Internet Appendix to
"The Effects of Oil Inventories on Growth Prospects, Futures Markets, and Risk Premia" *

A. Solution Method

To account for the non-linearities arising close to the occasionally binding constraint for oil inventories, the model is solved using a global non-linear solution algorithm. As in Judd (1992), we use projection methods to approximate the policy functions for investment (I_t^k, I_t^o, I_t^p) and optimal oil usage (O_t). Further, we approximate the households continuation utility ($\mathcal{U}_t$), as well as the value of a patent ($\mathcal{J}_t$) and the shadow price of one more unit of oil in inventory (λ_t^s). We thus determine

$$\{I_t^k, I_t^o, I_t^p, O_t, \mathcal{U}_t, \mathcal{J}_t^P, \lambda_t^s\}_{\{0 \leq t \leq \infty\}} \tag{IA.1}$$

when we solve for the equilibrium.

Due to recursive preference, a very small discount rate, persistent growth patterns, and of course, the non-negativity constraint on commodities storage, convergence proved to be very hard to achieve. Even though piece-wise approximation would usually be the suitably approach for problems involving occasionally binding constraints, we choose Chebyshev polynomials as our set of basis functions due to their nice properties of convergence. Ultimately, we solve the equilibrium conditions directly with `MATLAB`'s *fmincon*-solver, which proves to be extremely stable. To reduce the computational burden, we use orthogonal collocation, which allows us to have exactly as many points on our approximating grid as our degree of approximation. Finally, *fmincon* allows parallel function evaluation.
We detrend all variables with growth of the R&D stock and are thus left with 4 state variables in our stationary equilibrium. The vector of state variables in logs is given by

$$\{a_t, \hat{k}_t^k, \hat{k}_t^o, \hat{s}_t\}_{\{0 \leq t \leq \infty\}}, \tag{IA.2}$$

which completely describes the state of our economy at any point in time.

We find, that 4^{th} order approximations suffice for detrended physical capital ($\hat{k}_t^k$) and productivity (a_t). In contrast, policy functions become increasingly non-linear up to possibly being kinked in the directions of oil producing capital and the stock of inventories. Consequentially, the approximating degree in those two directions is of order 9 to obtain a sufficiently good fit to the equilibrium. We find that increasing the degree of approximation for the states a and $\hat{k}^k$ does not pose a significant burden on the solver, while $\hat{k}^o$ and $\hat{s}$ typically take more iteration steps and can only be increased jointly. Table IA.1 reports Euler equation errors for investments into the three types of capital in the model. We compute these errors on a grid with 50 points in each dimension. Euler equation errors are then reported as maximum and mean errors on the grid. As a third measure, we use a long time series of 10,000 periods and evaluate the average absolute deviation over the whole sample path. Overall, errors tend to be higher for investment into the stock of inventories than for the investment decisions. Figure IA.1 depicts absolute Euler equation errors four all four dimensions.

The interval bounds for the exogenous state variables are ± 3.25 times the unconditional volatility, such that they cover 99.95% of probability mass. The bounds for the endogenous state variables are chosen such that they will not be violated during the simulations. Expectations are computed with a 5-node *Gauss-Hermite* quadrature.

We simulate the model quarterly over 80 years (320 periods) and use an additional burn-in period of 500 quarters. The results are based on 10,000 paths. The model is solved using the algorithm developed in Gräber and Schumacher (2017).

Finally, in a second step, we approximate conditional financial variables, such as futures prices, the price of the aggregate market claim, the associated premia, and the risk-free rate in the same fashion as we have solved for the

*Internet Appendix to Branger, Gräber, and Schumacher (2017).

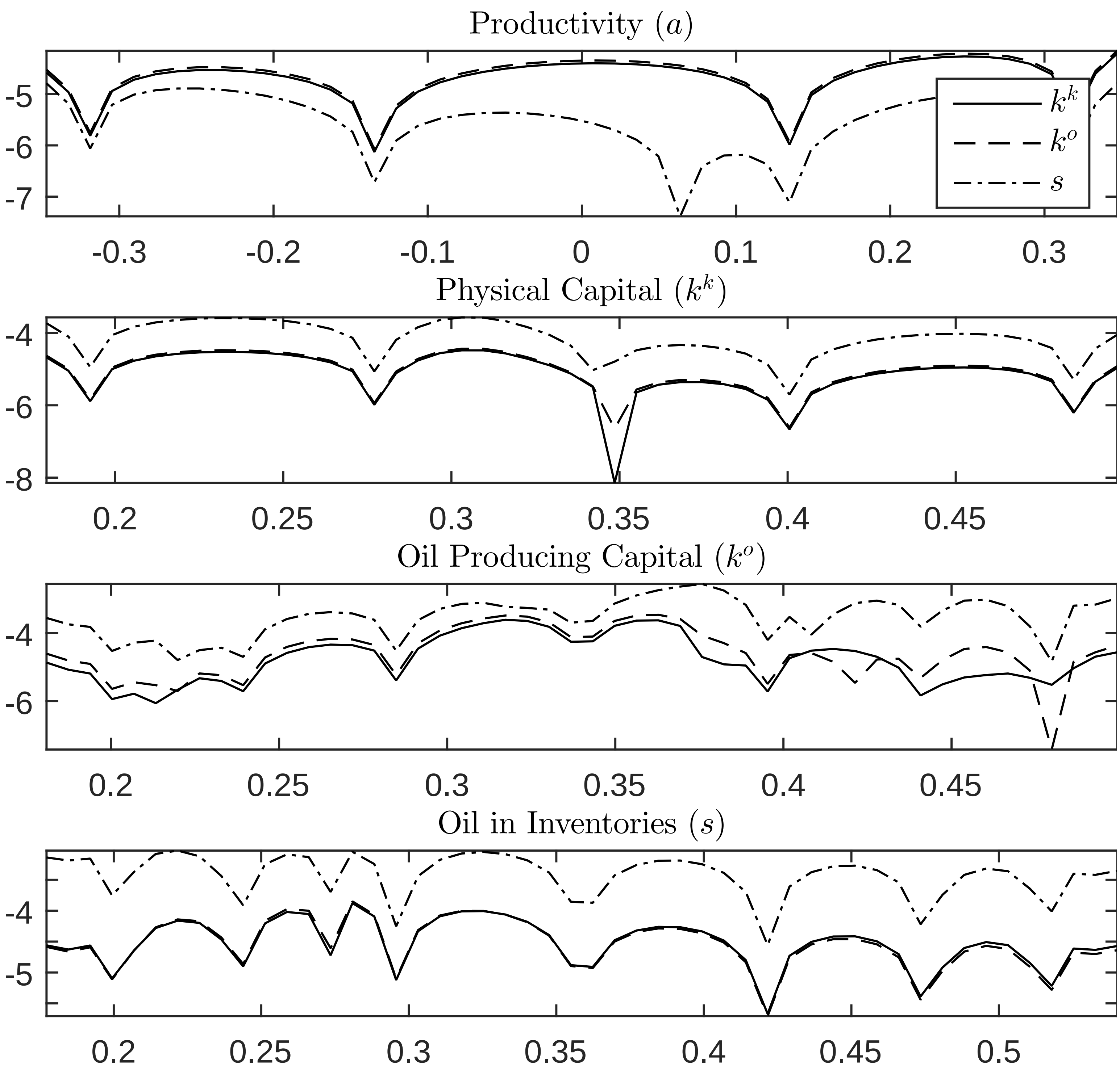

Figure IA.1. Euler Equation Errors (Full Model)

This figure depicts Euler equation errors as $\log_{10}(|EEE|)$ for the return on physical capital k^k, oil producing capital k^o, and oil in inventories s for the Full model. Euler equation errors are computed for a grid of 50 points in each direction. The upper panel depicts Euler errors for the grid of productivity (a). The second panel depicts Euler errors for the grid of physical capital (k^k). The third panel depicts Euler errors for the grid of oil producing capital capital (k^o). The fourth panel depicts Euler errors for the grid of oil in inventories (s). For productivity level upper and lower bounds are at ± 3.25 standard deviations.

	Model		
	Full	TtB	NoS
Approximation Order	4.4.9.9	4.4.9.9	4.4.7
Physical Capital			
$\log_{10}(\|\|EEE\|\|_\infty)$	-3.07	-5.89	-6.50
$\log_{10}(\|\|EEE\|\|_1)$	-4.04	-6.43	-6.79
$\log_{10}(\|\|EEE\|\|_1^{simul})$	-4.05	-6.70	-6.73
Oil Producing Capital			
$\log_{10}(\|\|EEE\|\|_\infty)$	-2.91	-5.72	-6.44
$\log_{10}(\|\|EEE\|\|_1)$	-3.94	-6.11	-6.75
$\log_{10}(\|\|EEE\|\|_1^{simul})$	-3.89	-6.24	-6.73
Oil in Inventories			
$\log_{10}(\|\|EEE\|\|_\infty)$	-1.92	-4.40	–
$\log_{10}(\|\|EEE\|\|_1)$	-3.01	-4.78	–
$\log_{10}(\|\|EEE\|\|_1^{simul})$	-2.98	-4.83	–

Table IA.1 Euler Equation Errors (All Growth Models)

This table depicts Euler equation errors for all three growth models. Full is the benchmark model, TtB the *time-to-build* model, and NoS the *no storage* model. The top panel depicts Euler equation errors for the return on physical capital. The mid panel depicts Euler equation errors for the return on oil related capital. The bottom panel depicts Euler equation errors for the return on oil in inventories. Euler equation errors are reported with respect to the $\mathcal{L}^1$-norm for the mean error on the grid and simulation. Errors reported are in order, the maximum error on the grid ($\log_{10}(||EEE||_\infty)$), average error on the grid ($\log_{10}(||EEE||_1)$), and average error for a sample path of 10,000 periods ($\log_{10}(||EEE||_1^{simul})$).

dynamic equilibrium.

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