paper.

Unlike insured depositors a bank's uninsured creditors only get the contracted returns in

[1] If insured deposits and uninsured debt are both of equal seniority, the impact of a more stringent capital rule on asset risk is (also) ambiguous. Cf. Homölle (2000). In this paper we confine ourselves to a scenario with *subordinated* uninsured debt.

full if the bank is solvent. Therefore uninsured creditors are interested in bank risk-taking and require a risk premium in compensation for insolvency risk. We use a state-preference model to distinguish between situations with different repayments to the bank's creditors due to bank solvency and insolvency. A model based on standard portfolio theory (e.g., Koehn and Santomero, 1980, Kim and Santomero, 1988) would not fit because the returns on uninsured debt are not exogenous but depend on bank behaviour. The structure of our state-preference model is introduced in section 2.

In section 3, we start our analysis describing the behaviour of a bank which has to comply with a leverage restriction. The optimal levels of insured deposits, uninsured debt, and asset risk are determined for two market scenarios: In the first scenario the bank has got some monopoly power on the market for insured deposits whereas the market for uninsured debt is perfectly competitive. In the second scenario both markets are imperfect.

The widely discussed scenario with only insured deposits is also included in our analysis because in some scenarios, as will be shown later, it is optimal not to hold uninsured debt. For several reasons we do not consider banks with uninsured debt and no insured deposits. Firstly and most importantly, an optimizing bank holds (as soon will become clear) insured deposits as long as the deposit insurance is subsidized which is a common assumption in the literature as well as in our model. Secondly, in our analytical framework the bank's profit would not be influenced by asset risk if the bank held solely uninsured debt. The bank could choose any level of asset risk. Thus, the question how asset risk changes as a reaction to a higher capital requirement could not be answered within this model. It could only be analysed how different assumptions of the market for uninsured debt change bank profits. But this is not the intention of this paper.

Finally, in section 4, we prove that a bank which only issues insured deposits and has a fixed amount of equity will decrease asset risk as a reaction to a more stringent capital regulation. However, a bank which in addition holds subordinated uninsured debt may raise asset risk if it has to pay a positive, fixed-rate deposit insurance premium. Moreover, the response of a bank with variable equity to a tightened capital requirement is not as clear as stated by Furlong and Keeley (1989). In this case an increased bank risk-taking cannot be denied either.

2 The Model

The analytical framework of this paper is a model based on state-preference theory (e.g., Arrow, 1964, Hirshleifer, 1965, Hirshleifer, 1966, Myers, 1968). As a starting point of our analysis, we use the approach of Furlong and Keeley (1989) which is extended and altered in several aspects. We consider two dates, time 0 and time 1, and three possible states of nature at time 1. The time 0 price of one dollar payout in state i, p_i, $i = 1, 2, 3$, is taken as given where $\sum_i p_i < 1$ indicating a positive rate of interest.

At time 0 the bank we are analysing decides on the structure of its capital and assets. The bank raises equity, C_0, which has to exceed a certain level.[2] Bank owners have limited liability. The bank can issue insured deposits, E_0, and uninsured debt, U_0.

For the deposit insurance guarantee the bank pays a fixed-rate premium depending on the amount of insured deposits:[3]

$$I_0 = vE_0, \qquad 1 > v \geq 0. \tag{1}$$

Furlong and Keeley (1989) assume a free deposit insurance. This case is included in our model by setting the deposit insurance premium rate, v, equal to zero.

The capital invested in assets, A_0, consists of equity, insured deposits, diminished by the

[2] A minimum amount of regulatory capital (equity) is often one of the requirements to obtain a bank charter. For example, German banks who want to hold deposits have to issue at least 5 million Euro as initial core capital.

[3] For example the deposit insurance of German private banks (Einlagensicherungsfonds des Bundesverbandes Deutscher Banken) required a uniform, yearly premium of 0,03 % of liabilities to non-banks until 1998. In that year some kind of risk-adjusted premium was adopted by introducing three risk classes for banks. So the premium rates differ according to the risk class, but they still refer to the amount of (insured) liabilities to non-banks.

insurance premium, and uninsured debt:

$$A_0 = C_0 + E_0(1 - v) + U_0.$$

The bank can allocate its funds among the securities X and Y. Let α, $0 \leq \alpha \leq 1$, be the share of X in the asset portfolio and $1 - \alpha$ the share of Y.[4] At time 1 the bank gets a return of x_i (y_i) per invested dollar in security X (Y) if state i occurs. As the capital market is assumed to be complete and perfect, the net present values of the assets equal zero. Security X is riskier than security Y because the spread of returns on X is higher than that of Y:[5]

$$-1 + p_1x_1 + p_2x_2 + p_3x_3 = -1 + p_1y_1 + p_2y_2 + p_3y_3 = 0, \tag{2}$$

$$x_1 < y_1 < x_2 < y_2 < y_3 < x_3. \tag{3}$$

From (3) we know that, independent of α, asset returns in state 1 are lower than those in state 2 which in turn fall below the asset returns in state 3: $A_1 < A_2 < A_3$ where $A_i = A_0(\alpha x_i + (1 - \alpha)y_i)$, $i = 1, 2, 3$.

The returns on X and Y in states 1 and 2 at time 1 are assumed to be so low that the bank cannot pay the promised returns E_0e and U_0u on insured deposits and uninsured debt, respectively, even if it only holds security Y ($\alpha = 0$):

$$A_0(\alpha x_i + (1 - \alpha)y_i) \leq A_0y_2 < E_0e + U_0u, \qquad i = 1, 2. \tag{4}$$

The bank does not meet its obligations to depositors and other creditors. Thus it is insolvent.

Insured deposits are senior to uninsured debt. From (3) we know that the return on the asset portfolio in state 2 is higher than that in state 1. Assume that in state 2 the

4 The bank cannot sell short X and Y. There exists a conflict between this implicit assumption of Furlong and Keeley (1989) and the state-preference theory because a capital market with these two securities and short-selling restrictions is not complete. Thus one should at least implicitly suppose that there exists a third security which may be sold short. For a complete capital market with short-selling restrictions cf. Raab and Schwager (1993).

5 Cf. the comment of Sharpe (1978) on the definition of "risk" in a state-preference model: "Risk is generally considered to have increased when a set of returns becomes more ‘spread’ out."

asset returns exceed the promised repayments to the insured depositors, $E_0 e$, whereas in state 1 the asset returns are so low that even the insured depositors do not fully receive the contracted returns from the insolvent bank. For $0 \leq \alpha \leq 1$ it holds that:[6]

$$A_0(\alpha x_1 + (1-\alpha)y_1) \leq A_0 y_1 < E_0 e, \tag{5}$$

$$E_0 e < A_0 x_2 \leq A_0(\alpha x_2 + (1-\alpha)y_2). \tag{6}$$

The deposit insurance guarantees that insured depositors receive full repayment at time 1 whatever state occurs. Hence, in state 1 the deposit insurance has to pay to insured depositors the difference between promised returns, $E_0 e$, and asset returns:

$$I_1 = E_0 e - A_0(\alpha x_1 + (1-\alpha)y_1). \tag{7}$$

The uninsured creditors (and the bank owners) get no return in state 1. In state 2 they receive:

$$U_0 u_2 = A_0(\alpha x_2 + (1-\alpha)y_2) - E_0 e. \tag{8}$$

Note that if the bank does not issue uninsured deposits, bank insolvency does not happen in state 2.

If state 3 occurs, the bank is solvent because asset returns are assumed to be higher than the promised repayments of total liabilities:

$$A_0(\alpha x_3 + (1-\alpha)y_3) \geq A_0 y_3 > E_0 e + U_0 u. \tag{9}$$

[6] If the uninsured debt, U_0, is sufficiently high, inequality (5) is violated. In this case there would not exist a positive net present value of the deposit insurance guarantee. A crucial assumption in our paper (see (14)) would be violated. Thus (5) implicitly defines an upper bound for U_0. In general, the insolvency and solvency conditions (4) and (9) as well as (5) and (6) are assumed to hold at least for some values of E_0 and U_0 irrespective of α. Thus, we implicitly define a starting point (or better: area) for determining the optimal levels of insured and uninsured liabilities and of asset risk.

The difference is paid to the owners ($C_3 = A_0(\alpha x_3 + (1-\alpha)y_3) - E_0 e - U_0 u$).

In this paper we consider a value-maximizing bank excluding agency problems, e.g., between bank managers and owners. The bank tries to maximize the net present value of the equity, put differently (the present value of) its profit, G_0:

$$\begin{aligned} &\max_{C_0, E_0, U_0, \alpha} \quad G_0 \\ &\text{where} \quad G_0 = C_0^B - C_0 \\ &\qquad\qquad\quad = I_0^B - I_0 + E_0 - E_0^B + U_0 - U_0^B. \end{aligned} \tag{10}$$

Note that the net present value of assets is set equal to zero (see (2)). With the help of (7) and (8), the present values of the deposit insurance subsidy, I_0^B, the insured deposits, E_0^B, and of uninsured debt, U_0^B, can be calculated as follows:

$$I_0^B = p_1[E_0 e - A_0(\alpha x_1 + (1-\alpha)y_1)], \tag{11}$$

$$E_0^B = (p_1 + p_2 + p_3)E_0 e, \tag{12}$$

$$U_0^B = p_2[A_0(\alpha x_2 + (1-\alpha)y_2) - E_0 e] + p_3 U_0 u. \tag{13}$$

The net present value of the deposit insurance guarantee is assumed to be positive:

$$I_0^B - I_0 > 0. \tag{14}$$

This assumption is implicitly made by all authors who model a free deposit insurance (e.g., Furlong and Keeley, 1989, Gennotte and Pyle, 1991, Gjerde and Semmen, 1995). A reason for assuming a positive net present value in a world without asymmetric information[7] is given by Buser et al. (1981): The deposit insurance sets the premium lower than the present value to persuade a value-maximizing bank to take part in the deposit insurance system. Thus the bank is willing to accept some limits to its activities set by the deposit insurance or

[7] Asymmetric information between deposit insurance institution and bank is often given as a reason for the underpricing of the deposit insurance subsidy. For the calculation of a theoretically correct premium cf. Merton (1977), Merton (1978), Pyle (1984), Chan et al. (1992), Duan and Yu (1999).

(other) regulatory authorities. The explicit premium, I_0, is supplemented by an implicit premium, e.g., via capital requirements.

In this paper we distinguish two situations concerning the markets for liabilities. For the first scenario we follow Kareken and Wallace (1978). The market for subordinated uninsured debt is perfectly competitive:

$$\mathbf{pu} = 1 \tag{15}$$

$$\text{where} \qquad \mathbf{p} = (p_1, p_2, p_3),$$

$$\mathbf{u} = (0, u_2, u)'.$$

The return on a riskless asset would be $1/(p_1 + p_2 + p_3)$ und thus independent of bank behaviour. However, the return u the bank has to promise to attract uninsured debt depends on the chosen values for the share of the riskier security, α, equity, C_0, insured deposits, E_0, and uninsured debt, U_0 (see (8) and (15)). From (15) together with (8) and (13) it follows that $U_0 = U_0^B$.

At the same time, the bank has some monopoly power on the market for insured deposits, e.g., because it supplies payment services together with insured deposits (Kareken and Wallace, 1978, Dothan and Williams, 1980). The demand equation for these deposits is:

$$E_0 = f(\mathbf{pe}) \tag{16}$$

$$\text{where} \qquad \mathbf{pe} \in [0, 1[, \qquad \mathbf{e} = (e, e, e)',$$

$$f(0) = 0, \qquad f'(\mathbf{pe}) > 0,$$

$$\lim_{\mathbf{pe} \to 1} f(\mathbf{pe}) = \infty, \qquad f''(\mathbf{pe}) > 0.$$

The demand for insured deposits is a strictly increasing, convex function of (the present value of) the return e. As a consequence of (16) the net present value of insured deposits is positive.

In the second scenario the market for subordinated uninsured debt is imperfect, too.

The bank may issue insured and subordinated uninsured deposits. In reality such uninsured deposits can be observed when only a part of each deposit is covered by the deposit insurance. In the EU depositors who have more information about the bank and do not have to be protected as much as others may even totally be excluded from deposit insurance. The demand for uninsured debt is a strictly increasing and convex function g of the present value of the state-dependent returns at time 1:

$$U_0 = g(\mathbf{pu}) \tag{17}$$

$$\text{where} \quad \mathbf{pu} \in [0, 1[, \qquad g'(\mathbf{pu}) > 0,$$
$$g(0) = 0, \qquad g''(\mathbf{pu}) > 0,$$
$$\lim_{\mathbf{pu} \to 1} g(\mathbf{pu}) = \infty.$$

Insured depositors as well as uninsured creditors accept repayments at time 1 whose present value is less than their payments to the bank at time 0 ($E_0^B < E_0$, $U_0^B < U_0$).

3 A Regulated Bank

3.1 Profit Function and Leverage Contraint

Knowing the basic structure of the model, we now look at the optimal behaviour of a regulated bank trying to separate situations in which banks issue both, insured deposits and subordinated uninsured debt, from situations with banks only holding insured deposits. At time 0 the bank determines the optimal share of security X, α, and the optimal levels of equity, C_0, insured deposits, E_0, and of uninsured debt, U_0. The bank maximizes its profit by maximizing the net present values of the deposit insurance guarantee, insured deposits, and uninsured debt. Inserting (1), (11), (16), and (17) into (10), the profit function can be written as:

$$G_0 = p_1 \left[\frac{1}{p_1 + p_2 + p_3} E_0 f^{-1}(E_0) - A_0(\alpha x_1 + (1-\alpha) y_1) \right] - vE_0 \\ + E_0(1 - f^{-1}(E_0)) + U_0(1 - g^{-1}(U_0)) \tag{18}$$

where $\quad A_0 = C_0 + E_0(1 - v) + U_0,$

$$f^{-1}(E_0) = (p_1 + p_2 + p_3)e,$$

$$g^{-1}(U_0) = p_2 u_2 + p_3 u.$$

Note that if the market for uninsured debt is perfect, it holds that $p_2 u_2 + p_3 u = 1$ (see (15)), i.e. the last term of (18) equals zero.

It can easily be shown that a profit-maximizing, *un*regulated bank would raise leverage risk as far as possible holding only the prescribed minimum amount of equity and expanding insured deposits ad infinitum. This is because the positive net present value of the deposit insurance subsidy is a strictly increasing function of E_0 and the net present value of the insured deposits is assumed to be positive (Homölle, 1999). This risk-shifting behaviour is optimal as long as the marginal gain is higher than the marginal cost. Regulatory restrictions may serve as a source of such (marginal) cost. We assume that the regulatory authority imposes some constraints on bank behaviour to restrict bank risk-taking and to prevent unappropriately high payments from the deposit insurance in case of bank insolvency.[8]

By assumption the regulatory authority introduces a leverage constraint which guarantees that assets are not only financed by liabilities. However, we do not consider – as would be intuitive – an upper bound for the ratio of assets to equity. Instead we assume, mainly for analytical convenience, that the bank has to comply with a lower bound, s, on the ratio of assets to liabilities:

$$\frac{A_0}{E_0 + U_0} \geq s > 1. \tag{19}$$

[8] As Kareken and Wallace (1978) put it: "Regulation is not an alternative to deposit insurance but rather a necessary complement." See also Crouhy and Galai (1991), Flannery (1982).

Because of $s > 1$, assets are not allowed to be solely financed by liabilities, i.e. bank equity (minus the deposit insurance premium) must be positive. For several reasons we analyse this quite simple form of capital constraint and do not include any risk-weights. First of all, using (19) our conclusions can quite easily be compared with former results presented in the literature, especially with Furlong and Keeley (1989). Moreover, in the discussion about the new proposal of the Basle Committee it is obvious that simple capital ratios have not become obsolete yet. The Shadow Financial Regulatory Committees (1999), e.g., believe that a simple leverage constraint would be superior to the current risk-weighted capital requirement. Finally, if we allow for asset risk-weights, our results would not change very much as long as we assume that both securities X and Y are in the same risk class.

3.2 Optimal Volumes of Insured Deposits and Subordinated Uninsured Debt

The capital rule (19) is binding because it restricts the incentive for a bank to increase leverage. Therefore (19) can be transformed as follows:

$$\frac{A_0}{E_0 + U_0} = s \quad \Leftrightarrow \quad C_0 - (s - 1 + v)E_0 - (s - 1)U_0 = 0. \tag{20}$$

When the bank chooses the optimal amounts of insured deposits, uninsured debt, and equity, it has to comply with (20) so that the optimal levels of E_0, U_0, and C_0 are not independent from each other. For example, the regulated bank can only issue additional insured deposits by reducing uninsured debt or raising equity. This can be shown by the total differential of (20) where $ds = 0$:

$$dE_0 = -\frac{s-1}{s-1+v}dU_0 + \frac{1}{s-1+v}dC_0. \tag{21}$$

Thus far we have at least implicitly assumed that banks can increase their equity without any problems. But this is not true in general. E.g., publicly owned savings banks are unable to issue new equity in the capital market. So it may be difficult for them to respond to a new capital regulation by increasing equity. For this reason we are going to differentiate between banks whose equity can be viewed as constant (at least in the short run) and banks which can raise their equity. This distinction is also drawn by Furlong and Keeley (1989).

1) Fixed equity ($dC_0 = 0$): If equity is supposed to be fixed, the leverage constraint defines a negative relation between changes of insured deposits and subordinated uninsured debt and, since U_0 cannot be negative, an upper bound for E_0. The question is whether insured deposits should be expanded up to this upper limit, $E_0 = C_0/(s-1+v)$. There are only two decision variables left: E_0 *or* U_0 and α. When the bank determines the optimal level of insured deposits, it simultaneously chooses the optimal amount of uninsured debt.

In the first market scenario the marginal gain from increasing insured deposits (and reducing uninsured debt) can be expressed by the total derivative of the profit function (18) (where $ds = 0$, $dC_0 = 0$, and $\mathbf{pu} = 1$):

$$\begin{aligned} \left.\frac{dG_0}{dE_0}\right|_{s,C_0} = {} & p_1\left[\frac{1}{p_1+p_2+p_3}f^{-1}(E_0) + \frac{vs}{s-1}(\alpha x_1 + (1-\alpha)y_1)\right] - v \\ & + 1 - f^{-1}(E_0) - \frac{p_2+p_3}{p_1+p_2+p_3}E_0 f^{-1\prime}(E_0). \end{aligned} \tag{22}$$

Since the net present value of insured deposits (per insured deposit) is assumed to be positive (see (14)),[9] this net present value strictly increases in E_0. Besides, the positive sign of the ratio $vs/(s-1)$ – if $v > 0$ – is noteworthy. Because of the positive deposit insurance premium assets shrink as long as the bank issues additional insured deposits and reduces uninsured debt: For $v > 0$ and $dC_0 = 0$, it holds that $|dE_0| < |dU_0|$ according to (21). Hence for $dE_0 > 0$ it follows that $dA_0 = (1-v)dE_0 + dU_0 < 0$. Thus asset returns in state 1 decline whereas the promised returns on insured deposits rise which has a positive influence on the net present value of the deposit insurance subsidy and the profit. Since this effect cannot be observed if the premium rate, v, equals zero, *the assumption of a free deposit insurance is not innocuous.* However, even with a zero premium rate the net present value of the deposit

[9] For very low values of E_0 the net present value of the deposit insurance guarantee might become negative because the promised return e is very low. As $f^{-1}(E_0) = (p_1+p_2+p_3)e$ is a strictly increasing function of E_0, however, we can assume that starting at a certain level of insured deposits the deposit insurance guarantee is positive. See also footnote 6.

insurance guarantee is strictly increasing in E_0 because the required return $E_0 e$ is positively related to the volume of insured deposits.

(22) may equal zero at a certain level of insured deposits, E_0^*, depending on the exact demand equation for insured deposits. This (local) maximum exists if the increase in the net present value of the deposit insurance subsidy equals the decrease in the net present value of insured deposits at $E_0^* < C_0/(s-1+v)$.

For example, if $v = 0$, we can reduce (22) to

$$\left.\frac{dG_0}{dE_0}\right|_{s,C_0} = \frac{p_2 + p_3}{p_1 + p_2 + p_3}[1 - f^{-1}(E_0) - E_0 f^{-1\prime}(E_0)] + \frac{p_1}{p_1 + p_2 + p_3}.$$

Suppose the net present value of insured deposits has a maximum with $1 - f^{-1}(E_0) - E_0 f^{-1\prime}(E_0) = 0$. It can easily be shown that this is true for $f(\mathbf{pe}) = |\ln(1 - \mathbf{pe})|^n$, $n \geq 1$ (see appendix A.1). Then it may be possible that an increase in insured deposits above this maximum reduces the net present value of insured deposits as much as it increases the net present value of the deposit insurance subsidy and hence $dG_0/dE_0|_{s,C_0} = 0$.

A global maximum of the profit function of the regulated bank with fixed equity occurs at E_0^* if

$$G_0(E_0^*) > G_0\left(\frac{1}{s-1+v} C_0\right).$$

Under this condition it is not optimal to issue insured deposits at a level of $C_0/(s-1+v)$, but rather to hold insured *and* uninsured liabilities (for an example see Fig. 1). Finally, the question whether this interior solution exists or whether the bank issues insured deposits up to $E_0 = C_0/(s-1+v)$ can only be answered if the values of the parameters of (18) are known.

Considering imperfect markets for all liabilities we get a similar result. Due to imperfections on the market for uninsured debt and the relationship between changes of insured and uninsured liabilities, we now have to take into account the influence on the positive net present value of uninsured debt. The total derivative of (18) can be written as:

$$\left.\frac{dG_0}{dE_0}\right|_{s,C_0} = p_1\left[\frac{1}{p_1+p_2+p_3}f^{-1}(E_0) + \frac{vs}{s-1}(\alpha x_1 + (1-\alpha)y_1)\right] - v$$
$$+ 1 - f^{-1}(E_0) - \frac{p_2+p_3}{p_1+p_2+p_3}E_0 f^{-1\prime}(E_0) \tag{23}$$
$$- \frac{s-1+v}{s-1}[1 - g^{-1}(U_0) - U_0 g^{-1\prime}(U_0)].$$

We already know that the net present value of the deposit insurance subsidy is strictly increasing in E_0. The net present values of insured deposits and uninsured debt are either strictly increasing in E_0 and U_0, respectively, (e.g., if $f(\mathbf{pe}) = 1/(1-\mathbf{pe})^n - 1$, $n \geq 1$) or functions with an interior maximum (see appendix A.1). The profit maximizing bank has to consider that the issuance of insured deposits up to the maximum amount, $E_0 = C_0/(s-1+v)$, may be non-optimal because of the possible low level of the net present values of insured and/or uninsured liabilities. As described above, there may be an interior solution for the optimal level of insured deposits, E_0^*. Hence, the regulated bank may hold uninsured debt.

Result 1 *If a regulated bank with a fixed amount of equity is sufficiently disciplined by the demand for insured deposits (and for uninsured debt), it issues both, insured deposits and subordinated uninsured debt.*

2) Variable equity ($dC_0 > 0$): If the bank's equity is supposed to be variable, the regulated bank may increase insured deposits while reducing uninsured debt *or* raising equity. In this case the leverage constraint (19) does not define an upper bound for insured deposits. Due to possible adjustments of the equity, the optimal amounts of insured deposits and uninsured debt are independent from each other. We start calculating the optimal value of U_0 before the optimal E_0 is determined. (Then the optimal volume of equity is given by (20).)

If the market for uninsured debt is perfect, it is not optimal for the bank to hold uninsured debt. As can be seen from the total derivative of (18),

$$\left.\frac{dG_0}{dU_0}\right|_{s,E_0} = -p_1 s(\alpha x_1 + (1-\alpha)y_1) < 0, \tag{24}$$

the profit decreases in U_0. The issue of uninsured debt (accompanied by a rise of equity) would lead to an increase in assets. This would not improve the profit, for the net present values of uninsured debt and of assets equal zero. Instead the profit would shrink because more assets would be available in state 1 to pay off the claims of creditors. The bank owners would have to take over more of the losses in state 1. The present value of the deposit insurance guarantee would decline.

Result 2 *A regulated bank which can raise additional equity does not issue subordinated debt as long as the market for uninsured debt is perfect.*

With imperfections on the market for uninsured debt, the derivative $dG_0/dU_0|_{s,E_0}$ is different from the one described above:

$$\left.\frac{dG_0}{dU_0}\right|_{s,E_0} = -p_1 s(\alpha x_1 + (1-\alpha)y_1) + 1 - g^{-1}(U_0) - U_0 g^{-1\prime}(U_0). \tag{25}$$

As opposed to the situation with a perfect market for uninsured debt, it might now be optimal for a bank to issue uninsured debt. Depending on the exact demand equation for uninsured debt, the increase in the net present value of uninsured debt may exceed the decrease in the net present value of the deposit insurance guarantee (beginning at a certain level of U_0). Then the profit, G_0, strictly increases in U_0.[10]

There may also exist an optimal amount of uninsured debt, U_0^*. That means the increase in the net present value of uninsured debt exactly compensates the decrease in the net present value of the deposit insurance guarantee. Additionally this net present value must

[10] In this paper we confine ourselves to situations with a positive net present value of the deposit insurance guarantee. For this reason scenarios with such high optimal level of U_0 that (5) and (14) are violated are not considered. However, notice that sufficiently high subordinated uninsured debt stops the moral hazard problem of a subsidized deposit insurance.

be a strictly concave function at U_0^*:

$$p_1 s(\alpha x_1 + (1-\alpha)y_1) = 1 - g^{-1}(U_0^*) - U_0^* g^{-1\prime}(U_0^*), \tag{26}$$

$$-2g^{-1\prime}(U_0^*) - U_0 g^{-1\prime\prime}(U_0^*) < 0. \tag{27}$$

E.g., $g(\mathbf{pu}) = 1/(1-\mathbf{pu})^n - 1$, $n \geq 1$, implies such a strictly concave function of U_0 and fulfills all the conditions of (17) (see appendix A.1).

Result 3 *If a regulated bank which can raise equity has some monopoly power on the market for uninsured debt, it may be optimal (depending on the concrete demand equation) to hold subordinated uninsured debt.*

The different assumptions about the market for uninsured debt do not influence the marginal gain of increasing E_0 (accompanied by rising equity):

$$\begin{aligned} \left.\frac{dG_0}{dE_0}\right|_{s,U_0} &= p_1 \left[\frac{1}{p_1+p_2+p_3} f^{-1}(E_0) - s(\alpha x_1 + (1-\alpha)y_1)\right] - v \\ &\quad + 1 - f^{-1}(E_0) - \frac{p_2+p_3}{p_1+p_2+p_3} E_0 f^{-1\prime}(E_0). \end{aligned} \tag{28}$$

The regulated bank increases its insured deposits (and its equity) as far as possible because the net present value of the deposit insurance subsidy (see (14)) is positively related to E_0 (see appendix A.2) and the net present value of insured deposits is assumed to be positive. Although there may exist a local maximum, it holds that $\lim_{E_0 \to \infty} G_0 = \infty$ (e.g., see Fig. 2).

Furlong and Keeley (1989) also describe a bank's incentive to increase insured deposits and equity, thus expanding asset size. They argue that the bank invests in assets until the marginal gain is balanced by the marginal cost and regard regulatory restrictions of asset size as a main source of this cost. In our more general analytical framework this crucial assumption is no longer convincing because the increase in insured deposits and *not* the rise of assets is responsible for the rising deposit insurance subsidy and because a rise of insured deposits does *not* necessarily lead to an increase in assets (due to the possible reduction of uninsured debt).

Result 4 *A regulated bank which can raise additional equity tends to expand insured deposits ad infinitum, thus increasing the payments of the deposit insurance. A supplementary restriction of insured deposits seems to be necessary to limit the deposit insurance subsidy.*

That is why we do not assume that there exist some regulatory cost depending on asset size. Instead, we suppose that the regulatory authority directly limits the incentive of a bank to increase E_0 by regulatory cost, K^E, as a function of E_0, where

$$\frac{dK^E}{dE_0} > 0, \qquad \frac{d^2K^E}{dE_0{}^2} > 0. \tag{29}$$

The assumption about restrictions of insured deposits is not only important for our analysis. In reality such limitations exist as well. For example, the European directive concerning deposit insurance requires an upper bound of 20,000 Euro per person.[11] Such an upper bound is not directly connected with regulatory cost but we can imagine that the circumvention of this limit, e.g., opening (insured) deposit accounts under the name of relatives if a person's insured deposits reach the limit, may induce some transaction cost.

Taking this cost into account the regulated bank issues insured deposits up to $\bar{E}_0$ where the marginal cost equals the marginal gain:

$$\frac{dK^E}{dE_0} = \left.\frac{dG_0}{dE_0}\right|_{s,U_0} \quad \text{where} \quad E_0 = \bar{E}_0. \tag{30}$$

3.3 Optimal Level of Asset Risk

Whether the bank's equity is fixed or variable does not influence the optimal share of the riskier security X. With $A_0 = s(E_0 + U_0)$, the marginal gain of increasing asset risk can be written as follows (where $E_0 > 0$, $U_0 \geq 0$):

$$\left.\frac{\partial G_0}{\partial \alpha}\right|_s = -p_1 s(E_0 + U_0)(x_1 - y_1) > 0. \tag{31}$$

[11] This is a kind of minimum upper limit because higher national deposit insurance guarantees may be maintained.

Independent of the market assumptions the bank only invests in the riskier asset X. This result also holds for an unregulated bank and is widely known as the moral hazard problem of a subsidized deposit insurance (e.g., Kareken and Wallace, 1978, Sharpe, 1978, Dothan and Williams, 1980, Gjerde and Semmen, 1995): The returns to the bank owners in state 3, C_3, rise whereas – due to limited liability – the bank owners do not lose more than the bank's equity in case of insolvency (states 1 and 2). As the insured depositors know that they will receive the promised return per dollar, e, in each state, they do not require a higher return. However, the payments to uninsured creditors in state 2, u_2, are reduced by increasing α. The bank compensates this decrease by raising the promised return, u, according to (15) and (17), respectively, so that the present value of uninsured debt and the profit are kept constant. Without this compensation the uninsured creditors would withdraw their money.

Result 5 *A bank which has to comply with a leverage constraint only invests in the riskiest asset to maximize profit. An additional limitation of asset risk is necessary to limit risk-taking and to protect the deposit insurance.*

Referring to Furlong and Keeley (1989) we assume that the regulatory authority imposes regulatory cost, K^α, to limit risk-taking. Let K^α be a function of α where

$$\frac{dK^\alpha}{d\alpha} > 0, \qquad \frac{d^2K^\alpha}{d\alpha^2} > 0.$$

Actual regulatory restrictions are influenced by the idea of limiting the share of a single asset. However, in reality it is not easy to determine the riskiest asset. For this reason the share of each asset is limited to guarantee that the portfolio is sufficiently diversified. In Germany, e.g., a loan to a single company is restricted by law to 25 % of the bank's regulatory capital. The banks are allowed to go beyond this limit but they must in this case hold capital covering 100 % of the exceeding amount. This capital cannot be used any more to fulfill other regulatory rules and therefore loan concentration does indeed lead to some cost.

The bank raises the share of the riskier security X until the marginal gain from increasing α equals the marginal cost. Suppose the regulatory authority does not want the share of X to exceed an upper bound, $\bar{\alpha} < 1$. The regulatory cost, K^{α}, is then designed so that the first-order condition for the profit maximum at $\bar{\alpha}$ holds:

$$\frac{dK^{\alpha}}{d\alpha} = \left.\frac{\partial G_0}{\partial \alpha}\right|_s \quad \text{where} \quad \alpha = \bar{\alpha}. \tag{32}$$

The results referring to the structure of assets and liabilities of a regulated bank are summarized in Table 1. The bank has to take into account the capital constraint (19) and the regulatory cost, K^{α} and possibly K^{E}. The various profit maxima are used as starting points for the analysis of the impact of a higher capital requirement in the next section.

4 The Enforcement of a More Stringent Capital Requirement

4.1 A Bank with Fixed Equity

After having shown in which situations regulated banks may issue subordinated uninsured debt, we now describe the effects of higher capital requirements on asset risk. Suppose the regulatory authority enforces a more stringent capital requirement ($ds > 0$) to improve bank solvency. We analyse whether the rise of s can induce the bank to take higher asset risk. For that analysis we once again distinguish between banks with fixed and variable equity considering two scenarios for each: First, we look at a bank which only holds insured deposits ($U_0 = 0$). Second, we analyse the behaviour of a bank whose liabilities consist of insured deposits and uninsured debt ($U_0 > 0$).

1) No uninsured debt ($U_0 = 0$): In the previous section we showed that under certain circumstances it may be optimal for a bank with fixed equity not to hold uninsured debt but solely insured deposits. In these situations the optimal level of insured deposits is defined by the leverage constraint (19):

$$E_0 = \frac{1}{s-1+v} C_0. \tag{33}$$

The bank chooses the optimal share of security X which is defined by (32). For $U_0 = 0$ this condition is reduced to:

$$\frac{dK^\alpha}{d\alpha} = -p_1 s E_0 (x_1 - y_1). \tag{34}$$

Applying the envelope theorem we know that if the regulatory authority increases the capital ratio ($ds > 0$), the bank changes its decision variables E_0 and α so that the first-order conditions for a maximum (33) and (34) still hold (remember $dC_0 = 0$):

$$\begin{aligned} dE_0 &= -\frac{1}{(s-1+v)^2} C_0 ds, \\ \frac{d^2 K^\alpha}{d\alpha^2} d\alpha &= -p_1 s (x_1 - y_1) dE_0 - p_1 E_0 (x_1 - y_1) ds. \end{aligned}$$

After some transformations we get:

$$\frac{dE_0}{ds} = -\frac{1}{(s-1+v)^2} C_0 < 0, \tag{35}$$

$$\frac{d\alpha}{ds} = \frac{-p_1 \frac{v-1}{(s-1+v)^2} C_0 (x_1 - y_1)}{\frac{d^2 K^\alpha}{d\alpha^2}} < 0. \tag{36}$$

Since equity is fixed and liabilities only consist of insured deposits, the bank has to reduce these deposits to comply with the new capital regulation (see (35)).

An increase in s shows one direct and one indirect effect on the marginal gain from increasing the share of risky X, $\partial G_0/\partial\alpha|_s$ (see also Table 2). From (31) we know that the marginal gain directly increases when s is raised by the regulatory authority. Additionally the decrease in insured deposits reduces $\partial G_0/\partial\alpha|_s$. The negative sign of (36) implies that the second, indirect effect dominates the first, direct effect. This is because in total asset size shrinks and therefore the marginal loss of an increase in asset risk which is borne by the deposit insurance declines, i.e. the bank's marginal gain from increasing asset risk is reduced. As the marginal cost, $dK^\alpha/d\alpha$, is unaffected by the change of the capital requirement, the bank reduces the share of the riskier security X until equation (34) holds again.

Result 6 *A more stringent capital requirement gives a bank with only fixed equity and insured deposits the incentive to reduce asset risk.*

This conclusion is quite similar to that described by Furlong and Keeley (1989).[12] We are now going to show that it does not necessarily hold for a regulated bank which also issues subordinated uninsured debt.

2) Uninsured debt ($U_0 > 0$): We showed above that it may be optimal for a bank to issue a certain amount of insured deposits, $E_0^* < C_0/(s-1+v)$, for which the following holds:

$$\left.\frac{dG_0}{dE_0}\right|_{s,C_0} = 0. \tag{37}$$

The optimal share of security X is defined by (32). The profit function (including regulatory cost) is strictly concave in E_0^* and $\bar{\alpha}$. Thus we assume that the necessary and sufficient conditions of an interior maximum at E_0^* and $\bar{\alpha}$ hold before the regulatory authority imposes a more stringent capital requirement (see appendix A.3).

The issue of uninsured debt may be optimal in two scenarios. We are going to consider them separately starting with a perfectly competitive market for uninsured debt. In the second case the market for uninsured debt is assumed to be imperfect.

2a) Perfect market for uninsured debt ($U_0 = U_0^B$): According to (15) and (16) the market for uninsured debt is still perfectly competitive whereas the bank has got some monopoly power on the market for insured deposits. The optimal variation of the share of security

[12] Furlong and Keeley (1989), p. 887, say that "a bank would not be expected to respond to higher capital requirements by increasing the riskiness of its asset portfolio." The difference is caused by our assumption about the regulatory cost, K^α. Furlong and Keeley do not argue using a continuous, twice differentiable cost function but only consider the upper part of the cost function. They state that the marginal cost of *exceeding* $\bar{\alpha}$ should be at least equal to the marginal gain.

X when s is raised is calculated by inserting (22) and (31) into (32) and (37) and totally differentiating:

$$\frac{d\alpha}{ds} = \frac{-p_1 \frac{1}{s-1}(x_1 - y_1)\left(-(E_0^* + U_0)a + p_1 \frac{v^2 s}{(s-1)^2}(\bar{\alpha}x_1 + (1-\bar{\alpha})y_1)\right)}{\frac{d^2 K^\alpha}{d\alpha^2}a - p_1{}^2 \left(\frac{vs}{s-1}\right)^2 (x_1 - y_1)^2} \tag{38}$$

where

$$a = \frac{p_2 + p_3}{p_1 + p_2 + p_3}[2f^{-1\prime}(E_0^*) + E_0^* f^{-1\prime\prime}(E_0^*)] > 0,$$
$$U_0 = -\frac{s-1+v}{s-1}E_0^* + \frac{1}{s-1}C_0 > 0.$$

The denominator of (38) must be positive to fulfill the second-order conditions for an original maximum at E_0^* and $\bar{\alpha}$ (see appendix A.3). Thus the sign of $d\alpha/ds$ depends on the numerator:

$$p_1 \frac{v^2 s}{(s-1)^2}(\bar{\alpha}x_1 + (1-\bar{\alpha})y_1) \gtreqless (E_0^* + U_0)a \quad \Leftrightarrow \quad \frac{d\alpha}{ds} \gtreqless 0.$$

If the regulatory authority increases the lower bound s, the bank has to reduce liabilities (the structure of liabilities still unchanged) and assets. Due to this reduction of asset size $\partial G_0/\partial\alpha|_s$, *decreases* (see also Table 2). This direct effect induces the bank to *reduce* asset risk. There are no other effects if the bank does not have to pay any deposit insurance premium. For $v = 0$ the marginal gain from increasing insured deposits (22) and the first-order condition for an maximum (37) are independent of s. Hence, there is no reason to alter the level of insured deposits as a reaction to a more stringent capital requirement. The bank may reduce these deposits or the uninsured debt. If deposit insurance is free, it holds that

$$\frac{d\alpha}{ds} = \frac{p_1 \frac{1}{s-1}(E_0^* + U_0)(x_1 - y_1)}{\frac{d^2 K^\alpha}{d\alpha^2}} < 0. \tag{39}$$

With a positive premium rate ($v > 0$) there is another direct effect of raising s. The marginal gain from increasing E_0, $dG_0/dE_0|_{s,C_0}$, decreases in s so that the maximizing bank reduces its insured deposits while increasing uninsured debt. This regrouping of liabilities (under compliance with the new capital requirement) causes a higher level of assets which leads to a *rise* of $\partial G_0/\partial\alpha|_s$. The total effect of the opposing direct and indirect effects on the marginal gain of increasing asset risk is ambiguous. At the same time it is not clear how the bank changes its insured deposits because the adjustment of the share of the riskier asset influences the optimal variation of E_0.

It can easily be shown (see appendix A.4) that the profit-maximizing bank reduces asset risk if it is optimal to issue additional insured deposits or to leave them unchanged (and retire uninsured debt):

$$\frac{dE_0}{ds} \geq 0 \quad \Rightarrow \quad \frac{d\alpha}{ds} < 0.$$

If the rise of s leads to a decline in insured deposits ($dE_0/ds < 0$), the bank may expand asset risk depending on the parameters of (38). E.g., the lower a and the higher v, the more likely is an increase in asset risk because the indirect effect on $\partial G_0/\partial\alpha|_s$ gets stronger. a is the negative value of the second derivative of the profit function with respect to E_0 at E_0^* (see also appendix A.3) and reflects the curvature of the profit function. With a small a the strictly concave curve is almost flat and the reduction of insured deposits to hold (37) is relatively large which reinforces the indirect effect on $\partial G_0/\partial\alpha|_s$. – A higher premium rate, v, also induces a stronger indirect effect. As

$$dU_0 = -\frac{s-1+v}{s-1}dE_0 \quad \text{and} \quad \frac{\partial(dU_0)}{\partial v} = -\frac{1}{s-1}dE_0 < 0,$$

a rise of v causes a higher increase in uninsured debt if insured deposits shrink which leads to a more expanded asset size. Besides, it is more likely that the indirect effect dominates the direct effect, i.e. $d\alpha/ds$ rises, if the prescribed lower bound of the capital ratio, s, and $\bar{\alpha}$ are low (see (38)).

Result 7 *With a perfect market for subordinated uninsured debt, a bank with fixed equity is more likely to raise asset risk if the premium rate of the deposit insurance and the allowed* *leverage are high, the original optimal level of asset risk is low and the curve of the profit function at the optimal amount of insured deposits before the introduction of the new capital regulation is nearly flat.*

2b) Imperfect market for uninsured debt ($U_0 > U_0^B$): With imperfections on both markets for liabilities (see (16) and (17)) it is also possible that there exists an interior solution for the optimal level of insured deposits. Referring to (23), (31), (32), and (37) we calculate the following variation of the optimal share of the riskier security X when s is changed:

$$\begin{aligned}\frac{d\alpha}{ds} = -p_1 \frac{1}{s-1}(x_1 - y_1) \Bigg(& \frac{-(E_0^* + U_0)\left[a + \left(\frac{s-1+v}{s-1}\right)^2 b\right]}{\frac{d^2K\alpha}{d\alpha^2}\left[a + \left(\frac{s-1+v}{s-1}\right)^2 b\right] - {p_1}^2 \left(\frac{vs}{s-1}\right)^2 (x_1 - y_1)^2} \\ & + \frac{vs\left[p_1 \frac{v}{(s-1)^2}(\bar{\alpha} x_1 + (1-\bar{\alpha}) y_1) - c\right]}{\frac{d^2K\alpha}{d\alpha^2}\left[a + \left(\frac{s-1+v}{s-1}\right)^2 b\right] - {p_1}^2 \left(\frac{vs}{s-1}\right)^2 (x_1 - y_1)^2} \Bigg)\end{aligned} \tag{40}$$

where

$$\begin{aligned} a &= \frac{p_2 + p_3}{p_1 + p_2 + p_3}[2f^{-1\prime}(E_0^*) + E_0^* f^{-1\prime\prime}(E_0^*)], \\ b &= 2g^{-1\prime}(U_0) + U_0 g^{-1\prime\prime}(U_0), \\ c &= \frac{v}{(s-1)^2}[1 - g^{-1}(U_0) - U_0 g^{-1\prime}(U_0)] - \frac{s-1+v}{(s-1)^2}(E_0^* + U_0) b, \\ U_0 &= -\frac{s-1+v}{s-1} E_0^* + \frac{1}{s-1} C_0 > 0. \end{aligned}$$

As before, the sign of (40) depends on the numerator because the denominator is again positive by our assumption to guarantee an interior solution at E_0^* and $\bar{\alpha}$ before the increase in s (see appendix A.3).

If the bank does not have to pay a deposit insurance premium ($v = 0$), the second ratio of (40) equals zero and we get the same result as in the situation with a perfect market for uninsured debt (see (39)). The higher capital requirement directly leads to a reduction of the marginal gain $\partial G_0 / \partial \alpha|_s$.

Result 8 *A bank with insured deposits, subordinated uninsured debt, and fixed equity tends to decrease asset risk as a response to a higher capital requirement if deposit insurance is free.*

This result holds irrespective of the assumptions about the (im-) perfection of the market for uninsured debt.

As long as we assume that equity is constant and deposit insurance is free, we can confirm the result of Furlong and Keeley (1989) even though we allow for subordinated uninsured debt. But if the deposit insurance requires a premium depending on the amount of insured deposits ($v > 0$), the sign of (40) is not clearly positive or negative. Without knowing the exact bank structure and demand equations, we cannot say how bank risk-taking changes as a reaction to a higher capital requirement.

Result 9 *A bank with fixed equity and subordinated uninsured debt has to sell assets and retire liabilities to comply with a more stringent capital regulation. However, the new capital regulation also leads to a new optimal structure of liabilities if the bank has to pay a positive deposit insurance premium. It may be optimal to reduce insured deposits and raise uninsured debt which induces an increase in assets. The total effect on asset size and finally on bank risk-taking depends on the concrete data.*

The direct and indirect effects of s are the same as in the scenario with a perfect market for uninsured debt except for the direct effect on the marginal gain of increasing insured deposits, $dG_0/dE_0|_{s,C_0}$ (see also Table 2). Since the impact of a change of insured deposits on the net present value of uninsured debt (and its variation caused by a higher capital requirement) has to be taken into account, the rise of s may directly induce an *increase* in this marginal gain and thus an expansion of insured deposits. In this case the indirect effect strengthens the direct effect on the marginal of increasing asset risk. The bank *reduces* asset risk. – Like in the case with a perfect market for uninsured debt the following relationship between the optimal adjustments of insured deposits and asset risk hold:

Result 10 *A bank which holds subordinated uninsured debt and cannot issue new equity always reduces asset risk when it is optimal to raise insured deposits or to keep them unchanged. With imperfections on the market for uninsured debt such a reduction is more likely.*

But when does a bank react with an undesired *increase* in asset risk? Firstly, insured deposits must be reduced. Secondly, this reduction must be so high that the indirect effect offsets the direct effect on $\partial G_0/\partial\alpha|_s$. From (40) we know

$$vs\left[p_1\frac{v}{(s-1)^2}(\bar{\alpha}x_1+(1-\bar{\alpha})y_1)-c\right] > (E_0^*+U_0)\left[a+\left(\frac{s-1+v}{s-1}\right)^2 b\right] \quad\Leftrightarrow\quad \frac{d\alpha}{ds}>0.$$

The term $[p_1[v/(s-1)^2](\bar{\alpha}x_1+(1-\bar{\alpha})y_1)-c]$ must be positive – i.e. the direct effect of s on $dG_0/dE_0|_{s,C_0}$ must be negative (see Table 2) – and sufficiently high. It has a positive sign if (for $U_0=-(s-1+v)E_0^*/(s-1)+C_0/(s-1)$) the marginal gain from increasing uninsured debt, $\partial G_0/\partial U_0$, is negative and the net present value of uninsured debt is concave, for

$$\begin{aligned}
&p_1\frac{v}{(s-1)^2}(\bar{\alpha}x_1+(1-\bar{\alpha})y_1)-\bar{c}\\
&=\frac{v}{(s-1)^2}[p_1(\bar{\alpha}x_1+(1-\bar{\alpha})y_1)-[1-g^{-1}(U_0)-U_0g^{-1\prime}(U_0)]]\\
&\quad+\frac{s-1+v}{(s-1)^2}(E_0^*+U_0)[2g^{-1\prime}(U_0)+U_0g^{-1\prime\prime}(U_0)]\\
&=-\frac{v}{(s-1)^2}\frac{\partial G_0}{\partial U_0}+\frac{s-1+v}{(s-1)^2}(E_0^*+U_0)[2g^{-1\prime}(U_0)+U_0g^{-1\prime\prime}(U_0).
\end{aligned}$$

When s is raised, the reduction of assets caused by a shift from uninsured debt to insured deposits (under compliance with the leverage restriction) decreases. For the same dE_0 uninsured debt has to be reduced less than before:

$$\frac{\partial(dU_0)}{\partial s}=\frac{v}{(s-1)^2}dE_0>0.$$

Hence, the positive effect of increasing insured deposits on the net present value of the deposit insurance guarantee is mitigated. – As the net present value of uninsured debt is a concave function, the marginal net present value rises when uninsured debt decreases while insured deposits are expanded. This positive impact is lessened by the enforcement of a more stringent capital requirement because the reduction of uninsured debt which is required to compensate a certain increase in insured deposits is less than before. This is another reason why the marginal gain $dG_0/dE_0|_s$ decreases with an increase in s. The bank tends to reduce insured deposits and to issue additional uninsured debt. Thus we recognize the indirect effect described above: The decline in insured deposits leads to an increase in assets.

Whether this increase is sufficiently high so that the indirect effect offsets the direct effect depends on the concrete data. The influence of the premium rate, v, and of the curvature of the profit function at E_0^* (here: $-[a + (s-1+v)^2 b/(s-1)^2] < 0$) is the same as in the scenario with a perfect market for uninsured debt, that is, a relatively high premium rate and a flat curve of the profit function strengthen the indirect effect.

4.2 A Bank with Variable Equity

1) No uninsured debt ($U_0 = 0$): To analyse the reaction of a bank which only issues insured deposits and can raise additional equity we use the first-order conditions (30) and (32). Additionally we assume that the second-order conditions for an maximum at $\bar{E}_0$ and $\bar{\alpha}$ are also fulfilled (see appendix A.5). Inserting and totally differentiating leads us to

$$\frac{d\alpha}{ds} = \frac{-p_1(x_1 - y_1)\left(\bar{E}_0\left[\frac{d^2K^E}{dE_0{}^2} + \bar{a}\right] - p_1 s(\bar{\alpha}x_1 + (1-\bar{\alpha})y_1)\right)}{\frac{d^2K^\alpha}{d\alpha^2}\left[\frac{d^2K^E}{dE_0{}^2} + \bar{a}\right] - p_1{}^2 s^2 (x_1 - y_1)^2} \qquad (41)$$

where

$$\bar{a} = \frac{p_2 + p_3}{p_1 + p_2 + p_3}[2f^{-1\prime}(\bar{E}_0) + \bar{E}_0 f^{-1\prime\prime}(\bar{E}_0)]. \qquad (42)$$

The denominator is positive by assumption (see appendix A.5) whereas the numerator is not obviously of a given sign. (Notice that $d^2K^E/dE_0{}^2 + \bar{a} > 0$ (see appendix A.5).)

A more stringent leverage constraint directly affects the marginal gain from increasing asset risk, $\partial G_0/\partial\alpha|_s$ (see also Table 3). Raising $s = A_0/E_0$ (without changing E_0) leads to an increase in asset returns in state 1 that are due to the insured depositors. This induces a rise of the marginal loss of an increase in asset risk which is borne by the deposit insurance.

The rise of additional payments from the deposit insurance in state 1 *increases* the bank's marginal gain from raising α.

However, insured deposits are not kept constant because at the same time the marginal gain from increasing E_0 (see (28)) decreases in s which gives the bank the incentive to reduce insured deposits (and raise equity). The decrease in E_0 leads to a *reduction* of $\partial G_0/\partial\alpha|_s$ because the deposit insurance has to take into account a smaller part of the additional losses in state 1 which are induced by an increase in asset risk.

Furlong and Keeley (1989) state that a bank which can issue new equity "would not hold more assets when required to reduce leverage". This result cannot be confirmed in our extended analytical framework. It is by no means clear whether a bank with variable equity sells assets and retires liabilities or invests in new assets (mainly) financed by equity. Therefore the question whether the indirect effect on $\partial G_0/\partial\alpha|_s$ (via dE_0) dominates the first, direct effect cannot be answered in general. As a rising share of the riskier security, α, has a positive impact on the marginal gain from raising insured deposits (see (28)), the change of E_0 is also not clear. As opposed to the scenario with fixed equity (and no uninsured debt) a bank with variable equity can *raise* insured deposits even though the capital constraint is tightened. To achieve the required reduction of leverage the increase in liabilities must be compensated by an issuance of new equity.

Result 11 *Due to the possibility of an increase in insured deposits and the interdependence of the optimal adjustments of insured deposits and asset risk, the reaction of a bank with variable equity and insured deposits to a tightened capital requirement is not as clear as for a bank with fixed equity. The reduced leverage induces an increase in asset risk if insured deposits do not shrink:*

$$\frac{dE_0}{ds} \geq 0 \qquad \Rightarrow \qquad \frac{d\alpha}{ds} > 0.$$ [13]

[13] See appendix A.4.

But even if the bank reduces insured deposits, the possibility of increased bank risk-taking cannot be rejected. If the value of the second derivative of the profit function with respect to E_0, $-[d^2K^E/dE_0^2 + \bar{a}]$, is low at $\bar{E}_0$, the bank does not alter the insured deposits very much to maintain condition (30). Thus the indirect effect on the marginal gain from increasing asset risk is quite small. Because of the dominating direct effect of increasing s, $\partial G_0/\partial\alpha|_s$ rises. It is optimal to increase the share of the riskier security in the asset portfolio.

2) Uninsured debt ($U_0 > 0$): In section 3 we showed that a regulated bank which can raise additional equity may issue a certain amount of uninsured debt if the market for uninsured debt is imperfect. How does the bank in this scenario respond to a rise of s? The bank's optimal amount of insured deposits and its optimal level of asset risk are determined by (30) and (32) together with (28) and (31). Additionally, the optimal level of uninsured debt is defined by (26).

The change of asset risk due to a rise of s can be written as:

$$\frac{d\alpha}{ds} = \frac{-p_1(x_1 - y_1)\left(\left(\bar{E}_0 + U_0^*\right)\left[\frac{d^2K^E}{dE_0{}^2} + \bar{a}\right]\bar{b} - p_1 s(\bar{\alpha}x_1 + (1-\bar{\alpha})y_1)\left[\frac{d^2K^E}{dE_0{}^2} + \bar{a} + \bar{b}\right]\right)}{\frac{d^2K^\alpha}{d\alpha^2}\left[\frac{d^2K^E}{dE_0{}^2} + \bar{a}\right]\bar{b} - p_1{}^2 s^2(x_1 - y_1)^2\left[\frac{d^2K^E}{dE_0{}^2} + \bar{a} + \bar{b}\right]} \tag{43}$$

where

$$\bar{a} = \frac{p_2 + p_3}{p_1 + p_2 + p_3}[2f^{-1\prime}(\bar{E}_0) + \bar{E}_0 f^{-1\prime\prime}(\bar{E}_0)],$$
$$\bar{b} = 2g^{-1\prime}(U_0^*) + U_0^* g^{-1\prime\prime}(U_0^*).$$

Once again the denominator is positive (see appendix A.5) whereas the sign of the numerator is not clear. Like in the scenario without uninsured debt, $d\alpha/ds$ is independent of the premium rate, v. This result differs from the reaction of a bank with constant equity because with variable equity there is no longer a fixed relationship between changes of insured deposits and uninsured debt which automatically causes adjustments of asset size.

Result 12 *The impact of the deposit insurance premium on the effect of a more stringent* *capital regulation on asset risk is different for banks with fixed and those with variable equity. For a bank which can raise new equity the effect of a tightened leverage constraint is the same for a positive flat-rate premium based on the amount of insured deposits and for a free deposit insurance.*

We can observe the same effects of a more stringent leverage constraint on asset risk and insured deposits as in the scenario without uninsured debt. However, unlike in the previous scenarios the requirement of reduced leverage also leads to an adjustment of subordinated uninsured debt which is not given by the change of insured deposits. For this reason there is another effect on the marginal gain of increasing asset risk caused by the adjustment of uninsured debt: The increase in s leads to a reduction of the marginal gain of subordinated uninsured debt (see also Table 3). The resulting decline in uninsured debt has a negative impact on the marginal gain of increasing α, i.e. $\partial G_0/\partial\alpha|_s$ *decreases.* This effect is similar to the indirect effect caused by a decrease in insured deposits. Both lead to a lower asset size so that the marginal losses of increased risk-taking which are born by the deposit insurance shrink. As the optimal adjustment of U_0 is influenced by the change of asset risk, it is not clear whether it is really optimal to retire uninsured debt.

Result 13 *A stronger capital requirement directly increases the marginal gain from raising asset risk. Besides, it directly leads to a decline in insured deposits as well as in subordinated uninsured debt which has a negative impact on the marginal gain from raising asset risk. As there exist interdependences between the change of asset risk and the adjustments of insured deposits and uninsured debt, respectively, the question if a bank with variable equity increases asset risk can only be answered if we know the exact bank structure and the demand equations.*

For a bank with variable equity the sign of $d\alpha/ds$ tends to be positive if the insured deposits and the share of security X before the imposition of a stronger capital constraint, $\bar{E}_0$ and $\bar{\alpha}$, are high and the lower bound s of the capital ratio is low (see (41) and (43)).

Result 14 *If the regulatory restrictions concerning asset risk and leverage risk are not too strict, it is quite possible that tightening the capital requirement induces a bank with variable equity to increase asset risk.*

In this section, the relevance of subordinated uninsured debt for the response of banks with variable equity to a higher capital requirement was shown. The conclusion of Furlong and Keeley (1989) that a bank which can issue new equity does not react with increased risk-taking cannot be confirmed. At least in some situations the possibility of rising asset risk may not be denied. Our results are summarized in Table 3.

5 Conclusion

In this paper we have analysed the impact of a higher capital requirement on bank risk-taking. Using a state-preference model we have identified at least three points which are worth emphasizing because they are crucial for the answer to this problem.

First and most importantly, in our model the value-maximizing bank can issue not only insured deposits but also subordinated uninsured debt. The issuance of uninsured debt is important for the reaction of the bank to a more stringent capital requirement. A bank which only holds a fixed amount of equity and insured deposits tends to decrease asset risk. However, this clear result cannot generally be obtained if a bank holds both, insured and subordinated uninsured liabilities, or if a bank can issue new equity. In these cases the bank increases its asset risk under certain conditions.

Second, the model includes assumptions about perfect as well as imperfect markets for liabilities. We showed under which conditions it is reasonable to deviate from the usual assumption of a bank with only equity and insured deposits. With imperfections on the market for insured deposits (and on the market for uninsured debt) it may be optimal for a regulated bank not only to hold insured deposits but also to issue subordinated uninsured debt.

Finally and perhaps most surprisingly, the question whether a subsidized deposit insurance charges a fixed-rate premium (depending on the amount of insured deposits) or does not charge a premium at all is sometimes essential for our results. E.g., a bank with fixed equity which issues insured deposits and subordinated uninsured debt decreases the risk of its assets if deposit insurance is free, but with a premium which rises proportionally to the insured deposits the reaction is not clear.

In this paper we have extended the framework of Furlong and Keeley (1989). Nevertheless, the model is still quite simplifying. Some other extensions or variations may be useful. For example, the assumption that insured deposits are senior to uninsured debt is dropped in a companion paper. If insured and uninsured liabilities are both of equal seniority, a bank's response to an enforcement of a more stringent capital requirement is ambiguous as well (Homölle, 2000).

The impact of a higher capital requirement on asset risk is described in this paper. However, the asset-substitution effect is only one effect of a more stringent bank capital regulation. If a bank increases asset risk, one can ask whether this effect exceeds the effect of decreased leverage so that insolvency risk is higher than before. To solve this problem within our analytical framework, it must be examined whether the number of insolvency states tends to vary because of the changing structure of bank capital and assets. But that is still another question left for further research.

A Appendix

A.1 Demand Equations

First Example

$$f(\mathbf{pe}) = \frac{1}{(1-\mathbf{pe})^n} - 1, \quad n \geq 1$$

This demand equation meets all the requirements of (16):

$$\begin{aligned} f(0) &= 0, \\ f'(\mathbf{pe}) &= \frac{n}{(1-\mathbf{pe})^{n+1}} > 0, \\ f''(\mathbf{pe}) &= \frac{n(n+1)}{(1-\mathbf{pe})^{n+2}} > 0, \\ \lim_{\mathbf{pe}\to 1} f(\mathbf{pe}) &= \infty. \end{aligned}$$

The inverse function $f^{-1}(E_0)$ and its first and second derivatives can be written as:

$$\begin{aligned} f^{-1}(E_0) &= 1 - \sqrt[n]{\frac{1}{E_0+1}}, \\ f^{-1\prime}(E_0) &= \frac{1}{n}\left(\frac{1}{E_0+1}\right)^{\frac{1+n}{n}}, \\ f^{-1\prime\prime}(E_0) &= -\frac{1+n}{n^2}\left(\frac{1}{E_0+1}\right)^{\frac{1+2n}{n}}. \end{aligned}$$

If this demand equation holds, the net present value of insured deposits is a strictly increasing and concave function of E_0, i.e. the first derivative of the net present value of insured deposits, $E_0(1 - f^{-1}(E_0))$, is positive for $E_0 \geq 0$ whereas the second derivative is negative:

$$\begin{aligned}
1 - f^{-1}(E_0) - E_0 f^{-1\prime}(E_0) &= \sqrt[n]{\frac{1}{E_0+1}} - E_0 \frac{1}{n}\left(\frac{1}{E_0+1}\right)^{\frac{1+n}{n}} \\
&= \left(\frac{1}{E_0+1}\right)^{\frac{1}{n}}\left(1 - \frac{1}{n}\frac{E_0}{E_0+1}\right) \\
&> 0, \\
-2f^{-1\prime}(E_0) - E_0 f^{-1\prime\prime}(E_0) &= -\frac{2}{n}\left(\frac{1}{E_0+1}\right)^{\frac{1+n}{n}} + E_0 \frac{1+n}{n^2}\left(\frac{1}{E_0+1}\right)^{\frac{1+2n}{n}} \\
&= -\frac{1}{n}\left(\frac{1}{E_0+1}\right)^{\frac{1+n}{n}}\left(2 - \frac{1+n}{n}\frac{E_0}{E_0+1}\right) \\
&< 0 \quad \text{q.e.d.}
\end{aligned}$$

Second Example

$$f(\mathbf{pe}) = |\ln(1-\mathbf{pe})|^n, \quad n \geq 1$$

This demand equation meets all the requirements of (16), too:

$$\begin{aligned}
f(0) &= 0, \\
f'(\mathbf{pe}) &= \frac{n}{1-\mathbf{pe}}|\ln(1-\mathbf{pe})|^{n-1} > 0, \\
f''(\mathbf{pe}) &= \frac{n}{(1-\mathbf{pe})^2}|\ln(1-\mathbf{pe})|^{n-2}((n-1) + |\ln(1-\mathbf{pe})|) > 0, \\
\lim_{\mathbf{pe}\to 1} f(\mathbf{pe}) &= \infty.
\end{aligned}$$

The inverse function $f^{-1}(E_0)$ and its first and second derivatives:

$$\begin{aligned}
f^{-1}(E_0) &= 1 - \mathrm{e}^{-\sqrt[n]{E_0}}, \\
f^{-1\prime}(E_0) &= \frac{1}{n}\mathrm{e}^{-\sqrt[n]{E_0}} E_0^{\frac{1-n}{n}},
\end{aligned}$$

$$f^{-1\prime\prime}(E_0) = \frac{1}{n^2} \mathrm{e}^{-\sqrt[n]{E_0}} E_0^{\frac{1-2n}{n}} \left(1 - n - E_0^{\frac{1}{n}}\right).$$

If this demand equation holds, the net present value of insured deposits reaches its absolute maximum at $E_0 = n^n$.

First-order condition:

$$\begin{aligned} 1 - f^{-1}(E_0) - E_0 f^{-1\prime}(E_0) &= 0 \\ \frac{1}{n} \mathrm{e}^{-\sqrt[n]{E_0}} \left(n - E_0^{\frac{1}{n}}\right) &= 0 \\ \Leftrightarrow \qquad E_0 &= n^n. \end{aligned}$$

Second-order condition:

$$\begin{aligned} -2f^{-1\prime}(E_0) - E_0 f^{-1\prime\prime}(E_0) &< 0 \qquad \text{where} \qquad E_0 = n^n \\ -\frac{1}{n^2} \mathrm{e}^{-\sqrt[n]{E_0}} E_0^{\frac{1-n}{n}} \left(n + 1 - E_0^{\frac{1}{n}}\right) &= -\frac{1}{n^{n+1} \mathrm{e}^n} \\ &< 0 \quad \text{q.e.d.} \end{aligned}$$

Note: These examples can easily be transferred to the market for subordinated uninsured debt by substituting **pu** for **pe** and U_0 for E_0.

A.2 The Net Present Value of the Deposit Insurance Subsidy

As the net present value of the deposit insurance guarantee is assumed to be positive, it is a strictly increasing function of E_0 (holding U_0 constant):

$$\begin{aligned} I_0^B - I_0 = p_1 \left[\frac{1}{p_1 + p_2 + p_3} E_0 f^{-1}(E_0) - A_0(\alpha x_1 + (1-\alpha) y_1) \right] - vE_0 > 0 \\ \Leftrightarrow \qquad p_1 \left[\frac{1}{p_1 + p_2 + p_3} f^{-1}(E_0) - \frac{A_0}{E_0}(\alpha x_1 + (1-\alpha) y_1) \right] - v > 0 \end{aligned}$$

$$\Rightarrow \qquad p_1\left[\frac{1}{p_1+p_2+p_3}f^{-1}(E_0)-s(\alpha x_1+(1-\alpha)y_1)\right]-v>0$$

$$\Leftrightarrow \qquad \left.\frac{d(I_0^B-I_0)}{dE_0}\right|_{s,U_0}>0$$

$$\text{where} \qquad s=\frac{A_0}{E_0+U_0}, \qquad E_0>0, \qquad U_0\geq 0.$$

A.3 Optimization Problems of Regulated Banks with Fixed Equity

Perfect Market for Uninsured Debt

$$\begin{aligned}\text{Maximize} \quad G_0^K &= p_1\left[\frac{1}{p_1+p_2+p_3}E_0 f^{-1}(E_0)-A_0(\alpha x_1+(1-\alpha)y_1)\right]-vE_0\\ &+E_0(1-f^{-1}(E_0))-K^\alpha\\ \text{subject to} \quad s&=\frac{A_0}{E_0+U_0}.\end{aligned}$$

First-order conditions for a maximum at E_0^* and $\bar{\alpha}$:

$$\begin{aligned}\frac{\partial G_0^K}{\partial E_0} &= p_1\left[\frac{1}{p_1+p_2+p_3}f^{-1}(E_0^*)+\frac{vs}{s-1}(\bar{\alpha}x_1+(1-\bar{\alpha})y_1)\right]-v\\ &+1-f^{-1}(E_0^*)-\frac{p_2+p_3}{p_1+p_2+p_3}E_0^* f^{-1\prime}(E_0^*)=0 \qquad \Leftrightarrow \qquad \left.\frac{dG_0}{dE_0}\right|_{s,C_0}=0,\\ \frac{\partial G_0^K}{\partial \alpha} &= -p_1 s(E_0^*+U_0)(x_1-y_1)-\frac{dK^\alpha}{d\alpha}=0 \qquad \Leftrightarrow \qquad \left.\frac{\partial G_0}{\partial\alpha}\right|_s=\frac{dK^\alpha}{d\alpha}.\end{aligned}$$

Second-order conditions:

$$\begin{aligned}0 &> -\frac{p_2+p_3}{p_1+p_2+p_3}[2f^{-1\prime}(E_0^*)+E_0 f^{-1\prime\prime}(E_0^*)],\\ 0 &< \frac{d^2K^\alpha}{d\alpha^2}\frac{p_2+p_3}{p_1+p_2+p_3}[2f^{-1\prime}(E_0^*)+E_0^* f^{-1\prime\prime}(E_0^*)]-{p_1}^2\left(\frac{vs}{s-1}\right)^2(x_1-y_1)^2.\end{aligned}$$

$$\text{Maximize} \quad G_0^K = p_1 \left[\frac{1}{p_1+p_2+p_3} E_0 f^{-1}(E_0) - A_0(\alpha x_1 + (1-\alpha) y_1) \right] - vE_0$$
$$+ E_0(1 - f^{-1}(E_0)) + U_0(1 - g^{-1}(U_0)) - K^\alpha$$
$$\text{subject to} \quad s = \frac{A_0}{E_0 + U_0}.$$

First-order conditions for a maximum at E_0^* and $\bar{\alpha}$:

$$\frac{\partial G_0^K}{\partial E_0} = p_1 \left[\frac{1}{p_1+p_2+p_3} f^{-1}(E_0^*) + \frac{vs}{s-1}(\bar{\alpha} x_1 + (1-\bar{\alpha}) y_1) \right] - v$$
$$+ 1 - f^{-1}(E_0^*) - \frac{p_2+p_3}{p_1+p_2+p_3} E_0^* f^{-1\prime}(E_0^*)$$
$$- \frac{s-1+v}{s-1}[1 - g^{-1}(U_0) - U_0 g^{-1\prime}(U_0)] = 0 \quad \Leftrightarrow \quad \left.\frac{dG_0}{dE_0}\right|_{s,C_0} = 0,$$
$$\frac{\partial G_0^K}{\partial \alpha} = -p_1 s(E_0^* + U_0)(x_1 - y_1) - \frac{dK^\alpha}{d\alpha} = 0 \quad \Leftrightarrow \quad \left.\frac{\partial G_0}{\partial \alpha}\right|_s = \frac{dK^\alpha}{d\alpha}.$$

Second-order conditions:

$$0 > -\left[\frac{p_2+p_3}{p_1+p_2+p_3}[2f^{-1\prime}(E_0^*) + E_0 f^{-1\prime\prime}(E_0^*)] + \left(\frac{s-1+v}{s-1}\right)^2 [2g^{-1\prime}(U_0) + U_0 g^{-1\prime\prime}(U_0)] \right],$$
$$0 < \frac{d^2K^\alpha}{d\alpha^2} \left[\frac{p_2+p_3}{p_1+p_2+p_3}[2f^{-1\prime}(E_0^*) + E_0^* f^{-1\prime\prime}(E_0^*)] \right.$$
$$\left. + \left(\frac{s-1+v}{s-1}\right)^2 [2g^{-1\prime}(U_0) + U_0 g^{-1\prime\prime}(U_0)] \right] - {p_1}^2 \left(\frac{vs}{s-1}\right)^2 (x_1 - y_1)^2.$$

A.4 Relation between Variations of Insured Deposits and Asset Risk

Optimal variations of the share of the riskier asset, α, and of insured deposits, E_0, when s is raised:

$$\frac{d\alpha}{ds} = \frac{\frac{\partial^2 G_0^K}{\partial E_0\,\partial s}\frac{\partial^2 G_0^K}{\partial E_0\,\partial \alpha} - \frac{\partial^2 G_0^K}{\partial \alpha\,\partial s}\frac{\partial^2 G_0^K}{\partial {E_0}^2}}{\frac{\partial^2 G_0^K}{\partial \alpha^2}\frac{\partial^2 G_0^K}{\partial {E_0}^2} - \left(\frac{\partial^2 G_0^K}{\partial E_0\,\partial \alpha}\right)^2},$$

$$\frac{dE_0}{ds} = \frac{\frac{\partial^2 G_0^K}{\partial\alpha\,\partial s}\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha} - \frac{\partial^2 G_0^K}{\partial E_0\,\partial s}\frac{\partial^2 G_0^K}{\partial\alpha^2}}{\frac{\partial^2 G_0^K}{\partial\alpha^2}\frac{\partial^2 G_0^K}{\partial {E_0}^2} - \left(\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha}\right)^2}$$

$$\text{where} \qquad \frac{\partial^2 G_0^K}{\partial\alpha^2} < 0,$$

$$\frac{\partial^2 G_0^K}{\partial {E_0}^2} < 0,$$

$$\frac{\partial^2 G_0^K}{\partial\alpha^2}\frac{\partial^2 G_0^K}{\partial {E_0}^2} - \left(\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha}\right)^2 > 0 \qquad \Leftrightarrow \qquad \frac{\partial^2 G_0^K}{\partial\alpha^2} < \frac{\left(\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha}\right)^2}{\frac{\partial^2 G_0^K}{\partial {E_0}^2}}.$$

A Bank with Fixed Equity

If the tightening of the capital requirement induces the bank not to reduce insured deposits, it is at the same time optimal to *reduce* asset risk:

$$\frac{dE_0}{ds} \geq 0 \qquad \Rightarrow \qquad \frac{d\alpha}{ds} < 0.$$

Proof:

$$\frac{dE_0}{ds} \geq 0$$

$$\Leftrightarrow \qquad \frac{\partial^2 G_0^K}{\partial\alpha\,\partial s}\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha} - \frac{\partial^2 G_0^K}{\partial E_0\,\partial s}\frac{\partial^2 G_0^K}{\partial\alpha^2} \geq 0$$

$$\Rightarrow \qquad \frac{\partial^2 G_0^K}{\partial\alpha\,\partial s}\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha} - \frac{\partial^2 G_0^K}{\partial E_0\,\partial s}\frac{\left(\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha}\right)^2}{\frac{\partial^2 G_0^K}{\partial {E_0}^2}} \geq 0$$

$$\Leftrightarrow \qquad \frac{\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha}}{\frac{\partial^2 G_0^K}{\partial {E_0}^2}}\left[\frac{\partial^2 G_0^K}{\partial\alpha\,\partial s}\frac{\partial^2 G_0^K}{\partial {E_0}^2} - \frac{\partial^2 G_0^K}{\partial E_0\,\partial s}\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha}\right] \geq 0$$

$$\Rightarrow \qquad \frac{\partial^2 G_0^K}{\partial\alpha\,\partial s}\frac{\partial^2 G_0^K}{\partial {E_0}^2} - \frac{\partial^2 G_0^K}{\partial E_0\,\partial s}\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha} > 0 \qquad \text{where} \qquad \frac{\partial^2 G_0^K}{\partial\alpha\,\partial s} < 0, \frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha} \leq 0$$

$$\Leftrightarrow \qquad \frac{\partial^2 G_0^K}{\partial E_0\,\partial s}\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha} - \frac{\partial^2 G_0^K}{\partial\alpha\,\partial s}\frac{\partial^2 G_0^K}{\partial {E_0}^2} < 0$$

$$\Leftrightarrow \qquad \frac{d\alpha}{ds} < 0 \quad \text{q.e.d.}$$

If the tightening of the capital requirement induces the bank not to reduce insured deposits, it is at the same time optimal to *raise* asset risk:

$$\frac{dE_0}{ds} \geq 0 \qquad \Rightarrow \qquad \frac{d\alpha}{ds} > 0.$$

Proof:

$$\begin{aligned}
& & \frac{dE_0}{ds} \geq 0 \\
\Leftrightarrow & & \frac{\partial^2 G_0^K}{\partial\alpha\,\partial s}\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha} - \frac{\partial^2 G_0^K}{\partial E_0\,\partial s}\frac{\partial^2 G_0^K}{\partial\alpha^2} \geq 0 \\
\Rightarrow & & \frac{\partial^2 G_0^K}{\partial\alpha\,\partial s}\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha} - \frac{\partial^2 G_0^K}{\partial E_0\,\partial s}\frac{\left(\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha}\right)^2}{\frac{\partial^2 G_0^K}{\partial {E_0}^2}} > 0 \qquad \text{where} \qquad \frac{\partial^2 G_0^K}{\partial E_0\,\partial s} < 0 \\
\Leftrightarrow & & \frac{\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha}}{\frac{\partial^2 G_0^K}{\partial {E_0}^2}}\left[\frac{\partial^2 G_0^K}{\partial\alpha\,\partial s}\frac{\partial^2 G_0^K}{\partial {E_0}^2} - \frac{\partial^2 G_0^K}{\partial E_0\,\partial s}\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha}\right] > 0 \\
\Rightarrow & & \frac{\partial^2 G_0^K}{\partial E_0\,\partial s}\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha} - \frac{\partial^2 G_0^K}{\partial\alpha\,\partial s}\frac{\partial^2 G_0^K}{\partial {E_0}^2} > 0 \qquad \text{where} \qquad \frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha} > 0 \\
\Leftrightarrow & & \frac{d\alpha}{ds} > 0 \quad \text{q.e.d.}
\end{aligned}$$

A.5 Optimization Problems of Regulated Banks with Variable Equity

Perfect Market for Uninsured Debt

$$\begin{aligned}
\text{Maximize} \quad G_0^K &= p_1\left[\frac{1}{p_1+p_2+p_3}E_0 f^{-1}(E_0) - A_0(\alpha x_1 + (1-\alpha)y_1)\right] - vE_0 \\
&\quad + E_0(1 - f^{-1}(E_0)) - K^\alpha - K^E \\
\text{subject to} \quad s &= \frac{A_0}{E_0 + U_0}.
\end{aligned}$$

Notice: $U_0^* = 0$ because

$$\frac{\partial G_0^K}{\partial U_0} = -p_1 s(\alpha x_1 + (1-\alpha)y_1) < 0.$$

First-order conditions for a maximum at $\bar{E}_0$ and $\bar{\alpha}$:

$$\begin{aligned} \frac{\partial G_0^K}{\partial E_0} &= p_1\left[\frac{1}{p_1+p_2+p_3}f^{-1}(\bar{E}_0) - s(\bar{\alpha}x_1 + (1-\bar{\alpha})y_1)\right] - v \\ &\quad + 1 - f^{-1}(\bar{E}_0) - \frac{p_2+p_3}{p_1+p_2+p_3}\bar{E}_0 f^{-1\prime}(\bar{E}_0) - \frac{dK^E}{dE_0} = 0 \quad \Leftrightarrow \quad \left.\frac{dG_0}{dE_0}\right|_{s,U_0} = \frac{dK^E}{dE_0}, \\ \frac{\partial G_0^K}{\partial \alpha} &= -p_1 s\bar{E}_0(x_1-y_1) - \frac{dK^\alpha}{d\alpha} = 0 \quad \Leftrightarrow \quad \left.\frac{\partial G_0}{\partial \alpha}\right|_{s} = \frac{dK^\alpha}{d\alpha}. \end{aligned}$$

Second-order conditions:

$$\begin{aligned} 0 &> -\left[\frac{d^2K^E}{dE_0{}^2} + \frac{p_2+p_3}{p_1+p_2+p_3}[2f^{-1\prime}(\bar{E}_0) + \bar{E}_0 f^{-1\prime\prime}(\bar{E}_0)]\right], \\ 0 &< \frac{d^2K^\alpha}{d\alpha^2}\left[\frac{d^2K^E}{dE_0{}^2} + \frac{p_2+p_3}{p_1+p_2+p_3}[2f^{-1\prime}(\bar{E}_0) + \bar{E}_0 f^{-1\prime\prime}(\bar{E}_0)]\right] - p_1{}^2 s^2(x_1-y_1)^2. \end{aligned}$$

Imperfect Market for Uninsured Debt

$$\begin{aligned} \text{Maximize} \quad G_0^K &= p_1\left[\frac{1}{p_1+p_2+p_3}E_0 f^{-1}(E_0) - A_0(\alpha x_1 + (1-\alpha)y_1)\right] - vE_0 \\ &\quad + E_0(1-f^{-1}(E_0)) + U_0(1-g^{-1}(U_0)) - K^\alpha - K^E \\ \text{subject to} \quad s &= \frac{A_0}{E_0+U_0}. \end{aligned}$$

First-order conditions for a maximum at $\bar{E}_0$, $\bar{\alpha}$, and U_0^*[14]:

$$\begin{aligned} \frac{\partial G_0^K}{\partial E_0} &= p_1\left[\frac{1}{p_1+p_2+p_3}f^{-1}(\bar{E}_0) - s(\bar{\alpha}x_1 + (1-\bar{\alpha})y_1)\right] - v \\ &\quad + 1 - f^{-1}(\bar{E}_0) - \frac{p_2+p_3}{p_1+p_2+p_3}\bar{E}_0 f^{-1\prime}(\bar{E}_0) - \frac{dK^E}{dE_0} = 0 \quad \Leftrightarrow \quad \left.\frac{dG_0}{dE_0}\right|_{s,U_0} = \frac{dK^E}{dE_0}, \end{aligned}$$

14 Depending on the concrete data $U_0 = 0$ may also be optimal because of $\partial G_0^K/\partial U_0 < 0$. In this case the same conditions of a maximum hold as in the perfect market scenario (see above).

$$\frac{\partial G_0^K}{\partial \alpha} = -p_1 s(\bar{E}_0 + U_0^*)(x_1 - y_1) - \frac{dK^\alpha}{d\alpha} = 0 \qquad \Leftrightarrow \qquad \left.\frac{\partial G_0}{\partial \alpha}\right|_s = \frac{dK^\alpha}{d\alpha},$$

$$\frac{\partial G_0^K}{\partial U_0} = -p_1 s(\bar{\alpha} x_1 + (1-\bar{\alpha}) y_1) + 1 - g^{-1}(U_0^*) - U_0^* g^{-1\prime}(U_0^*) = 0 \quad \Leftrightarrow \quad \left.\frac{dG_0}{dU_0}\right|_{s,E_0} = 0.$$

Second-order conditions:

$$0 > -\left[\frac{d^2 K^E}{d{E_0}^2} + \frac{p_2 + p_3}{p_1 + p_2 + p_3}[2f^{-1\prime}(\bar{E}_0) + \bar{E}_0 f^{-1\prime\prime}(\bar{E}_0)]\right],$$

$$0 < \frac{d^2 K^\alpha}{d\alpha^2}\left[\frac{d^2 K^E}{d{E_0}^2} + \frac{p_2 + p_3}{p_1 + p_2 + p_3}[2f^{-1\prime}(\bar{E}_0) + \bar{E}_0 f^{-1\prime\prime}(\bar{E}_0)]\right] - p_1^2 s^2 (x_1 - y_1)^2,$$

$$0 > -\frac{d^2 K^\alpha}{d\alpha^2}\left[\frac{d^2 K^E}{d{E_0}^2} + \frac{p_2 + p_3}{p_1 + p_2 + p_3}[2f^{-1\prime}(\bar{E}_0) + \bar{E}_0 f^{-1\prime\prime}(\bar{E}_0)]\right][2g^{-1\prime}(U_0^*) + U_0^* g^{-1\prime\prime}(U_0^*)]$$
$$+ p_1^2 s^2 (x_1 - y_1)^2 \left[\frac{d^2 K^E}{d{E_0}^2} + \frac{p_2 + p_3}{p_1 + p_2 + p_3}[2f^{-1\prime}(\bar{E}_0) + \bar{E}_0 f^{-1\prime\prime}(\bar{E}_0)]\right.$$
$$\left. + 2g^{-1\prime}(U_0^*) + U_0^* g^{-1\prime\prime}(U_0^*)\right].$$

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Table 1: Optimal Behaviour of a Regulated Bank

	fixed equity ($dC_0 = 0$)	variable equity ($dC_0 > 0$)	
		$U_0^B = U_0$	$U_0^B < U_0$
insured deposits	$E_0 = \frac{1}{s-1+v}C_0$ $\vee \quad E_0 = E_0^* < \frac{1}{s-1+v}C_0$	$E_0 = \bar{E}_0$	
uninsured debt	$U_0 = 0 \quad \vee \quad U_0 > 0$	$U_0 = 0$	$U_0 = 0$ $\vee \quad U_0 > 0$
asset risk	$\alpha = \bar{\alpha}$		

Table 2: Bank with Fixed Equity: Effects of a Higher Capital Requirement

<table>
<tr><td colspan="2" rowspan="2">effects on</td><td rowspan="2">1) $U_0 = 0$</td><td colspan="2">2) $U_0 > 0$</td></tr>
<tr><td>2a) $U_0^B = U_0$</td><td>2b) $U_0^B > U_0$</td></tr>
<tr><td rowspan="2">$\left.\frac{\partial G_0}{\partial \alpha}\right|_s$</td><td>direct</td><td>$-p_1 E_0(x_1 - y_1) > 0$</td><td colspan="2">$p_1 \frac{1}{s-1}(E_0^* + U_0)(x_1 - y_1) < 0$</td></tr>
<tr><td>indirect
(via dE_0)</td><td>$-p_1 s(x_1 - y_1) > 0$</td><td colspan="2">$p_1 \frac{vs}{s-1}(x_1 - y_1) \leq 0$</td></tr>
<tr><td rowspan="2">$\left.\frac{dG_0}{dE_0}\right|_{s,C_0}$</td><td>direct</td><td>—</td><td>$-p_1 \frac{v}{(s-1)^2}(\bar{\alpha} x_1$
$+(1-\bar{\alpha})y_1) \leq 0$</td><td>$-p_1 \frac{v}{(s-1)^2}(\bar{\alpha} x_1$
$+(1-\bar{\alpha})y_1) + c \gtreqless 0$</td></tr>
<tr><td>indirect
(via $d\alpha$)</td><td>—</td><td colspan="2">$p_1 \frac{vs}{s-1}(x_1 - y_1) \leq 0$</td></tr>
<tr><td colspan="2">total effect: $\frac{d\alpha}{ds}$</td><td>< 0</td><td colspan="2">< 0 if $v = 0$, $\gtreqless 0$ if $v > 0$</td></tr>
</table>

Table 3: Bank with Variable Equity: Effects of a Higher Capital Requirement

effects on		*1)* $U_0 = 0$	*2)* $U_0 > 0$
$\left.\frac{\partial G_0}{\partial \alpha}\right\|_s$	direct	$-p_1 \bar{E}_0(x_1 - y_1) > 0$	$-p_1(\bar{E}_0 + U_0^*)(x_1 - y_1) > 0$
	indirect (via dE_0)	$-p_1 s(x_1 - y_1) > 0$	$-p_1 s(x_1 - y_1) > 0$
	indirect (via dU_0)	—	$-p_1 s(x_1 - y_1) > 0$
$\left.\frac{dG_0}{dE_0}\right\|_{s,U_0}$	direct	$-p_1(\bar{\alpha} x_1 + (1 - \bar{\alpha}) y_1) < 0$	$-p_1(\bar{\alpha} x_1 + (1 - \bar{\alpha}) y_1) < 0$
	indirect (via $d\alpha$)	$-p_1 s(x_1 - y_1) > 0$	$-p_1 s(x_1 - y_1) > 0$
$\left.\frac{dG_0}{dU_0}\right\|_{s,E_0}$	direct	—	$-p_1(\bar{\alpha} x_1 + (1 - \bar{\alpha}) y_1) < 0$
	indirect (via $d\alpha$)	—	$-p_1 s(x_1 - y_1) > 0$
total effect: $\frac{d\alpha}{ds}$		$\gtreqless 0$	$\gtreqless 0$

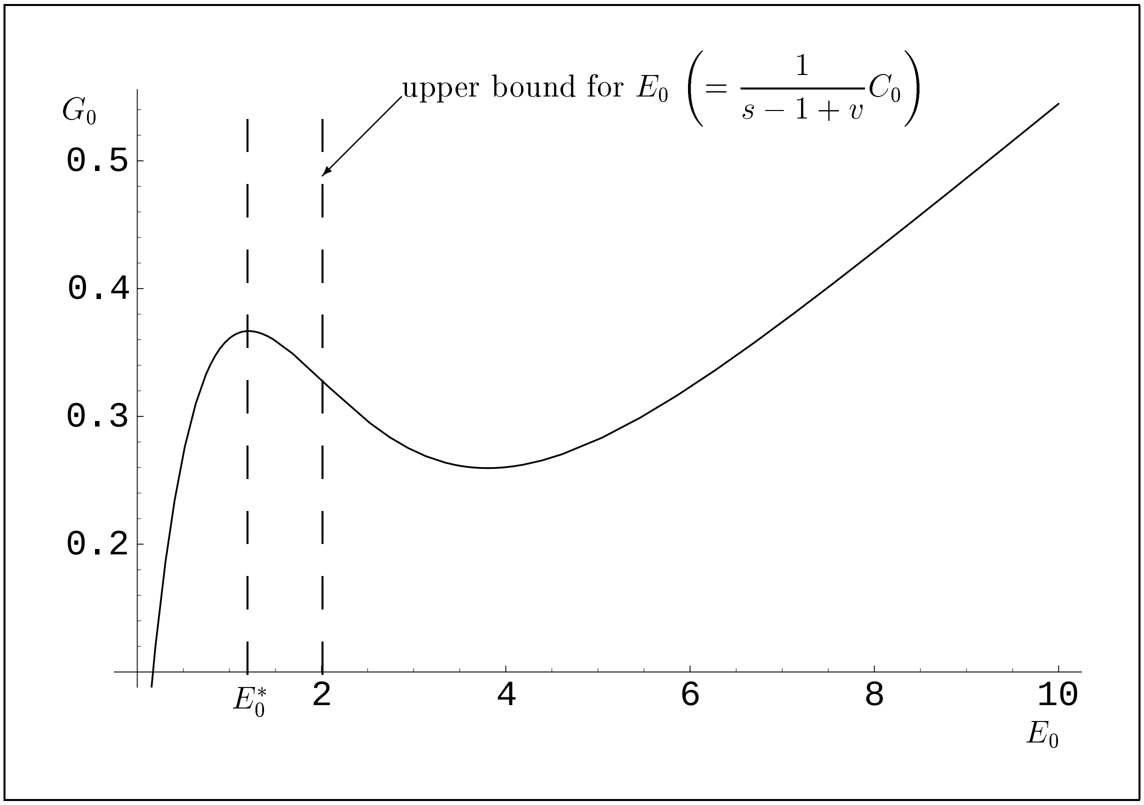

Figure 1: Profit Function G_0 of a Bank with Fixed Equity ($C_0 = 0.2$, $p_1 = 0.05$, $p_2 = 0.1$, $p_3 = 0.7$, $(\alpha x_1 + (1-\alpha)y_1) = 0.4$, $s = 1.1$, $v = 0$, $f(\mathbf{pe}) = \ln(1 - \mathbf{pe})$, $\mathbf{pu} = 1$)

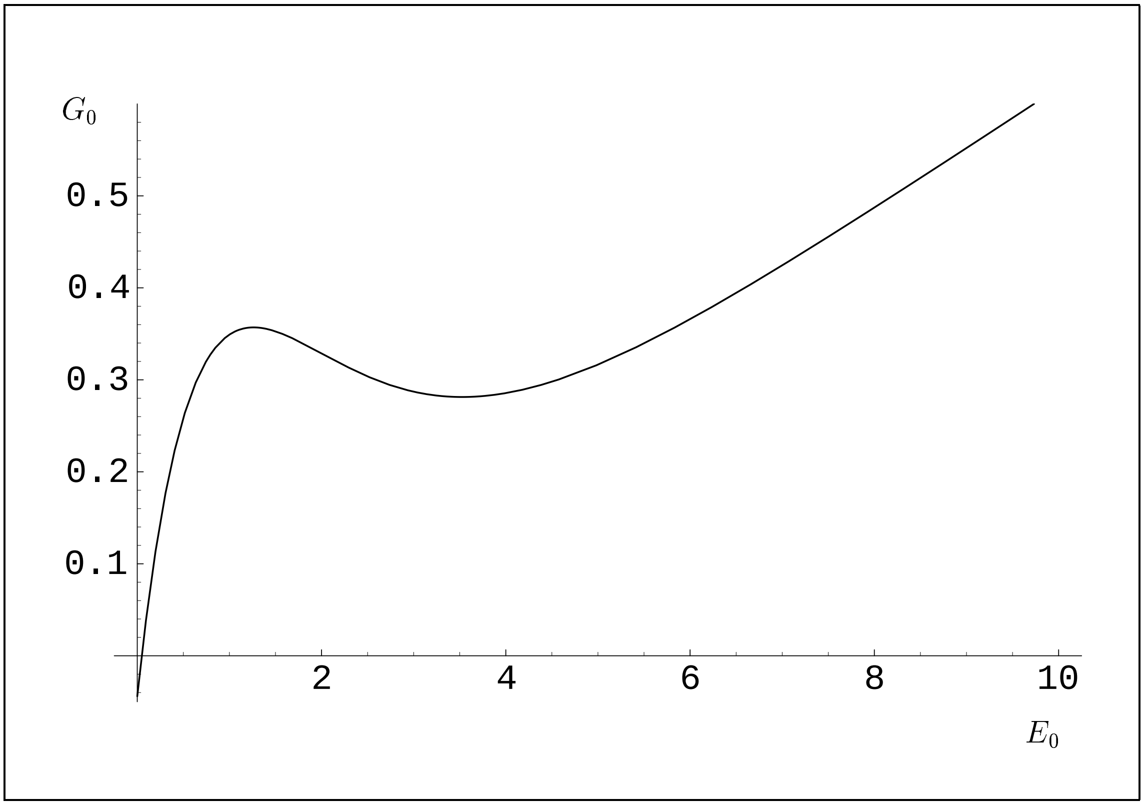

Figure 2: Profit Function G_0 of a Bank with Variable Equity ($U_0 = 1$, $p_1 = 0.1$, $p_2 = 0.1$, $p_3 = 0.7$, $(\alpha x_1 + (1-\alpha)y_1) = 0.4$, $s = 1.1$, $v = 0.001$, $f(\mathbf{pe}) = \ln(1-\mathbf{pe})$, $\mathbf{pu} = 1$)