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Abstract

Bank capital regulation is intended to promote bank safety and limit the risk of bank insolvency. Whether more stringent capital requirements lead to reduced or to (undesired) increased risk-taking of banks has been discussed controversially in the academic literature. It has so far been common to assume in corresponding models, contrary to what is being observed in reality, that all liabilities are guaranteed by a subsidized deposit insurance. In our paper we will allow for uninsured debt, too. Two main new observations emerge. First, assuming a free deposit insurance is not as innocuous as was suggested by previous papers. In particular, only a positive insurance premium induces a bank to refrain from issuing uninsured debt. Second, the results concerning the reaction of a value-maximizing bank to a higher capital requirement may change substantially if the bank additionally issues uninsured debt. We identify situations in which the bank responds to the new capital regulation by increasing asset risk.

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Capital Regulation and Bank Risk-taking: The Role of Uninsured Debt

1 Introduction

Regulators attempt to limit the default risk of banks by capital requirements in order to prevent bank insolvency. In the eighties the impact of a rise of capital requirements on bank behaviour was discussed controversially in the United States because of numerous modifications of the US capital regulation. Portfolio models as well as state-preference models and models based on option pricing theory were used to analyse the question whether a more stringent capital requirement reducing the possible leverage induces the intended reduction in banks' risk-taking. The answers vary, not so surprisingly, due to the different analytic frameworks.

For example, *Kahane* (1977), *Koehn/Santomero* (1980), and *Kim/Santomero* (1988) apply portfolio theory to show that higher capital requirements can lead a utility-maximizing bank to increase the risk of its portfolio. The increase in asset risk may offset the desired effect of reduced leverage. Thus the regulatory authority cannot be sure to achieve a reduction of insolvency risk. *Furlong/Keeley* (1989) point out the relevance of the deposit insurance guarantee. In models based on state-preference theory and option pricing theory, respectively, they prove that it is not optimal for value-maximizing banks to increase asset risk if the capital requirement is tightened.

The theoretical discussion is still going on[1] accompanied by numerous empirical investigations.[2] During the last years agency problems were additionally taken into account when analysing the effects of more stringent capital regulation. Considering asymmetric information between bank managers and bank owners *Besanko/Kanatas* (1996) identify a third effect. Besides the reduction of leverage (buffer effect) and the asset-substitution effect mentioned above, they describe a so-called dilution or outsider equity effect[3]: The bank raises equity from bank outsiders to comply with the new capital requirement. Thus the insiders' (managers') share of bank profits

[1] Cf. *Lam/Chen* (1985); *Flannery* (1989); *Zarruk* (1989); *Gennotte/Pyle* (1991); *Kendall* (1991); *Rochet* (1992); *Gjerde/Semmen* (1995); *Blum* (1999).

[2] Cf. for example the survey article of *Wall/Peterson* (1996) and the literature quoted therein; *Furlong* (1988); *Karels et al.* (1989); *Avery/Berger* (1991); *Bock* (1995); *Grenadier/Hall* (1995); *Ediz et al.* (1998).

[3] Cf. *Greenbaum/Thakor* (1995), pp. 524-525.

Table 1: Liabilities to banks and non-banks on December 31, 1999 (million Euro)

	liabilities to		
	banks	non-banks	total assets
Deutsche Bank	174,655	291,042	839,865
Dresdner Bank	71,876	142,160	396,846
Commerzbank	72,661	91,042	372,040
Hypovereinsbank	85,078	143,052	503,255

decreases which in turn reduces their incentives to expend efforts in bank management. The consequent insufficient risk management can lead to increased bank risk-taking.[4]

In the EU, capital regulation was altered several times during recent years. Hence, the question about the impact of more stringent capital requirements on banks' risk choice seems to be even more relevant than before. We are going to analyse this problem once again, for "the theoretical issue of how higher required equity ratios affect bank risk-taking is [still] unresolved."[5] In our analysis we concentrate on the asset-substitution effect: Does the enforcement of a more stringent capital requirement lead a bank to increase asset risk? In our opinion this effect has not been examined sufficiently yet: Looking at the literature mentioned above, it can be stated that some aspects, which are interesting not only from the theoretical point of view but also from an empirical perspective, have not been taken into account yet. In particular the capital structure of a bank is often assumed to be quite simple, consisting only of equity and insured deposits. It is not considered that a bank may issue both, deposits whose repayments are guaranteed by a deposit insurance and uninsured debt.

In the EU, different deposit insurance systems were discussed in great depth during recent years. From the new European directive concerning deposit insurance we know that not all liabilities are allowed to be insured. For example, liabilities to banks are excluded. In *Table 1* we collect some information on liabilities to banks and non-banks of some large German banks at the end of 1999. The uninsured liabilities to banks are lower than the (insured) deposits of non-banks but are still of substantial amount.

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[4] See also *Weiland* (1999) who analyses various impacts of capital regulation on bank managers' decisions.

[5] *Berger et al.* (1995), p. 409.

Whether uninsured creditors receive the promised returns depends on the future bank solvency or insolvency. Hence, they care about banks' risk-taking and require risk premia. We use a state-preference model to distinguish between a situation in which the bank is solvent and a situation with an insolvent bank.[6] Explaining bank behaviour within a state-preference model can be critized because in a world with complete and perfect capital markets banks do not have any special function.[7] However, we are not going to answer the question: "Why do banks exist?". Instead we assume that the bank (as well as the deposit insurance and the regulatory authority) already exists. Being aware of the possible objection just mentioned we use state-preference theory because it allows us, unlike models based on standard portfolio theory[8], to model a return on uninsured debt which is not exogenous but depends on bank behaviour. Besides, we do not have to specify the utility function of a bank but can consider a value-maximizing bank.

In this paper it will become clear that it may be reasonable for a bank to hold uninsured debt depending on the deposit insurance premium. We show that the assumption of a free deposit insurance is not as innocuous as sometimes stated[9]. Moreover, we prove that the consideration of uninsured debt changes the answer to the question how the incentive for a bank to expand the risk of its asset portfolio varies with a rise in capital requirements.

In section 2, our model is introduced. In section 3, we analyse the decisions of a value-maximizing bank in a world without regulation. We show under which circumstances the bank does not only hold insured but also uninsured liabilities. In section 4, the optimal behaviour of a bank which has to comply with a leverage constraint is examined. We describe the effect of this capital rule and suggest some supplementary regulatory restrictions. Finally, in section 5, we prove that a bank which only issues insured deposits while having a fixed amount of equity will decrease asset risk as a reaction to a more stringent capital requirement. However, a bank which in addition holds uninsured debt or which can raise equity may increase asset risk under certain conditions.

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6 For the state-preference theory see for example *Arrow* (1964); *Hirshleifer* (1965); *Hirshleifer* (1966); *Myers* (1968).

7 Cf. *Merton* (1989), p. 229; *Hellwig* (1997), pp. 213 f. To justify the existence of financial intermediaries one can assume that there are some transaction costs for at least some of the investors. A model with such imperfections on the market for insured deposits or uninsured debt is described in more detail by *Homölle* (1999).

8 Cf. *Kahane* (1977); *Koehn/Santomero* (1980); *Kim/Santomero* (1988).

9 Cf. *Furlong/Keeley* (1987), p. 23; *Furlong/Keeley* (1989), p. 885.

2 The Model

2.1 Payments at Time 0

Our analytic framework is a state-preference model with two dates, time 0 and time 1, and two possible states of nature at time 1. The time 0 price of a dollar payout at time 1 in state i, p_i, $i = 1, 2$, is taken as given where $\sum_i p_i < 1$. As a starting point of our analysis, we use the approach of *Furlong/Keeley* (1989) which is altered in some important aspects.[10]

At time 0 the bank we are analysing decides on the structure of its capital and its assets. The bank raises some amount of equity, C_0, which by assumption is not allowed to fall below a certain level.[11] The bank owners have limited liability. The bank can issue insured deposits, E_0, as well as uninsured debt, U_0. The capital raised is invested in assets, A_0. Besides, the bank has to pay a premium, I_0, for the deposit insurance guarantee. For the payments of the bank at time 0 the following holds:

$$A_0 + I_0 = C_0 + E_0 + U_0. \tag{1}$$

The assets consist of the securities X and Y. Let α, $0 \leq \alpha \leq 1$, be the share of X in the asset portfolio and $1 - \alpha$ the share of Y.[12] At time 1 the bank gets a return of x_i (y_i) per invested dollar in security X (Y) if state i occurs. As the capital market is assumed to be complete and perfect, the net present values of the assets equal zero. Security X is riskier than security Y because the spread of returns on X is higher than that on Y:[13]

$$p_1 x_1 + p_2 x_2 = p_1 y_1 + p_2 y_2 = 1, \tag{2}$$

$$x_1 < y_1 < y_2 < x_2. \tag{3}$$

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10 Partial-equilibrium state-preference models are also used, e.g., by *Kareken/Wallace* (1978) and *Dothan/Williams* (1980).

11 German banks, who issue deposits, have to raise at least 5 million ECU as initial core capital. See § 33 I 1d) KWG.

12 Cf. *Furlong/Keeley* (1989), pp. 884-885. The bank cannot sell short X and Y. There exists a conflict between this implicit assumption of *Furlong* and *Keeley* and the state-preference theory because a capital market with these two securities and short-selling restrictions is not complete. Thus one should at least implicitly suppose that there exists a third security which may be sold short. For a complete capital market with short-selling restrictions cf. *Raab/Schwager* (1993).

13 Cf. the comment by *Sharpe* (1978), p. 710, on the definition of "risk" in a state-preference model: "Risk is generally considered to have increased when a set of returns becomes more 'spread' out."

From (2) and (3) we know:

$$x_1 < y_1 < \frac{1}{p_1 + p_2} < y_2 < x_2.$$

In state 1 (2) the returns on X and Y are lower (higher) than the return on a riskless asset, $1/(p_1 + p_2)$.

By assumption the bank pays a fixed-rate premium depending on the amount of insured deposits.[14] To show some important implications of the (non-)existence of a premium, we alternatively assume – as *Furlong/Keeley* (1989) do – a free deposit insurance. That is, the premium rate, v, is set equal to zero:

$$I_0 = vE_0, \qquad 1 > v \geq 0. \tag{4}$$

From (1) and (4) we know that the capital invested in assets, A_0, consists of equity, insured deposits, diminished by the insurance premium, and uninsured debt:

$$A_0 = C_0 + E_0(1 - v) + U_0.$$

2.2 Insolvency and Solvency at Time 1

In state 1 at time 1 the bank is insolvent by assumption. Since asset returns fall below the promised returns on insured deposits, E_0e, and on uninsured debt, U_0u,

$$A_0(\alpha x_1 + (1 - \alpha)y_1) < E_0e + U_0u, \tag{5}$$

the bank cannot meet its obligations. (5) even holds if the bank only invests in security Y.

Insured deposits and uninsured debt are both of equal seniority.[15] Asset returns of the insolvent bank are distributed among creditors proportional to their share of total liabilities at time 0, $E_0 + U_0$. The share of insured depositors, k, and uninsured

[14] E.g., the deposit insurance of German private banks (Einlagensicherungsfonds des Bundesverbandes Deutscher Banken) required a uniform, yearly premium of 0,03 % of liabilities to non-banks until 1998. In that year some kind of risk-adjusted premium was adopted by introducing three risk classes for banks. So the premium rates differ according to the risk class, but they still refer to the amount of (insured) liabilities to non-banks. See also *Holtorf/Rudolf* (1999).

[15] A scenario with subordinated uninsured debt also leads to some interesting results. Cf. *Homölle* (2000).

creditors, $1-k$, respectively, can be written as:[16]

$$k = \frac{E_0}{E_0 + U_0}, \quad 1 - k = \frac{U_0}{E_0 + U_0}. \tag{6}$$

Notice that k is not a constant but is determined by the chosen amount of insured and uninsured liabilities. The insolvent bank pays $kA_0(\alpha x_1 + (1-\alpha)y_1)$ to insured depositors, whereas uninsured creditors receive $(1-k)A_0(\alpha x_1 + (1-\alpha)y_1)$ in state 1. In addition to (5) we assume that in state 1 insured depositors receive from the bank less than the promised return:[17]

$$kA_0(\alpha x_1 + (1-\alpha)y_1) < E_0 e \quad \Leftrightarrow \quad \frac{A_0}{E_0 + U_0}(\alpha x_1 + (1-\alpha)y_1) < e.^{18} \tag{7}$$

The deposit insurance guarantees that insured depositors receive full repayment at time 1 even if bank insolvency occurs. In state 1 the deposit insurance has to pay the difference between contracted returns, $E_0 e$, and asset returns which belong to the insured depositors ($I_1 = E_0 e - kA_0(\alpha x_1 + (1-\alpha)y_1)$). In case of insolvency insured depositors get:

$$\begin{aligned} E_1 = & \underbrace{kA_0(\alpha x_1 + (1-\alpha)y_1)}_{\text{share of the assets of the insolvent bank}} + \underbrace{E_0 e - kA_0(\alpha x_1 + (1-\alpha)y_1)}_{\text{payment from the deposit insurance } I_1} \\ = & E_0 e, \end{aligned} \tag{8}$$

whereas uninsured creditors only receive:

$$U_1 = (1-k)A_0(\alpha x_1 + (1-\alpha)y_1) = U_0 u_1. \tag{9}$$

The bank owners get no return ($C_1 = 0$).

The bank is assumed to be solvent in state 2. Asset returns exceed the promised

16 For analytical convenience the creditors' claims on the assets of the insolvent bank are related to the payments at time 0, $E_0 + U_0$, and not to the promised returns, $E_0 e + U_0 u$. This second case is analysed by *Homölle* (1999).

17 From (5) it is not clear that the promised return on insured deposits is lower than the insured depositors' share of assets of the insolvent bank. But it can easily be shown that the (unregulated) bank raises its leverage risk and asset risk so much that equation (7) holds. Otherwise the present value of the deposit insurance guarantee (see equation (12) below) would not be positive and hence not optimal for a value-maximizing bank.

18 Later it will become clear that $u > e$. Thus we know that the insolvent bank does not pay the promised return to uninsured creditors either: $[A_0/(E_0 + U_0)](\alpha x_1 + (1-\alpha)y_1) < u$.

repayments of all liabilities:

$$A_0(\alpha x_2 + (1-\alpha)y_2) > E_0 e + U_0 u. \tag{10}$$

The difference is paid to the bank owners ($C_2 = A_0(\alpha x_2 + (1-\alpha)y_2) - E_0 e - U_0 u$).

2.3 Profit Function

In our model, we consider a value-maximizing bank. Agency problems, e.g. between bank managers and owners, are excluded. The bank maximizes the net present value of equity, put differently (the present value of) its profit, G_0:[19]

$$\begin{aligned} G_0 &= C_0^B - C_0 \\ &= I_0^B - I_0 + E_0 - E_0^B + U_0 - U_0^B \quad \rightarrow \quad \max. \end{aligned} \tag{11}$$

The present values of the deposit insurance subsidy, I_0^B, insured deposits, E_0^B, and uninsured debt, U_0^B, can be calculated as follows:

$$I_0^B = p_1[E_0 e - kA_0(\alpha x_1 + (1-\alpha)y_1)], \tag{12}$$

$$E_0^B = (p_1 + p_2)E_0 e, \tag{13}$$

$$U_0^B = p_1(1-k)A_0(\alpha x_1 + (1-\alpha)y_1) + p_2 U_0 u. \tag{14}$$

The net present value of the deposit insurance guarantee is assumed to be positive:[20]

$$I_0^B - I_0 > 0. \tag{15}$$

A reason for this assumption in a world without asymmetric information[21] is given by *Buser et al.* (1981): The deposit insurance sets the premium lower than the present value to persuade a value-maximizing bank to take part in the deposit insurance system. Thus the bank is willing to accept some limits to its activities which are set by the deposit insurance or (other) regulatory authorities. The explicit premium, I_0, is supplemented by an implicit premium, e.g. via capital requirements.

19 The net present value of assets is set equal to zero (see (2)).

20 This assumption is implicitly made by all authors who model a free deposit insurance, for example *Furlong/Keeley* (1989); *Gennotte/Pyle* (1991); *Gjerde/Semmen* (1995).

21 Asymmetric information between the deposit insurance institution and the bank is often given as a reason for the underpricing of the deposit insurance subsidy. For the calculation of a theoretically correct, risk-adjusted premium cf. *Merton* (1977); *Merton* (1978); *Pyle* (1984); *Chan et al.* (1992). Risk-adjusted premiums for German banks are calculated by *Fischer/Grünbichler* (1991).

Whether the net present values of insured deposits, $E_0 - E_0^B$, and uninsured debt, $U_0 - U_0^B$, are positive or not, depends on the assumptions about the markets for these liabilities. In this paper we confine ourselves to a scenario with perfectly competitive markets for insured deposits and uninsured debt:[22]

$$p_1 e + p_2 e = 1, \tag{16}$$

$$p_1 u_1 + p_2 u = 1. \tag{17}$$

As $e = 1/(p_1 + p_2)$, the return on insured deposits equals the return on a riskless asset. From the insured depositor's point of view insured deposits are a riskless investment because in any case he receives the promised return at time 1.

Inserting (6) and (9) into (17) we get:

$$u = \frac{1}{p_2}\left[1 - p_1 \frac{A_0}{E_0 + U_0}(\alpha x_1 + (1-\alpha) y_1)\right]. \tag{18}$$

Since the bank acts as a perfect market competitor, it has to promise a return on uninsured debt (per invested dollar) according to (18). u is determined by the chosen values for equity, insured deposits, uninsured debt, and the share of the riskier security X: the lower C_0 and the higher E_0 and α, respectively, the higher the return required by uninsured creditors in state 2. Higher leverage risk and asset risk reduce the return in state 1, u_1. This must be compensated by an increased return in state 2. The sign of the adjustment of u due to a change of uninsured debt, U_0, depends on whether equity exceeds the deposit insurance premium or not.[23] If the premium rate, v, equals zero, the bank has to raise the promised return to attract more uninsured debt.

From (16) and (17) together with (13) and (14) it follows that $E_0 = E_0^B$ and $U_0 = U_0^B$. The net present values of insured deposits and uninsured debt are zero. Thus the bank maximizes the net present value of the deposit insurance guarantee.

3 An Unregulated Bank

In this section we look at the optimal behaviour of a bank in the absence of any capital regulation (except the prescribed minimum amount of equity) to show that a

[22] For a scenario with a bank which has got some monopoly power on the market for insured deposits (and for uninsured debt) cf. *Homölle* (1999). Similar assumptions are discussed by *Kareken/ Wallace* (1978), p. 421; *Dothan/ Williams* (1980), p. 70; *Furlong/Keeley* (1989), p. 884.

[23] $\frac{\partial u}{\partial U_0} = \frac{p_1}{p_2}\frac{C_0 - vE_0}{(E_0+U_0)^2}(\alpha x_1 + (1-\alpha)y_1) \gtreqless 0 \quad \text{if} \quad C_0 \gtreqless vE_0.$

setting with a free deposit insurance and without uninsured debt is not convincing. At time 0 the bank simultaneously optimizes its capital structure and asset size choosing the optimal levels of equity, C_0, insured deposits, E_0, and uninsured debt, U_0. Moreover it determines the optimal share of the riskier security X, α.

Inserting (4), (6), (12), (13), (14), (16), (17) into (11), the profit function of the unregulated bank can be written as follows:

$$\begin{aligned} G_0 &= p_1 \left[E_0 e - \frac{E_0 A_0}{E_0 + U_0} (\alpha x_1 + (1-\alpha) y_1) \right] - v E_0 \\ &= E_0 \left[p_1 \left[e - \frac{A_0}{E_0 + U_0} (\alpha x_1 + (1-\alpha) y_1) \right] - v \right] \end{aligned} \tag{19}$$

$$\text{where} \quad A_0 = C_0 + E_0 (1-v) + U_0.$$

This profit function is quite similar to the one of *Furlong/Keeley* (1989). But there are two important differences. First, *Furlong/Keeley* (1989) assume a free deposit insurance ($v = 0$). Thus they maximize the present value (and not the *net* present value) of the deposit insurance guarantee. Second, they do not model uninsured debt ($U_0 = 0$). Asset returns of the insolvent bank are paid in full to insured depositors. We will see why these two differences are important.

The profit-maximizing bank only holds the prescribed minimum amount of equity because profit shrinks if the bank raises additional equity. This can be seen from the partial derivative of (19) with respect to C_0 which is negative (where $E_0 > 0$, $U_0 \geq 0$):

$$\frac{\partial G_0}{\partial C_0} = -p_1 \frac{E_0}{E_0 + U_0} (\alpha x_1 + (1-\alpha) y_1) < 0. \tag{20}$$

An increase in equity would not lead to a higher profit as the net present value of the assets equals zero. Instead the profit would decrease because there would be more assets in state 1 to pay off creditors. The deposit insurance subsidy would decrease whereas the bank would take into account more of its losses in state 1.[24] Thus the net present value of the deposit insurance guarantee would decline.

From (3) we know that the return on security X in state 1 is lower than the corresponding return on security Y: $x_1 < y_1$. For this reason the marginal gain from increasing α is positive:

$$\frac{\partial G_0}{\partial \alpha} = -p_1 \frac{E_0 A_0}{E_0 + U_0} (x_1 - y_1) > 0. \tag{21}$$

[24] See *Avery/Berger* (1991), p. 850.

The unregulated bank only invests in the riskier security X. This result is widely known as the moral hazard problem of a subsidized deposit insurance.[25] The returns to the bank owners in state 2, C_2, rise ($x_2 > y_2$) whereas – due to limited liability – the bank's payments to creditors in state 1 do not change. Since the insured depositors know that they will receive the contracted return per dollar, e, in each state, they do not require a higher return if asset risk increases.

However, the return on uninsured debt (per dollar) in state 1, u_1, decreases if α is raised. The bank has to alter the promised return, u, to comply with (18). This adjustment of u is implicitly included in the profit function so that profit does not change. From (21) we know:[26]

Result 1 *The issue of uninsured debt does not prevent the increase in asset risk caused by a subsidized deposit insurance. It only lessens the marginal gain from raising asset risk.*

If the bank issues uninsured debt, the deposit insurance only takes a part of the (additional) losses in state 1. The other part must be borne by uninsured creditors.

An interior solution for the optimal amount of insured deposits does not exist. Since the net present value of the deposit insurance guarantee is assumed to be positive (see (15)), the partial derivative of (19) with respect to E_0 is positive as well:

$$\begin{aligned}\frac{\partial G_0}{\partial E_0} &= p_1\left[e - \frac{A_0}{E_0+U_0}(\alpha x_1 + (1-\alpha)y_1)\right] - v \\ &\quad + p_1\frac{E_0(C_0+vU_0)}{(E_0+U_0)^2}(\alpha x_1 + (1-\alpha)y_1) > 0.\end{aligned} \tag{22}$$

We can identify two effects of increasing E_0. First, as the net present value of the deposit insurance subsidy per insured deposit is assumed to be positive, profit rises with each additional insured deposit. Second, the ratio $A_0/(E_0+U_0)$ decreases if E_0 is raised. In state 1 the creditors get a lower return (per invested dollar) which leads to a higher payment from the deposit insurance than before. The (net) present value of the deposit insurance guarantee rises. It is optimal to increase the insured deposits as far as possible.[27]

Thus far we can summarize our results as follows:

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[25] Cf. for example *Kareken/Wallace* (1978), pp. 414, 428; *Sharpe* (1978), pp. 710-716; *Dothan/Williams* (1980), pp. 75-81; *Carisano* (1992), pp. 79-85; *Gjerde/Semmen* (1995), p. 1165.

[26] See also *Furlong/Keeley* (1989), p. 886, equation (8).

[27] *Kareken/Wallace* (1978), p. 428, obtain a similar result.

Result 2 *The unregulated bank tries to maximize asset risk and – by increasing insured deposits while holding only the minimum amount of equity – leverage risk as well.*

Increasing insured deposit and hence asset size ad infinitum is a quite unrealistic scenario. However, the resulting increase of leverage risk confirms the already known risk-shifting effects of a fixed-rate deposit insurance in a perfect market scenario. In the next section we will see how bank behaviour changes if we introduce some market imperfections caused by regulatory restrictions.

Whether the profit-maximizing bank issues uninsured debt can be seen from the partial derivative of the profit function (19) with respect to U_0. The sign of this derivative depends on the amount of equity, C_0, and – with a constant premium rate $v > 0$ – on the level of insured deposits, E_0. For $E_0 > 0$, $C_0 > 0$ the following holds:

$$\frac{\partial G_0}{\partial U_0} = p_1 \frac{E_0(C_0 - vE_0)}{(E_0 + U_0)^2}(\alpha x_1 + (1-\alpha)y_1) \gtreqless 0 \qquad \Leftrightarrow \qquad C_0 \gtreqless vE_0. \tag{23}$$

There are two effects of raising uninsured debt. On the one hand, the uninsured creditors' share, $1-k$, of asset returns in state 1 increases, i.e. the insured depositors' share shrinks. On the other hand, the total returns on assets in state 1 rise. The first effect increases profit whereas the second effect induces a profit reduction.

The total effect is positive as long as equity exceeds the premium of the deposit insurance. This can be rationalized as follows: An increase in U_0 leads to a reduction of $A_0/(E_0 + U_0)$ if $C_0 > vE_0$. If uninsured debt is raised, the uninsured creditors' claim against the insolvent bank increases. As the part of assets which is financed by equity (diminished by the premium), $C_0 - vE_0$, remains unchanged, the insured depositors get less payments from the insolvent bank in state 1 and higher payments from the deposit insurance. The net present value of the deposit insurance subsidy increases. Hence, it is optimal to issue additional uninsured debt even if the net present value of this debt equals zero. Due to the minimum level of equity ($C_0 > 0$), $C_0 > vE_0$ always holds if $v = 0$.

Result 3 *If the bank does not have to pay any deposit insurance premium, it issues uninsured debt to maximize its profit.*[28]

We get a different result if the premium rate is positive ($v > 0$). It has already been shown that the bank tries to increase insured deposits as far as possible. In

[28] The marginal gain from increasing U_0 approaches zero as the amount of insured deposits tends to infinity: $\lim_{E_0 \to \infty} \partial G_0/\partial U_0 = 0$.

this situation equity falls below the premium of the deposit insurance. For $C_0 < vE_0$ the marginal gain from increasing U_0 is negative.

Result 4 *If the bank pays a positive fixed-rate deposit insurance premium related to the amount of insured deposits, it is optimal not to hold uninsured debt.*

Up to now we have changed the model of *Furlong/Keeley* (1989) in two important aspects: a positive deposit insurance premium and uninsured debt. However, an unregulated profit-maximizing bank does not hold uninsured debt if it has to pay a premium of $vE_0 > 0$. This result can be used to justify analytical frameworks with a bank only holding equity and insured deposits if the premium rate is positive. It does *not* explain (and in a sense even contradicts) the assumptions in models like those of *Furlong/Keeley* (1989) because these authors consider a free deposit insurance while only allowing for insured deposits and equity. If the deposit insurance is free, one should model a bank which can also issue uninsured debt. To include this difference in our analysis, we are going to consider two scenarios:

(1) $U_0 = 0$, $v > 0$,

(2) $U_0 > 0$, $v = 0$.

In the first case we analyse a bank whose liabilities only consist of insured deposits. In the second case the bank issues both, insured deposits and uninsured debt.

In this section we described the incentive of an unregulated bank to increase asset risk as well as leverage risk. This behaviour is optimal as long as the marginal gain exceeds the marginal cost which, however, has not been considered yet. Regulatory restrictions may serve as a source of such (marginal) cost. To restrict bank risk-taking and to prevent unappropriately high payments from the deposit insurance in case of bank insolvency, the regulatory authority may impose some constraints on bank behaviour.[29] How a bank behaves if it has to comply with some kind of capital regulation will be shown in the next section.

[29] As *Kareken/Wallace* (1978), p. 415, put it: "Regulation is not an alternative to deposit insurance but rather a necessary complement." See also *Crouhy/Galai* (1991), p. 87; *Flannery* (1982).

4 A Regulated Bank

4.1 Leverage Constraint and the Need for Supplementary Regulation

In this section we describe the behaviour of a profit-maximizing bank when it has to comply with a leverage constraint. A leverage constraint is usually designed as an upper bound for the ratio of liabilities or assets to equity. Thus it is guaranteed that assets are not only financed by liabilities. This can also be achieved by a lower bound on the ratio of assets to liabilities. We assume that the regulatory authority imposes such a lower bound, s, to limit leverage risk:

$$\frac{A_0}{E_0 + U_0} \geq s > 1. \tag{24}$$

For $s > 1$ it holds that assets are not allowed to be solely financed by liabilities, i.e. bank equity (minus the deposit insurance premium) must be positive.[30] This leverage constraint is quite a simple form of capital regulation. In reality more comprehensive rules including different asset risk-weights can be observed. In our model it would not make a (big) difference using risk-adjusted capital regulation instead of a simple leverage constraint as long as we assume that securities X and Y are in the same risk class (for example both are issued by private companies and thus get a risk weight of 100 %, see § 13 Grundsatz I).

The leverage constraint is binding because it restricts the incentive of an unregulated bank to increase insured deposits while holding the required minimum amount of equity. Thus (24) can be transformed as follows:

$$\frac{A_0}{E_0 + U_0} = s \quad \Leftrightarrow \quad C_0 - (s - 1 + v)E_0 - (s - 1)U_0 = 0. \tag{24'}$$

However, the incentive to increase asset risk exists irrespective of capital regulation. With (24′), i.e. keeping s constant,[31] the derivative (21) can be reduced to:

$$\left.\frac{\partial G_0}{\partial \alpha}\right|_s = -p_1 E_0 s (x_1 - y_1) > 0. \tag{25}$$

[30] As already mentioned above one can also consider an upper bound on the ratio of liabilities or assets to equity. The conclusions would be the same as long as deposit insurance is free. However, for $v > 0$ these constraints do not necessarily show the same effects on bank behaviour.

[31] Comparing (25) with (21) we see that, due to the constant ratio $A_0/(E_0 + U_0) = s$, a change of U_0 (or C_0) does not influence the marginal gain from increasing asset risk any more.

Like *Furlong/Keeley* (1989) we argue that the regulatory authority has to restrict the share of security X to limit asset risk.

When the bank chooses its optimal amounts of equity, insured deposits, and uninsured debt, it has to comply with the capital constraint (24). The optimal levels of C_0, E_0, and U_0 are no longer independent of each other. For example, the regulated bank can only expand insured deposits by reducing uninsured debt or raising additional equity. This can be shown by the total differential of (24′) where $ds = 0$:

$$dE_0 = -\frac{s-1}{s-1+v} dU_0 + \frac{1}{s-1+v} dC_0. \tag{26}$$

It is optimal to hold only insured deposits because the marginal gain from increasing insured deposits (and reducing uninsured debt or increasing equity) is positive. Holding s constant and considering (26), totally differentiating the profit function (19) leads to:[32]

$$\left.\frac{dG_0}{dE_0}\right|_s = p_1 \left[e - s(\alpha x_1 + (1-\alpha) y_1)\right] - v > 0. \tag{27}$$

According to (27) the marginal gain from increasing E_0 equals the net present value of the deposit insurance guarantee per insured deposit. Since it is assumed that this net present value is positive (see (15)), the sign of the derivative is positive as well. A regulated bank still maximizes its profit by increasing insured deposits.

At the end of the previous section we have distinguished two situations which we will now describe consecutively. We start with an unregulated bank which does not hold uninsured debt due to a positive premium rate, v.

(1) $U_0 = 0$*,* $v > 0$: We showed that an unregulated bank does not issue uninsured debt if the premium rate, v, is positive. If this bank has to comply with the capital requirement (24), its behaviour depends on whether it can increase its equity or not. Like *Furlong/Keeley* (1989) we are going to distinguish between a bank which can raise additional equity to comply with the imposed capital constraint ($dC_0 > 0$) and a bank whose equity can be considered constant ($dC_0 = 0$).

(1a) $dC_0 = 0$: If the equity of a bank is fixed, e.g. because the bank is a publicly owned savings bank being unable to raise additional equity in capital markets, an upper limit of insured deposits is defined by the capital constraint $A_0/E_0 \geq s$. The

[32] This total derivative holds irrespective of whether dC_0 or dU_0 are set equal to zero: $dG_0/dE_0|_{s,C_0} = dG_0/dE_0|_{s,U_0} = dG_0/dE_0|_s$.

bank could decrease insured deposits and issue uninsured debt but this would reduce profit because the marginal gain from increasing E_0 (and decreasing U_0) is positive (see (27)). Thus the regulated bank holds the maximum amount of insured deposits:

$$E_0 = \frac{1}{s-1+v} C_0. \tag{28}$$

(1b) $dC_0 > 0$: The assumption of fixed equity is not appropriate for a large bank with access to capital markets.[33] Such a bank increases its equity and insured deposits in accordance with (26). To prevent the rise of insured deposits which lead to higher payments from the deposit insurance, the regulatory authority has to restrict insured deposits.

Furlong/Keeley (1989) also identify the incentive of a regulated bank to increase insured deposits and equity and to invest this capital in assets. They argue that the bank increases assets until the marginal gain is balanced by the marginal cost, and consider regulatory restrictions of asset size as a main source of this cost. We will see that this assumption is crucial for their results and is no longer convincing if liabilities do not only consist of insured deposits.

(2) $U_0 > 0$, $v = 0$: If deposit insurance is free, an unregulated, profit maximizing bank holds insured deposits as well as uninsured debt. To describe the behaviour of this bank when it has to take into account the capital constraint (24) we once again distinguish between two scenarios.

(2a) $dC_0 = 0$: The bank holds the required minimum amount of equity but cannot increase its equity to comply with (24). This leverage constraint defines an upper bound of liabilities but does not determine the composition of these liabilities. Since (27) is positive, the regulated bank reduces its uninsured debt to increase insured deposits up to $E_0 = 1/(s-1+v)C_0$.

Result 5 *If a bank with fixed equity has to comply with the capital constraint* (24), *it does not hold any uninsured debt even if deposit insurance is free.*

The regulatory authority can prevent this shifting by imposing an adequate upper bound for insured deposits.

(2b) $dC_0 > 0$: The bank can issue insured deposits and increase its equity or decrease uninsured debt according to (26). Both, the increase in C_0 and the

[33] Cf. *Furlong/Keeley* (1989), p. 887.

Table 2: Limitation of insured deposits

	(a) $dC_0 = 0$	(b) $dC_0 > 0$
(1) $U_0 = 0$, $v > 0$	unnecessary: $E_0 = \frac{1}{s-1+v} C_0$	required: $\bar{E}_0$
(2) $U_0 > 0$, $v = 0$	required to guarantee $U_0 > 0$: $\bar{E}_0$	required: $\bar{E}_0$

reduction of U_0, do not (directly) alter the profit of the regulated bank because the ratio of assets to liabilities is fixed ($A_0/(E_0 + U_0) = s$).

The assumption of *Furlong/Keeley* (1989) concerning regulatory constraints of asset size is no longer convincing because the increase in insured deposits and *not* the increase in assets is responsible for the rising deposit insurance subsidy and because a rise of insured deposits does *not* necessarily lead to an increase in assets. With a limitation of asset size the shifting from uninsured debt to insured deposits cannot be prevented.

Result 6 *As the increase in insured deposits results in rising payments of the deposit insurance, the regulatory authority should directly restrict these deposits and not the assets. An adequate upper limit of insured deposits stops the bank's incentive to shift uninsured debt to insured deposits as well as to raise insured deposits and equity simultaneously.*

The results concerning the need for a supplementary limitation of insured deposits are summarized in *Table 2.* In the next sections we are going to distinguish between a scenario in which an additional restriction of insured deposits is not necessary and another scenario with an additional limit of insured deposits, $\bar{E}_0$. In the first case we consider a bank with fixed equity which does not hold uninsured debt: $U_0 = 0 \wedge dC_0 = 0$ (see (1a)). In the second scenario, the bank can either raise additional equity or hold uninsured debt: $U_0 > 0 \vee dC_0 > 0$ (see (1b), (2a), (2b)).[34]

4.2 Design of Supplementary Regulatory Constraints

In section 4.1 we showed that it is reasonable to supplement the leverage constraint (24) by a restriction of asset risk. Following the idea of *Furlong/Keeley* (1989) we

[34] The assumption whether the premium rate is positive or not is not crucial for the analysis in the next sections. Thus we will not treat the cases $v = 0$ and $v > 0$ separately any more.

assume that the regulatory authority limits the share of security X in the asset portfolio by imposing regulatory cost, K^α, as a function of α, where

$$\frac{dK^\alpha}{d\alpha} > 0, \qquad \frac{d^2 K^\alpha}{d\alpha^2} > 0. \tag{29}$$

Actual regulatory restrictions have been influenced by the idea of limiting the share of a single asset. However, in reality it is not easy to determine the riskiest asset. Thus the share of each asset is limited to make sure that the portfolio is sufficiently diversified. In Germany, e.g., a loan to a single person or company is restricted by law to 25 % of the bank's equity. It is allowed to go beyond this limit but the bank must hold equity covering 100 % of the exceeding amount. This equity cannot be used any more to fulfill other regulatory restrictions so that the bank loses the chance of getting profits from other possible transactions, e.g. granting other loans.

In the previous section, it was also shown that in some cases an upper bound for insured deposits seems reasonable. Before we describe the design of such a restriction we take a look at case (1a). In this benchmark scenario, a sufficient upper limit of insured deposits is already given by the leverage constraint itself.

$U_0 = 0 \wedge dC_0 = 0$: We are looking at a bank similar to one of those modelled by *Furlong/Keeley* (1989) which only holds insured deposits and a fixed amount of equity. The regulatory cost, K^α, must be taken into account to determine the optimal behaviour of the regulated bank. Its objective function can be written as (remember $k = 1$):

$$\begin{aligned} G_0^{K^\alpha} &= p_1 \left[E_0 e - A_0(\alpha x_1 + (1-\alpha) y_1) \right] - vE_0 - K^\alpha \\ \text{subject to} \quad s &= \frac{A_0}{E_0}. \end{aligned}$$

Since the capital constraint is binding and $dC_0 = 0$, $U_0 = 0$, the optimal amount of insured deposits is given by $E_0 = 1/(s - 1 + v)C_0$. The optimal level of asset risk can be determined by inserting the constraint into the objective function. Hence, we get the first-order condition for an interior maximum as follows:

$$\begin{aligned} \frac{\partial G_0^{K^\alpha}}{\partial \alpha} &= -p_1 E_0 s(x_1 - y_1) - \frac{dK^\alpha}{d\alpha} = 0 \\ \Leftrightarrow \qquad \frac{dK^\alpha}{d\alpha} &= \left. \frac{\partial G_0}{\partial \alpha} \right|_s . \end{aligned} \tag{30}$$

Let us assume that there exist regulatory cost so that condition (30) holds for

$\alpha = \bar{\alpha} < 1$. The bank rises the share of security X as far as the marginal gain (without regulatory cost) is balanced by the marginal regulatory cost. Since K^α is strictly convex in α (see (29)), we get an interior solution at $\bar{\alpha}$.

$U_0 > 0 \vee dC_0 > 0$: In this second case we consider a bank which can raise addional equity or which holds uninsured debt. Suppose that the regulatory authority limits the incentive of a bank to increase E_0 by regulatory cost, K^E, as a function of E_0, where

$$\frac{dK^E}{dE_0} > 0, \qquad \frac{d^2K^E}{d{E_0}^2} > 0. \tag{31}$$

The assumption about restrictions of insured deposits is not only important for our formal analysis. In reality such limitations exist as well. For example, in the EU insured deposits are limited to (at least) 20,000 ECU per person.[35] This upper bound is not directly connected with regulatory cost but we can imagine that the circumvention of this limit, e.g. opening (insured) deposit accounts under the name of children or other relatives if a person's insured deposits reach the limit, may induce some transaction cost.

The cost, K^E, also has to be included in the profit function to determine the optimal behaviour of the regulated bank. The objective function is extended to:[36]

$$G_0^K = p_1 \left[E_0 e - \frac{E_0 A_0}{E_0 + U_0}(\alpha x_1 + (1-\alpha) y_1) \right] - vE_0 - K^\alpha - K^E$$
$$\text{subject to} \quad s = \frac{A_0}{E_0 + U_0}.$$

First-order conditions for an interior maximum are:[37]

$$\begin{aligned} \frac{\partial G_0^K}{\partial \alpha} &= -p_1 E_0 s(x_1 - y_1) - \frac{dK^\alpha}{d\alpha} = 0 \\ \Leftrightarrow \qquad \frac{dK^\alpha}{d\alpha} &= \left.\frac{\partial G_0}{\partial \alpha}\right|_s , \end{aligned} \tag{30}$$

35 In Germany, the "Entschädigungseinrichtung der deutschen Banken GmbH" restricts its guarantee to 90 % of a deposit and to a maximum amount of 40,000 DM per depositor. Cf. *Holtorf/Rudolf* (1999), p. 250.

36 For a bank, which does not hold uninsured debt but can raise additional equity, U_0 must be set equal to zero.

37 If equity is supposed to be fixed ($dC_0 = 0$), the bank has only two choice variables: α and E_0 *or* U_0. By calculating the optimal level of insured deposits the optimal amount of uninsured debt is simultaneously determined according to (24′). If the bank can raise additional equity ($dC_0 > 0$), there exists a third decision variable. However, as $\partial G_0^K / \partial C_0 = 0$ and $\partial G_0^K / \partial U_0 = 0$ (because s is to be held constant), equity and unsinsured debt do not influence the attainable maximum profit.

$$\frac{\partial G_0^K}{\partial E_0} = p_1[e - s(\alpha x_1 + (1-\alpha)y_1)] - v - \frac{dK^E}{dE_0} = 0$$

$$\Leftrightarrow \qquad \frac{dK^E}{dE_0} = \left.\frac{dG_0}{dE_0}\right|_s . \tag{32}$$

From the Hessian Matrix

$$H = \begin{bmatrix} -\frac{d^2K^\alpha}{d\alpha^2} & -p_1 s(x_1 - y_1) \\ -p_1 s(x_1 - y_1) & -\frac{d^2K^E}{dE_0{}^2} \end{bmatrix}$$

we get the second-order conditions for a maximum:

$$|H_1| = -\frac{d^2K^\alpha}{d\alpha^2} < 0, \tag{33}$$

$$|H_2| = \frac{d^2K^E}{dE_0{}^2}\frac{d^2K^\alpha}{d\alpha^2} - p_1{}^2 s^2 (x_1 - y_1)^2 > 0. \tag{34}$$

We assume that the functions of regulatory costs are designed so that the conditions (30), (32), (33), and (34) are fulfilled at $\alpha = \bar{\alpha} < 1$ and at a certain amount of insured deposits, $E_0 = \bar{E}_0$. The bank raises insured deposits and the share of security X until the marginal gain is balanced by the marginal regulatory cost. Of course, one can say that these rather specific assumptions are unrealistic. But what is the main idea of the introduction of regulatory cost? This cost serves as some sort of imperfection also observed in reality making sure that banks do not increase asset risk and insured deposits ad infinitum. Including regulatory cost we get a strictly concave profit function and thus an interior maximum at $\bar{E}_0$ and $\bar{\alpha}$.

In section 4.1 we considered a bank which also issues uninsured debt (if $v = 0$). To guarantee that this bank is not forced by the capital constraint to reduce uninsured debt completely, the upper bound for insured deposits, $\bar{E}_0$, must satisfy another condition:

$$U_0 = -\frac{s-1+v}{s-1}\bar{E}_0 + \frac{1}{s-1}C_0 > 0,$$

$$\Leftrightarrow \qquad \bar{E}_0 < \frac{1}{s-1+v}C_0.$$

This condition defines either a fixed limit for $\bar{E}_0$ (if $dC_0 = 0$) or a positive relation between the maximum amount of $\bar{E}_0$ and the variable equity (if $dC_0 > 0$).

5 The Enforcement of a More Stringent Capital Requirement

In this section we answer the question how a bank responds to a more stringent capital requirement. Suppose the regulatory authority decides to increase s to improve bank solvency. Does the bank reduce its asset risk or does it increase the share of the riskier security in its portfolio?

$U_0 = 0 \wedge dC_0 = 0$: Once again we start our analysis with a regulated bank with fixed equity and insured deposits. The optimum is defined by

$$E_0 = \frac{1}{s-1+v} C_0, \tag{28}$$

$$\frac{dK^\alpha}{d\alpha} = -p_1 E_0 s (x_1 - y_1). \tag{30}$$

If the regulatory authority increases the leverage ratio ($ds > 0$), the bank changes the insured deposits, E_0, and the share of the riskier security, α, so that the conditions (28) and (30) still hold:

$$dE_0 = -\frac{1}{(s-1+v)^2} C_0 ds,$$

$$\frac{d^2 K^\alpha}{d\alpha^2} d\alpha = -p_1 s (x_1 - y_1) dE_0 - p_1 E_0 (x_1 - y_1) ds$$

and finally

$$\frac{dE_0}{ds} = -\frac{1}{(s-1+v)^2} C_0 < 0, \tag{35}$$

$$\frac{d\alpha}{ds} = \frac{-p_1 \frac{v-1}{(s-1+v)^2} C_0 (x_1 - y_1)}{\frac{d^2 K^\alpha}{d\alpha^2}} < 0. \tag{36}$$

Since equity is fixed and liabilities only consist of insured deposits, the bank has to reduce these deposits to comply with the new capital requirement.

An increase in s shows one direct and one indirect effect on the marginal gain from increasing the share of X, $\partial G_0 / \partial \alpha|_s$. From (25) we know that the marginal gain directly increases when s is raised by the regulatory authority. Additionally the rise of s leads to a decrease in insured deposits (see (35)) which in turn induces a reduction of $\partial G_0 / \partial \alpha|_s$. Since the sign of (36) is negative, this indirect effect is stronger than the first, direct effect. The marginal gain from increasing asset risk decreases. As the marginal cost, $dK^\alpha / d\alpha$, is unaffected by the change of the capital requirement, the bank reduces the share of the riskier security X until equation (30) holds again.

Result 7 *A more stringent capital requirement gives a bank with fixed equity and insured deposits the incentive to reduce asset risk.*

This result is quite similar to that described by *Furlong/Keeley* (1989).[38] We are now going to show that it does *not* hold for a regulated bank which also issues uninsured debt or can raise its equity.

$U_0 > 0 \vee dC_0 > 0$: As a starting point we use the optimality conditions (30) and (32) which hold for $\alpha = \bar{\alpha}$ and $E_0 = \bar{E}_0$. The optimal variation of the share of the riskier security X when s is raised is calculated by totally diffentiating these conditions:

$$\frac{d^2K^\alpha}{d\alpha^2}d\alpha = -p_1 s(x_1 - y_1)dE_0 - p_1\bar{E}_0(x_1 - y_1)ds,$$
$$\frac{d^2K^E}{d{E_0}^2}dE_0 = -p_1 s(x_1 - y_1)d\alpha - p_1(\bar{\alpha}x_1 + (1-\bar{\alpha})y_1)ds.$$

After some transformations we get

$$\frac{d\alpha}{ds} = \frac{-p_1\left(\bar{E}_0\frac{d^2K^E}{d{E_0}^2} - p_1 s(\bar{\alpha}x_1 + (1-\bar{\alpha})y_1)\right)(x_1 - y_1)}{\frac{d^2K^\alpha}{d\alpha^2}\frac{d^2K^E}{d{E_0}^2} - {p_1}^2 s^2(x_1 - y_1)^2}. \tag{37}$$

The sign of (37) is not clear. Since the denominator is positive by assumption (see (34)), it holds that

$$\bar{E}_0\frac{d^2K^E}{d{E_0}^2} \gtreqless p_1 s(\bar{\alpha}x_1 + (1-\bar{\alpha})y_1) \qquad \Leftrightarrow \qquad \frac{d\alpha}{ds} \gtreqless 0. \tag{38}$$

If the regulatory authority increases the lower bound, s, the marginal gain from increasing asset risk, $\partial G_0/\partial\alpha|_s$, rises. The rise of $A_0/(E_0 + U_0)$ leads to an increase in the share of insured depositors in asset returns in state 1. This induces a rise of the marginal loss of an increase in asset risk which is borne by the deposit insurance. This rise of additional payments from the deposit insurance in state 1 is good for the bank.

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[38] *Furlong/Keeley* (1989), p. 887, say that "a bank would not be expected to respond to higher capital requirements by increasing the riskiness of its asset portfolio." The difference is caused by our assumption about the regulatory cost, K^α. *Furlong* and *Keeley* do not argue using a continuous, twice differentiable cost function but only consider the upper part of the cost function. They state that the marginal cost of *exceeding* $\bar{\alpha}$ should be at least equal to the marginal gain.

We can recognize another direct effect of raising s. The marginal gain from increasing E_0, $dG_0/dE_0|_s$, decreases in s which gives the bank the incentive to reduce insured deposits. This decrease in E_0 leads to a reduction of $\partial G_0/\partial\alpha|_s$ because the deposit insurance has to take into account a smaller part of the additional losses in state 1 which are induced by an increase in asset risk.

The question is whether this indirect effect on $\partial G_0/\partial\alpha|_s$ dominates the first, direct effect. The total effect is positive, that is, $\partial G_0/\partial\alpha|_s$ increases if insured deposits do *not* shrink. It can easily be shown that the following relation holds:[39]

$$\frac{dE_0}{ds} \geq 0 \qquad \Rightarrow \qquad \frac{d\alpha}{ds} > 0.$$

However, the change of E_0 is not clear because the marginal gain from increasing E_0, $dG_0/dE_0|_s$, is also influenced by the variation of α. A rise of asset risk leads to a rise of $dG_0/dE_0|_s$. Thus the direct effect and the indirect effect point in different directions. It is possible that the profit-maximizing bank *increases* insured deposits. In this case the bank has to raise equity or to diminish uninsured debt to comply with the new capital requirement.

Result 8 *The enforcement of a stronger capital requirement directly increases the marginal gain from raising asset risk. Moreover it induces a change of the insured deposits which also influences the marginal gain from raising asset risk. Due to this interdependence it is not clear whether it is optimal for a bank which can raise additional equity or holds uninsured debt to decrease or increase asset risk.*

From (38) we know that the sign of $d\alpha/ds$ tends to be positive if the insured deposits and the share of security X before the imposition of a stronger capital constraint, $\bar{E}_0$ and $\bar{\alpha}$, are high and the lower bound s of the capital ratio is low.

Result 9 *If the regulatory restrictions concerning asset risk and leverage risk are not too strict, it is quite possible that tightening the capital requirement induces the bank to increase asset risk.*

The effects of an increase in s are summarized in *Table 3*. The importance of the functions of regulatory cost can be seen in this table as well. For example, the higher the change of the marginal cost of increasing asset risk, the lower the change

[39] See appendix.

Table 3: Effects of an enforcement of a more stringent capital requirement

Effects on	Description	Derivative
α	*direct effect:* marginal gain from increasing asset risk rises; incentive to increase asset risk	$\frac{\partial^2 G_0^K}{\partial\alpha\,\partial s} = -p_1\bar{E}_0(x_1 - y_1) > 0$
	indirect effect: marginal gain from increasing asset risk rises (decreases) if insured deposits rise (decrease); incentive to increase (decrease) asset risk	$\frac{\partial^2 G_0^K}{\partial\alpha\,\partial E_0} = -p_1 s(x_1 - y_1) > 0$
	the higher the change of marginal cost of increasing asset risk if asset risk changes, the lower the change of asset risk	$\frac{\partial^2 G_0^K}{\partial\alpha^2} = -\frac{d^2 K^\alpha}{d\alpha^2} < 0$
E_0	*direct effect:* marginal gain from increasing insured deposits decreases; incentive to reduce insured deposits	$\frac{\partial^2 G_0^K}{\partial E_0\,\partial s} = -p_1(\bar{\alpha}x_1 + (1-\bar{\alpha})y_1) < 0$
	indirect effect: marginal gain from increasing insured deposits increases (decreases) if asset risk increases (decreases); incentive to increase (decrease) insured deposits	$\frac{\partial^2 G_0^K}{\partial E_0\,\partial\alpha} = -p_1 s(x_1 - y_1) > 0$
	the higher the change of marginal cost of increasing insured deposits if insured deposits change, the lower the change of insured deposits	$\frac{\partial^2 G_0^K}{\partial {E_0}^2} = -\frac{d^2 K^E}{d{E_0}^2} < 0$

Note: The regulated bank holds uninsured debt or can raise additional equity.

of the share of the riskier security X, for

$$\lim_{\frac{d^2K^\alpha}{d\alpha^2}\to\infty} \frac{d\alpha}{ds} = 0.$$

If the value of the second derivative of the regulatory cost, K^E, is high (in $\bar{E}_0$), the bank does not alter the insured deposits very much to hold condition (30). Thus the indirect effect on the marginal gain from increasing asset risk is quite small. Because of the dominating direct effect of increasing s, $\partial G_0/\partial\alpha|_s$ rises. It is optimal to increase the share of the riskier security in the asset portfolio. The sign of (37) is positive:

$$\lim_{\frac{d^2K^E}{dE_0{}^2}\to\infty} \frac{d\alpha}{ds} = \frac{-p_1\bar{E}_0(x_1-y_1)}{\frac{d^2K^\alpha}{d\alpha^2}} > 0.$$

These results hold for a bank which can raise additional equity or which holds uninsured debt. Hence, if a bank issues uninsured debt, the results are independent of whether the bank can increase its equity or not. In both cases the bank can comply with the new capital constraint by diminishing uninsured debt. Equity can but does not have to be raised. This change as well as a change of uninsured debt does not directly alter profit.

6 Conclusion

In this paper we have described the impact of a higher capital requirement on bank risk-taking. Using a state-preference model we analysed a profit-maximizing bank which can issue not only insured deposits but also uninsured debt. Thus we got some new information of bank behaviour in a world with deposit insurance and capital regulation.

We showed that it is sometimes not sufficient for clear results to assume that the deposit insurance is subsidized because an unregulated bank issues uninsured debt if the deposit insurance does not charge any premium. But with a positive fixed-rate premium (depending on the amount of insured deposits) it is not optimal to hold uninsured debt.

The result of *Furlong/Keeley* (1989) concerning the reaction of a bank to a more stringent capital requirement can only be confirmed for a bank which solely holds insured deposits and a fixed amount of equity. This bank tends to decrease asset risk. However, for a bank which also holds uninsured debt or which can raise equity it is optimal to increase asset risk under certain circumstances, e.g. if the regulatory restrictions (before tightening the capital requirement) are not too strict.

We confined our analysis to the impact of a higher capital requirement on asset risk. The increase or decrease in asset risk is only one effect of a more stringent bank capital regulation. If a bank increases asset risk, one can ask whether this effect exceeds the effect of decreased leverage so that insolvency risk is higher than before. To solve this problem within our analytic framework, it must be examined whether the number of insolvency states tends to vary because of the changing structure of bank capital and assets. The analysis goes far beyond the scope of this paper. Thus, we can only give some hints. The insolvency risk of a bank with fixed equity and insured deposits tends to decrease whereas the results concerning banks with uninsured debt and variable equity are not clear. The variations of insured deposits as well as uninsured debt influence the change of insolvency risk.

Although our model already includes some new aspects, it is still quite simplifying. Some other variations may be useful, e.g. the asset returns of the insolvent bank might be distributed among the creditors proportional to their share of the promised returns on all liabilities.[40] Alternatively one can assume that insured deposits are senior to uninsured debt. This case leads to some interesting results, too. However, this will not be considered in this paper, but in a companion paper.[41]

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[40] Cf. *Homölle* (1999).

[41] Cf. *Homölle* (2000).

Appendix

Optimal variations of the share of the riskier asset, α, and of insured deposits, E_0, when s is raised:

$$\frac{d\alpha}{ds} = \frac{\frac{\partial^2 G_0^K}{\partial E_0\,\partial s}\frac{\partial^2 G_0^K}{\partial E_0\,\partial \alpha} - \frac{\partial^2 G_0^K}{\partial \alpha\,\partial s}\frac{\partial^2 G_0^K}{\partial {E_0}^2}}{\frac{\partial^2 G_0^K}{\partial \alpha^2}\frac{\partial^2 G_0^K}{\partial {E_0}^2} - \left(\frac{\partial^2 G_0^K}{\partial E_0\,\partial \alpha}\right)^2},$$

$$\frac{dE_0}{ds} = \frac{\frac{\partial^2 G_0^K}{\partial \alpha\,\partial s}\frac{\partial^2 G_0^K}{\partial E_0\,\partial \alpha} - \frac{\partial^2 G_0^K}{\partial E_0\,\partial s}\frac{\partial^2 G_0^K}{\partial \alpha^2}}{\frac{\partial^2 G_0^K}{\partial \alpha^2}\frac{\partial^2 G_0^K}{\partial {E_0}^2} - \left(\frac{\partial^2 G_0^K}{\partial E_0\,\partial \alpha}\right)^2}$$

$$\text{where} \qquad \frac{\partial^2 G_0^K}{\partial \alpha^2} < 0,$$

$$\frac{\partial^2 G_0^K}{\partial {E_0}^2} < 0,$$

$$\frac{\partial^2 G_0^K}{\partial \alpha^2}\frac{\partial^2 G_0^K}{\partial {E_0}^2} - \left(\frac{\partial^2 G_0^K}{\partial E_0\,\partial \alpha}\right)^2 > 0 \qquad \Leftrightarrow \qquad \frac{\partial^2 G_0^K}{\partial \alpha^2} < \frac{\left(\frac{\partial^2 G_0^K}{\partial E_0\,\partial \alpha}\right)^2}{\frac{\partial^2 G_0^K}{\partial {E_0}^2}}.$$

If the tightening of the capital requirement induces the bank not to reduce insured deposits, it is at the same time optimal to raise asset risk:

$$\frac{dE_0}{ds} \geq 0 \qquad \Rightarrow \qquad \frac{d\alpha}{ds} > 0.$$

Proof:

$$\frac{dE_0}{ds} \geq 0$$

$$\Leftrightarrow \qquad \frac{\partial^2 G_0^K}{\partial \alpha\,\partial s}\frac{\partial^2 G_0^K}{\partial E_0\,\partial \alpha} - \frac{\partial^2 G_0^K}{\partial E_0\,\partial s}\frac{\partial^2 G_0^K}{\partial \alpha^2} \geq 0$$

$$\Rightarrow \qquad \frac{\partial^2 G_0^K}{\partial \alpha\,\partial s}\frac{\partial^2 G_0^K}{\partial E_0\,\partial \alpha} - \frac{\partial^2 G_0^K}{\partial E_0\,\partial s}\frac{\left(\frac{\partial^2 G_0^K}{\partial E_0\,\partial \alpha}\right)^2}{\frac{\partial^2 G_0^K}{\partial {E_0}^2}} > 0 \qquad \text{where} \qquad \frac{\partial^2 G_0^K}{\partial E_0\,\partial s} < 0$$

$$\Leftrightarrow \qquad \frac{\frac{\partial^2 G_0^K}{\partial E_0\,\partial \alpha}}{\frac{\partial^2 G_0^K}{\partial {E_0}^2}}\left[\frac{\partial^2 G_0^K}{\partial \alpha\,\partial s}\frac{\partial^2 G_0^K}{\partial {E_0}^2} - \frac{\partial^2 G_0^K}{\partial E_0\,\partial s}\frac{\partial^2 G_0^K}{\partial E_0\,\partial \alpha}\right] > 0$$

$$\Rightarrow \qquad \frac{\partial^2 G_0^K}{\partial E_0\,\partial s}\frac{\partial^2 G_0^K}{\partial E_0\,\partial \alpha} - \frac{\partial^2 G_0^K}{\partial \alpha\,\partial s}\frac{\partial^2 G_0^K}{\partial {E_0}^2} > 0 \qquad \text{where} \qquad \frac{\partial^2 G_0^K}{\partial E_0\,\partial \alpha} > 0$$

$$\Leftrightarrow \qquad \frac{d\alpha}{ds} > 0 \quad \text{q.e.d.}$$

References

Arrow, Kenneth J. (1964), The Role of Securities in the Optimal Allocation of Risk-bearing, in: Review of Economic Studies, Vol. 31, pp. 91–96.

Avery, Robert B./Berger, Allen N. (1991), Risk-based Capital and Deposit Insurance, in: Journal of Banking and Finance, Vol. 15, pp. 847–874.

Berger, Allen N./Herring, Richard J./Szegö, Giorgio P. (1995), The Role of Capital in Financial Institutions, in: Journal of Banking and Finance, Vol. 19, pp. 393–430.

Besanko, David/Kanatas, George (1996), The Regulation of Bank Capital: Do Capital Standards Promote Bank Safety?, in: Journal of Financial Intermediation, Vol. 5, pp. 160–183.

Blum, Jürg (1999), Do Capital Requirements Reduce Risks in Banking?, in: Journal of Banking and Finance, Vol. 23, pp. 755–771.

Bock, Kurt F. (1995), The Impact of Risk-based Capital Requirements on Bank Risk: An Empirical Analysis.

Buser, Stephen A./Chen, Andrew H./Kane, Edward J. (1981), Federal Deposit Insurance, Regulatory Policy, and Optimal Bank Capital, in: Journal of Finance, Vol. 35, pp. 51–60.

Carisano, Rita (1992), Deposit Insurance: Theory, Policy and Evidence.

Chan, Yuk-Shee/Greenbaum, Stuart I./Thakor, Anjan V. (1992), Is Fairly Priced Deposit Insurance Possible?, in: Journal of Finance, Vol. 47, pp. 227–245.

Crouhy, Michel/Galai, Dan (1991), A Contingent Claim Analysis of a Regulated Depository Institution, in: Journal of Banking and Finance, Vol. 15, pp. 73–90.

Dothan, Uri/Williams, Joseph (1980), Banks, Bankruptcy, and Public Regulation, in: Journal of Banking and Finance, Vol. 4, pp. 65–87.

Ediz, Tolga/Michael, Ian/Perraudin, William (1998), Bank Capital Dynamics and Regulatory Policy, Working Paper.

Fischer, Edwin O./Grünbichler, Andreas (1991), Risikoangepaßte Prämien für die Einlagensicherung in Deutschland: Eine empirische Studie, in: Zeitschrift für betriebswirtschaftliche Forschung, Vol. 43, pp. 747–758.

Flannery, Mark J. (1982), Deposit Insurance Creates a Need for Bank Regulation, in: Business Review, Federal Reserve Bank of Philadelphia, pp. 17–27.

Flannery, Mark J. (1989), Capital Regulation and Insured Banks' Choice of Individual Loan Default Risks, in: Journal of Monetary Economics, Vol. 24, pp. 235–258.

Furlong, Frederick T. (1988), Changes in Bank Risk-taking, in: Economic Review, Federal Reserve Bank of San Francisco, pp. 45–56.

Furlong, Frederick T./Keeley, Michael C. (1987), Bank Capital Regulation and Asset Risk, in: Economic Review, Federal Reserve Bank of San Francisco, pp. 20–40.

Furlong, Frederick T./Keeley, Michael C. (1989), Capital Regulation and Bank Risk-taking: A Note, in: Journal of Banking and Finance, Vol. 13, pp. 883–891.

Gennotte, Gerard/Pyle, David (1991), Capital Controls and Bank Risks, in: Journal of Banking and Finance, Vol. 15, pp. 805–824.

Gjerde, Øystein/Semmen, Kristian (1995), Risk-based Capital Requirements and Bank Portfolio Risk, in: Journal of Banking and Finance, Vol. 19, pp. 1159–1173.

Greenbaum, Stuart I./Thakor, Anjan V. (1995), Contemporary Financial Intermediation.

Grenadier, Steven R./Hall, Brian J. (1995), Risk-based Capital Standards and the Riskiness of Bank Portfolios: Credit and Factor Risks, NBER Working Paper 5178, National Bureau of Economic Research.

Hellwig, Martin (1997), Unternehmensfinanzierung, Unternehmenskontrolle und Ressourcenallokation: Was leistet das Finanzsystem?, in: *Gahlen, Bernhard/Hesse, Helmut/Ramser, Hans Jürgen* (eds.), Finanzmärkte, pp. 211–243.

Hirshleifer, Jack (1965), Investment Decision under Uncertainty: Choice-theoretic Approaches, in: Quarterly Journal of Economics, Vol. 79, pp. 509–536.

Hirshleifer, Jack (1966), Investment Decision under Uncertainty: Applications of the State-preference Approach, in: Quarterly Journal of Economics, Vol. 80, pp. 252–277.

Holtorf, Claudia/Rudolf, Markus (1999), Editorial: Zum 25. Geburtstag des Einlagensicherungsfonds in Deutschland, in: Finanzmarkt und Portfolio Management, Vol. 13, pp. 249–254.

Homölle, Susanne (1999), Eigenkapitalregulierung und Risikoübernahme von Kreditinstituten.

Homölle, Susanne (2000), Bank Capital Regulation, Subordinated Uninsured Debt, and Asset Risk, Working Paper.

Kahane, Yehuda (1977), Capital Adequacy and the Regulation of Financial Intermediaries, in: Journal of Banking and Finance, Vol. 1, pp. 207–218.

Kareken, John H./Wallace, Neil (1978), Deposit Insurance and Bank Regulation: A Partial-equilibrium Exposition, in: Journal of Business, Vol. 51, pp. 413–438.

Karels, Gordon V./Prakash, Arun J./Roussakis, Emmanuel (1989), The Relationship between Bank Capital Adequacy and Market Measures of Risk, in: Journal of Business Finance and Accounting, Vol. 16, pp. 663–673.

Kendall, Sarah B. (1991), Bank Regulation under Nonbinding Capital Guidelines, in: Journal of Financial Services Research, Vol. 5, pp. 275–286.

Kim, Daesik/Santomero, Anthony M. (1988), Risk in Banking and Capital Regulation, in: Journal of Finance, Vol. 43, pp. 1219–1233.

Koehn, Michael/Santomero, Anthony M. (1980), Regulation of Bank Capital and Portfolio Risk, in: Journal of Finance, Vol. 35, pp. 1235–1244.

Lam, Chun H./Chen, Andrew H. (1985), Joint Effects of Interest Rate Deregulation and Capital Requirements on Optimal Bank Portfolio Adjustments, in: Journal of Finance, Vol. 40, pp. 563–575.

Merton, Robert C. (1977), An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantees, in: Journal of Banking and Finance, Vol. 1, pp. 3–11.

Merton, Robert C. (1978), On the Cost of Deposit Insurance When There Are Surveillance Costs, in: Journal of Business, Vol. 51, pp. 439–452.

Merton, Robert C. (1989), On the Application of the Continuous-Time Theory of Finance to Financial Intermediation and Insurance, in: The Geneva Papers on Risk and Insurance, Vol. 14, pp. 225–261.

Myers, Stewart C. (1968), A Time-state-preference Model of Security Valuation, in: Journal of Financial and Quantitative Analysis, Vol. 3, pp. 1–34.

Pyle, David H. (1984), Deregulation and Deposit Insurance Reform, in: Economic Review, Federal Reserve Bank of San Francisco, pp. 5–15.

Raab, Martin/Schwager, Robert (1993), Spanning with Short-Selling Restrictions, in: The Journal of Finance, Vol. 48, pp. 791–793.

Rochet, Jean-Charles (1992), Capital Requirements and the Behaviour of Commercial Banks, in: European Economic Review, Vol. 36, pp. 1137–1170.

Sharpe, William F. (1978), Bank Capital Adequacy, Deposit Insurance and Security Values, in: Journal of Financial and Quantitative Analysis, Vol. 13, pp. 701–718.

Wall, Larry D./Peterson, Pamela P. (1996), Banks' Responses to Binding Capital Regulatory Capital Requirements, in: Economic Review, Federal Reserve Bank of Atlanta, Vol. 81, pp. 1–17.

Weiland, Christiane (1999), Die Wirkung bankaufsichtlicher Eigenkapitalanforderungen - eine Untersuchung induzierter Verhaltensänderungen.

Zarruk, Emilio R. (1989), Bank Spread with Uncertain Deposit Level and Risk Aversion, in: Journal of Banking and Finance, Vol. 13, pp. 797–810.