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Abstract

We propose four multiplicative-component volatility MIDAS models for disentangling short- and long-term volatility sources. Three of our models specify short-term volatility as Markov-switching processes. We establish statistical properties, covariance-stationarity conditions, and an estimation framework, using regime-switching filter techniques. A simulation study shows the robustness of the estimates against several mis-specifications. An out-of-sample forecasting analysis with daily S&P500 returns and quarterly-sampled (macro)economic variables yields two major results. (i) Specific long-term variables in the MIDAS models significantly improve forecast accuracy (over the non-MIDAS benchmarks). (ii) We find superior and robust performance of one Markov-switching MIDAS specification (among a set of competitor models) when using the 'term structure' as the long-term variable.

Keywords: MIDAS volatility modeling; Hierarchical hidden Markov models; Markov-switching; Forecasting; Model confidence sets.

JEL classification: C51, C53, C58, E44.

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1 Introduction

The last four decades have expedited the already substantial progress in modeling and forecasting financial-market volatility, predominantly inspired by stylized empirical facts like *volatility clustering* and *kurtotic nature* of return-distributions (Cont, 2001). The most popular frameworks—the GARCH-type and Stochastic Volatility (SV) models, both in univariate and multivariate variants—have contributed profoundly to our understanding of financial-market volatility at relatively short horizons (for a recent overview and current research lines, see Zaharieva et al., 2020). While these approaches are capable of reproducing important statistical volatility features, they are less suited (i) to identifying major volatility sources, and (ii) to explaining why volatility typically changes over time. Aiming to tackle these latter issues, several authors suggest modeling volatility as multiple heterogeneous components and correspondingly document notable forecast-accuracy gains via this approach (Ding and Granger, 1996; Engle and Lee, 1999; Gallant et al., 1999; Alizadeh et al., 2002; Chernov et al., 2003; Adrian and Rosenberg, 2008). Despite such improvements, controversy remains on how to specify the dynamics of the respective components. Early proposals already emphasize the role of the macroeconomic environment in driving long-term movements of financial-market volatility (Officer, 1973; Schwert, 1989) and, following this line of argument, we propose some novel multi-component volatility specifications with explicit economic volatility drivers.

In line with the multiplicative-component Mixed Data Sampling (MIDAS) framework in Engle et al. (2013), our modeling approach also decomposes financial-market volatility into (i) a short-term high-frequency component reflecting financial-market characteristics, and (ii) a long-term (low-frequency) component (representing macroeconomic impacts). Engle et al. (2013), Conrad and Loch (2015), and Conrad and Kleen (2020) represent short-term volatility with standard GARCH and GJR-GARCH processes (Bollerslev, 1986; Glosten et al., 1993), thus constituting the GARCH-MIDAS and GJR-GARCH-MIDAS models.[1] In this paper, we complement this family of volatility MIDAS models by employing the following four short-term specifications: (1) the Markov-Switching-Multifractal (MSM) model (Calvet and Fisher, 2004), (2) the Factorial Hidden Markov Volatility (FHMV) model (Augustyniak et al., 2019), (3) the Markov-Switching GARCH (MSGARCH) model (Haas et al., 2004), and (4) the Hyperbolic GARCH (HYGARCH) model (Davidson, 2004). Beyond the volatility MIDAS

[1] See Wei et al. (2017), Fang et al. (2020) for recent applications.

framework, the first two models (MSM, FHMV) have turned out to possess salient volatility forecasting properties (Calvet and Fisher, 2004; Lux, 2008; Lux et al., 2016; Segnon et al., 2017; Augustyniak et al., 2019), and—per design—offer a unified framework for modeling (i) persistence in the volatility process, and (ii) structural breaks through regime-switching. In order to account for nonlinearity and long memory in the volatility process, we additionally embed the models (3) and (4) (MSGARCH, HYGARCH) into the multiplicative-component MIDAS framework.

In the subsequent sections, we (i) formalize our four volatility MIDAS specifications (MSM-, FHMV-, MSGARCH-, HYGARCH-MIDAS), (ii) explore their statistical properties (stationarity conditions, autocorrelation and moment structures), and (iii) establish the estimation framework. In our empirical out-of-sample investigation, we analyze daily cumulative variance forecasts (at quarterly forecast horizons) of S&P500 returns (observed over a 40-year time span). We apply predicitve ability tests (Giacomini and White, 2006) and model confidence sets (Hansen et al., 2011) to compare the forecast accuracy of our volatility MIDAS specifications with those of several conventional benchmark models. Using the robust MSE and QLike loss functions, we make inferences as to whether the inclusion of macroeconomic information leads to forecast accuracy gains. Our investigation has three major findings. (i) Several macroeconomic variables have substantial predictive content for financial-market volatility. (ii) The MSGARCH-MIDAS model has generally good forecasting properties at the 1-quarter (the shortest) horizon. (iii) There are two (macroeconomic) long-term variables ('Term spread', 'Housing starts'), which have the potential to improve the forecast accuracy of several volatility MIDAS models. This is unambiguously true for the FHMV-MIDAS specification across all forecast horizons.

The remainder of the paper is organized as follows. Section 2 presents the modeling framework and statistical properties of the novel volatility MIDAS specifications. Section 3 establishes the estimation approach and provides simulation results. In Section 4, we describe our data set, present the forecast evaluation strategy, and conduct the out-of-sample forecasting analysis, using S&P500 returns and macroeconomic data. Section 5 concludes.

2 Basic volatility MIDAS setup

Let $\{r_{i,t}\}$ represent a financial process of daily (log) returns, where the subscript i denotes a day within period t ($i = 1, \ldots, N^{(t)}; t = 1, \ldots, T$). We assume the conditional mean return, $\mu \equiv \mathbb{E}_{i-1,t}(r_{i,t})$, to be constant over time. As in Engle and Rangel (2008) and Engle et al. (2013), we specify

$$r_{i,t} - \mu = \sqrt{\tau_t \cdot g_{i,t}} \cdot \varepsilon_{i,t}, \tag{1}$$

with innovation process $\{\varepsilon_{i,t}\} \overset{\text{i.i.d.}}{\sim} (0, 1)$. The volatility term, $\sigma_{i,t} \equiv \sqrt{\tau_t \cdot g_{i,t}}$, consists of the long-term and short-term components τ_t and $g_{i,t}$. We consider long-term volatility movements as constant during period t, and write $\tau_t = \tau_{i,t}$ for $i = 1, \ldots, N^{(t)}$. We adopt the convention that the start of period t exactly coincides with the end of the previous period $t-1$ plus 1 day, i.e. $r_{N^{(t-1)}+1,t-1} = r_{1,t}$.

Following the MIDAS framework in Engle et al. (2013), we assume the long-term component τ_t to be the weighted K-lag average of an explanatory variable $\{X_t\}$. To ensure positivity, we specify the logarithm of the long-term component as

$$\log(\tau_t) = m + \theta \sum_{k=1}^{K} \varphi_k(\omega_1, \omega_2) X_{t-k}, \tag{2}$$

with $\varphi_k(\omega_1, \omega_2)$ chosen as the positive beta-weights

$$\varphi_k(\omega_1, \omega_2) = \frac{\left(\frac{k}{K+1}\right)^{\omega_1 - 1} \cdot \left(1 - \frac{k}{K+1}\right)^{\omega_2 - 1}}{\sum_{j=1}^{K} \left(\frac{j}{K+1}\right)^{\omega_1 - 1} \cdot \left(1 - \frac{j}{K+1}\right)^{\omega_2 - 1}}, \tag{3}$$

which sum to unity for $k = 1, \ldots, K$. This beta-weighting scheme generates extremely flexible weights (Ghysels et al., 2007), with (i) equal weights for all lags k when $\omega_1 = 1$ and $\omega_2 = 1$, (ii) monotonically decreasing weights for increasing lags k when $\omega_1 = 1$ and $\omega_2 > 1$, and (iii) hump-shaped and convex weights for unrestricted parameters. In Case (ii), ω_2 determines the rate of weight-decline, i.e. the larger ω_2, the faster the decrease of the weights φ_k. In Case (iii), the ratio of ω_1/ω_2 indicates whether higher weights are allocated to more recent or more distant lags.

Next, we address the modeling of the short-term component $\{g_{i,t}\}$ in Eq. (1), leading to the MSM-, FHMV-, MSGARCH-, and HYGARCH-MIDAS specifications.

2.1 MSM-MIDAS

In the Markov-Switching Multifractal (MSM) model, the short-term volatility process $\{g_{i,t}\}$ is modeled as the product of the N_M random volatility components called multipliers, $\{M_{i,t}^{(1)}\}, \{M_{i,t}^{(2)}\}, \ldots, \{M_{i,t}^{(N_M)}\}$,

$$g_{i,t} = \prod_{j=1}^{N_M} M_{i,t}^{(j)}, \tag{4}$$

where the specific dynamics of the multipliers constitute the multifractality of the model. At date i in period t, each multiplier $M_{i,t}^{(j)}$ is drawn from a base distribution M with positive support (specified below). Depending on its rank in the hierarchy of multipliers, $M_{i,t}^{(j)}$ changes from one date (index i) to the next with probability γ_j, or remains unchanged with probability $1 - \gamma_j$, which generates a broad spectrum of multiplier renewal.

Specifically, the parametric approach of Calvet and Fisher (2004), which ensures the convergence of the discrete-time MSM model to the continuous-time Poisson multifractal process, specifies the N_M transition probabilities as

$$\gamma_j = 1 - (1 - \gamma_{N_M})^{(b^{j-N_M})}, \qquad (j = 1, \ldots, N_M) \tag{5}$$

where $\gamma_j \in (0,1), b > 1$. Eq. (5) establishes a hierarchical structure among the multipliers, as $\gamma_1 < \gamma_2 < \ldots < \gamma_{N_M} < 1$. Thus, the multiplier $M_{i,t}^{(N_M)}$ has the highest probability of being renewed, $M_{i,t}^{(1)}$ has the lowest, and the transition matrix of the j-th multiplier is given by

$$\mathbf{P}^{(j)} = \begin{pmatrix} 1 - 0.5\gamma_j & 0.5\gamma_j \\ 0.5\gamma_j & 1 - 0.5\gamma_j \end{pmatrix}. \tag{6}$$

To complete the MSM-MIDAS setup, we need to specify the base distribution M. In case of a change, each multiplier $M_{i,t}^{(j)}$ is drawn from a discrete distribution with the two mass points $\{m_0, 2 - m_0\}, 1 < m_0 < 2$, and point probabilities $\mathbb{P}\text{rob}(M_{i,t}^{(j)} = m_0) = \mathbb{P}\text{rob}(M_{i,t}^{(j)} = 2 - m_0) = 0.5$, implying the unconditional expectation $\mathbb{E}(M_{i,t}^{(j)}) = 1$ for $j = 1, \ldots, N_M$.[2] Assuming stochastic independence among the contemporaneous multipliers, we obtain the transition probability matrix of the short-term volatility process $\{g_{i,t}\}$ as the $2^{N_M} \times 2^{N_M}$ matrix $\mathbf{P}_M = \mathbf{P}^{(1)} \otimes \mathbf{P}^{(2)} \otimes \ldots \otimes \mathbf{P}^{(N_M)}$, and $\{g_{i,t}\}$ has the finite support $\mathcal{X}_M = \{(m_0, 2 - m_0)^{\otimes N_M}\}$.[3]

[2] Liu et al. (2008) report that alternative base distributions like the lognormal and gamma yield very similar results in empirical applications.

[3] We denote the Kronecker product by $\otimes$, and the N-fold Kroecker power of $\mathbf{X}$ by $\mathbf{X}^{\otimes N} \equiv \mathbf{X} \otimes \ldots \otimes \mathbf{X}$. We

To state important properties of the Markov chain $\{g_{i,t}\}$, let $\mathbf{1}_n$ denote the $n \times 1$ column vector of ones. The parameter restrictions $\gamma_j \in (0,1), b > 1$ in Eqs. (5) and (6) ensure that the transition probability matrix $\mathbf{P}_M$ satisfies Requirement (21) in Shiryaev (1996, Theorem 1, pp. 118-120). It follows directly that $\{g_{i,t}\}$ (i) has the stationary distribution $\boldsymbol{\pi}^{\text{MSM}} = (1/2)^{2^{N_M}} \mathbf{1}_{2^{N_M}}$, and (ii) is geometrically ergodic. We collect all parameters of the MSM-MIDAS model in the vector $\boldsymbol{\Theta}^{\text{MSM}} = (m_0, \gamma_{N_M}, b, \mu, m, \theta, \omega_1, \omega_2)'$.

2.2 FHMV-MIDAS

In the Factorial Hidden Markov Volatility (FHMV) model (Augustyniak et al., 2019), the short-term volatility process $\{g_{i,t}\}$ is represented as the product of two stochastically independent components,

$$g_{i,t} = C_{i,t} \cdot Z_{i,t}. \tag{7}$$

$\{C_{i,t}\}$ has the function of capturing persistent impacts on the volatility, while the jump component $\{Z_{i,t}\}$ captures non-persistent impacts. In Eq. (7), $\{C_{i,t}\}$ is modeled as the product of the N_C independent volatility multipliers $\{C_{i,t}^{(1)}\}, \{C_{i,t}^{(2)}\}, \ldots, \{C_{i,t}^{(N_C)}\}$,

$$C_{i,t} = c_0 \prod_{j=1}^{N_C} C_{i,t}^{(j)}, \tag{8}$$

where the normalizing constant $c_0 = \left[\mathbb{E}\left(\prod_{j=1}^{N_C} C_{i,t}^{(j)}\right)\right]^{-1}$ ensures $\mathbb{E}(C_{i,t}) = 1$. The N_C multipliers $\{C_{i,t}^{(j)}\}$ are all assumed to have the transition probability matrix

$$\mathbf{P} = \begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix}, \qquad p \in (0,1). \tag{9}$$

A hierarchical structure among the multipliers is induced by their distinct supports, which (for $j = 1, \ldots, N_C$ and given parameter $c_1 > 1$) are recursively defined as

$$\text{supp}(C_{i,t}^{(j)}) = \{c_j, 1\} \quad \text{with} \quad c_j = 1 + \theta_C^{j-1}(c_1 - 1), \quad \theta_C \in [0,1]. \tag{10}$$

Eq. (10) implies $c_1 \geq c_2 \geq \ldots \geq c_{N_C} \geq 1$, so that the first multiplier $C_{i,t}^{(1)}$ has the highest impact on the volatility, when activated. The component $\{C_{i,t}\}$ from Eq. (8) constitutes a

use the $\otimes$-operator in matrix expressions, and likewise in the representation of sets.

Markov chain with state space (of cardinality 2^{N_C})

$$\mathcal{X}_C = \{c_0 \cdot (c_1, 1) \otimes (c_2, 1) \otimes \ldots \otimes (c_{N_C}, 1)\}, \tag{11}$$

and the associated $2^{N_C} \times 2^{N_C}$ transition probability matrix

$$\mathbf{P}_C = \mathbf{P}^{\otimes N_C}. \tag{12}$$

The component $\{Z_{i,t}\}$ in Eq. (7) is assumed to have the support

$$\mathcal{X}_Z = \{z_0 \cdot z_1, z_0 \cdot z_2, \ldots, z_0 \cdot z_{N_z-1}, z_0 \cdot z_{N_Z}\}, \tag{13}$$

(z_0 a normalizing constant defined below), where—for a given $q \in (0,1)$—(i) each of the first $(N_Z - 1)$ outcomes occurs with probability $\frac{q}{N_Z-1}$, and (ii) the outcome $z_0 z_{N_Z}$ occurs with probability $1-q$. Per construction, the probability of a $\{Z_{i,t}\}$-renewal is path-independent, so that $\{Z_{i,t}\}$ is able to capture sudden and non-persistent volatility jumps. Formally, for a given prespecified individual outcome $z_1 > 1$, the $N_Z - 2$ non-normalized outcomes $z_2, \ldots, z_{N_Z-1}$ are specified as

$$z_j = 1 + \theta_Z^{j-1}(z_1 - 1), \quad \theta_Z \in [0, 1]. \tag{14}$$

In order to ensure $\mathbb{E}(Z_{i,t}) = 1$, the normalizing constant is set to

$$z_0 = \left[1 + q\frac{(z_1 - 1)(1 - \theta_Z^{N_Z-1})}{(N_Z - 1)(1 - \theta_Z)}\right]^{-1}.$$

The final outcome is set to $z_{N_Z} = 1$, so that the entire specification exhibits the hierarchical structure $z_1 \geq z_2 \geq \ldots \geq z_{N_Z} = 1$. $\{Z_{i,t}\}$ can be represented as a Markov chain with state space $\mathcal{X}_Z$ from Eq. (13), and $N_Z \times N_Z$ transition probability matrix

$$\mathbf{P}_Z = \mathbf{1}_{N_Z}\boldsymbol{\pi}'_Z, \tag{15}$$

with $N_Z \times 1$ vector $\boldsymbol{\pi}_Z = (\frac{q}{N_Z-1}, \ldots, \frac{q}{N_Z-1}, 1 - q)'$.

Following the approach of Augustyniak et al. (2019), we assume that the short-term volatility process $\{g_{i,t}\}$ from Eq. (7) is itself a Markov chain. In view of Eqs. (11), (12), (13), and (15), its state space (of cardinality $N_Z \cdot 2^{N_Z}$) is given by $\mathcal{X}_g = \mathcal{X}_C \otimes \mathcal{X}_Z$, and $\{g_{i,t}\}$ has the $N_Z \cdot 2^{N_C} \times N_Z \cdot 2^{N_C}$ transition probability matrix $\mathbf{P}_g = \mathbf{P}_C \otimes \mathbf{P}_Z$. Owing to the parameter restrictions $p, q \in (0,1)$ in Eqs. (9), (13), Theorem 1 in Shiryaev (1996, pp. 118-120), shows that the Markov chain $\{g_{i,t}\}$ is (i) geometrically ergodic, and (ii) has the stationary distribution $\boldsymbol{\pi}^{\text{FHMV}} = \boldsymbol{\pi}_C \otimes \boldsymbol{\pi}_Z$, where $\boldsymbol{\pi}_C = (1/2)^{2^{N_C}} \mathbf{1}_{2^{N_C}}$. We collect all parameters of the FHMV-MIDAS model in the vector $\boldsymbol{\Theta}^{\text{FHMV}} = (c_1, \theta_C, p, z_1, \theta_Z, q, \mu, m, \theta, \omega_1, \omega_2)'$.

2.3 MSGARCH-MIDAS

Haas et al. (2004) propose a k-regime Markov-switching GARCH (MSGARCH) framework, which has proved to be extremely powerful in empirical applications. Using their setup, we specify the MSGARCH(k)-MIDAS model with (k-dimensional) short-term volatility process

$$\mathbf{g}_{i,t} = \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 \frac{(r_{i-1,t} - \mu)^2}{\tau_t} + \boldsymbol{\beta} \mathbf{g}_{i-1,t}, \tag{16}$$

$\mathbf{g}_{i,t} = \begin{pmatrix} g_{i,t}^{(1)} & \dots & g_{i,t}^{(k)} \end{pmatrix}'$, $\boldsymbol{\alpha}_0 = \begin{pmatrix} \alpha_{01} & \dots & \alpha_{0k} \end{pmatrix}'$, $\boldsymbol{\alpha}_1 = \begin{pmatrix} \alpha_{11} & \dots & \alpha_{1k} \end{pmatrix}'$, $\boldsymbol{\beta} = \text{diag}(\beta_1, \beta_2, \dots, \beta_k)$, with $\boldsymbol{\alpha}_0 = \mathbf{1}_k - \boldsymbol{\alpha}_1 - \boldsymbol{\beta}\mathbf{1}_k$, and where the (element-wise) inequalities $\boldsymbol{\alpha}_0 > \mathbf{0}_k$, $\boldsymbol{\alpha}_1, \boldsymbol{\beta}\mathbf{1}_k \geq \mathbf{0}_k$ are assumed to hold in order ensure the positivity of the variance process.[4]

The regime dynamics are governed by the Markov-chain $\{\Delta_{i,t}\}$ with state space $S = \{1, 2, \dots, k\}$, and we assume that $\{\Delta_{i,t}\}$ and the innovation process $\{\varepsilon_{i,t}\}$ from Eq. (1) are independent. Subsequently, we denote the irreducible, aperiodic $k \times k$ transition probability matrix of $\{\Delta_{i,t}\}$ by

$$\mathbf{P}_\Delta = (p_{jl}) = \left(\Pr\{\Delta_{i,t} = l | \Delta_{i-1,t} = j\}\right), \qquad j, l = 1, \dots, k. \tag{17}$$

2.4 HYGARCH-MIDAS

Based on Davidson's (2004) hyperbolic GARCH (HYGARCH) definition, we formalize our HYGARCH-MIDAS model via its ARCH(∞) representation (Conrad 2010, Eq. (9)). Using the lag-operator polynomials $B(\mathbb{L}) = 1 - \beta\mathbb{L}$ and $\Phi(\mathbb{L}) = 1 - \phi\mathbb{L}$ with $|\beta| < 1, |\phi| < 1$, we specify the dynamics of the short-term volatility process via the HYGARCH$(1,1)$ representation

$$g_{i,t} = (1 - \Psi(1)) + \Psi(\mathbb{L}) \frac{(r_{i,t} - \mu)^2}{\tau_t}, \tag{18}$$

[4] The operator $\text{diag}(\beta_1, \beta_2, \dots, \beta_k)$ creates the diagonal $k \times k$ matrix $\boldsymbol{\beta}$ with $\boldsymbol{\beta}_{ii} = \beta_i$ and $\boldsymbol{\beta}_{ij} = 0$ for $i \neq j$ $(i, j = 1, \dots, k)$. $\mathbf{0}_k$ is the k-dimensional vector of zeros (analogous to $\mathbf{1}_k$).

where

$$\Psi(\mathbb{L}) = 1 - \frac{\Phi(\mathbb{L})}{B(\mathbb{L})}\left\{1 - \kappa\left[1 - (1-\mathbb{L})^d\right]\right\}$$
$$= \psi_1 \mathbb{L} + \psi_2 \mathbb{L}^2 + \psi_3 \mathbb{L}^3 + \cdots, \tag{19}$$

for $d \in (0,1), \kappa \in (0,1)$.

The non-negative ψ_i-coefficients in the series expansion (19) may be obtained via the recursion $\psi_1 = \phi - \beta + \kappa d$, $\psi_i = \beta\psi_{i-1} + \left(\frac{i-1-d}{i} - \phi\right)\chi_{d,i-1}$ for $i = 2, \ldots, \infty$, where $\chi_{d,i} = \chi_{d,i-1}(i-1-d)/i$ with $\chi_{d,1} = \kappa d$.[5] Parameters d and κ represent the degree of fractional differentiation and the process amplitude. Parameters β, ϕ, d, κ satisfy the following sufficient conditions to guarantee the non-negativity of the ψ_i coefficients:

$$\beta - \kappa d \leq \phi \leq \frac{(2-d)}{3} \qquad \text{and} \qquad \kappa d\left[\phi - \frac{(1-d)}{2}\right] \leq \beta(\kappa d - \beta + \phi). \tag{20}$$

(For parameter restrictions of general HYGARCH specifications, see Conrad, 2010.) For $d = 0$ or $\kappa = 1$, which are inadmissible parameter values in our setup, the HYGARCH-MIDAS model would reduce to GARCH-MIDAS.

2.5 Statistical properties

We establish some statistical properties of the four volatility MIDAS models (Propositions 1–3). The proofs, which we give in the Appendix, are based on the following assumptions.

Assumption 1. *For the innovation process in Eq. (1), we stipulate* $\{\varepsilon_{i,t}\} \overset{\text{i.i.d.}}{\sim} \mathbb{N}(0,1)$ *(standard normal distribution).* $\{\varepsilon_{i,t}\}$ *is independent of the long-term and short-term volatility components* $\{\tau_t\}, \{g_{i,t}\}$*.*

Assumption 2. *For the long-term component* $\{\tau_t\}$ *and the involved explanatory variable* $\{X_t\}$ *in Eqs. (1)–(3), we assume that* $\{X_t\}$ *is a strictly stationary and ergodic process with* $\mathbb{E}(|X_t|^k) = \text{const.} < \infty$*, where (the integer-valued)* k *is sufficiently large to ensure* $\mathbb{E}(\tau_t^2) = \text{const.} < \infty$*.* $\{X_t\}$ *is assumed to be independent of* $\{\varepsilon_{i,t}\}$ *for all* i, t*.*

Assumption 3. *For each of the MSM-MIDAS and FHMV-MIDAS models in Sections 2.1*

[5] In our applications, we impose a truncation at lag 1000.

and 2.2, we assume the following. (i) The short-term and the long-term components $\{g_{i,t}\}$ and $\{\tau_t\}$ are stochastically independent for all i, t. (ii) The Markov chain $\{g_{i,t}\}$ is initialized from its stationary distribution (implying strict stationarity).

We obtain the following stationarity result for the MSM-, FHMV-, and HYGARCH-MIDAS models.

Proposition 1. *Under Assumptions 1–3, the MSM-MIDAS, FHMV-MIDAS, and the HYGARCH-MIDAS models from Sections 2.1, 2.2, and 2.4 are covariance stationary.*

Similarly, for the two-regime MSGARCH-MIDAS model, we have the following result.

Proposition 2. *Under Assumptions 1–3, the two-regime ($k = 2$) MSGARCH(2)-MIDAS model from Section 2.3 is covariance stationary.*

The final proposition characterizes the squared-return autocorrelation functions (ACFs) for the MSM- and FHMV-MIDAS models.

Proposition 3. *Under Assumptions 1–3, the ACFs of the squared returns in the MSM-MIDAS and FHMV-MIDAS models for the lags of u days and v periods ($\min\{u, v\} > 0$) can be decomposed as*

$$\begin{aligned} \rho_{r^2}(u, v) &\equiv \mathbb{C}\mathrm{orr}(r_{i,t}^2, r_{i-u,t-v}^2) \\ &= \rho_\tau(v) \frac{\mathbb{V}\mathrm{ar}(\tau_t)}{\mathbb{V}\mathrm{ar}(r_{i,t}^2)} + \rho_g(u, v) \frac{\mathbb{V}\mathrm{ar}(g_{i,t})}{\mathbb{V}\mathrm{ar}(r_{i,t}^2)} \left(\rho_\tau(v) \mathbb{V}\mathrm{ar}(\tau_t) + \mathbb{E}(\tau_t)^2 \right), \end{aligned} \tag{21}$$

where $\rho_\tau(v), \rho_g(u, v)$ denote the ACFs of the long-term and short-term volatility components, respectively. For MSM-MIDAS, we have

$$\begin{aligned} \rho_g(u, v) &\equiv \mathbb{C}\mathrm{orr}(g_{i,t}, g_{i-u,t-v}) \\ &= \frac{\prod_{j=1}^{N_M} \left(1 + (1 - \gamma_j)^{N^{(t-v)} + \ldots + N^{(t-1)} + u} (m_0 - 1)^2 \right) - 1}{((m_0 - 1)^2 + 1)^{N_M} - 1}, \end{aligned} \tag{22}$$

and for FHMV-MIDAS,

$$\begin{aligned}\rho_g(u,v) &\equiv \mathbb{C}\mathrm{orr}(g_{i,t}, g_{i-u,t-v}) \\ &= \frac{\prod_{j=1}^{N_C}\left(1+(2p-1)^{N^{(t-v)}+\ldots+N^{(t-1)}+u}\left(\frac{c_j-1}{c_j+1}\right)^2\right)-1}{\prod_{j=1}^{N_C}\left(1+\left(\frac{c_j-1}{c_j+1}\right)^2\right)\cdot z_0^2\left(\frac{q}{N_Z-1}\sum_{j=1}^{N_Z-1}z_j^2+(1-q)\right)-1}. \end{aligned} \tag{23}$$

Remark. *Eqs. (22), (23) imply that the MSM-MIDAS and FHMV-MIDAS models are characterized by short-memory, since the ACFs decline exponentially rapidly. However, Calvet and Fisher (2004, Proposition 1) and Augustyniak et al. (2019, Theorem 1) show that both non-MIDAS variants are capable of mimicking hyperbolic decay over a wide range of lags.*

3 Simulation study

In this section, we analyze the finite-sample estimation performance of the four volatility MIDAS models (MSM, FHMV, MSGARCH, HYGARCH) within a uniform simulation setup.

3.1 Estimation procedures

We estimate the MSM- and FHMV-MIDAS specifications via analogous regime-switching filtering techniques (Hamilton, 1994) and describe the main steps in one algorithm. We denote the elements of the respective state spaces χ_M (MSM) and χ_g (FHMV) by $h^{(1)},\ldots,h^{(d_1)}$ (MSM) and $h^{(1)},\ldots,h^{(d_2)}$ (FHMV), where $d_1 = 2^{N_M}$ and $d_2 = N_Z \cdot 2^{N_C}$. For $N_M = N_C$, the cardinality of χ_g exceeds that of χ_M, principally rendering the iteration of the Hamilton filter for the FHMV-MIDAS model computationally more demanding. We reduced this burden, making it comparable to that of MSM-MIDAS, by integrating the FHMV jump-component out of the predictive distribution, so that the filter only needs to iterate over 2^{N_C} states. We implemented the filtering and computation of the log-likelihood functions as follows (Hamilton, 1994, pp. 692-694):

1. Initialization:

(MSM) Set $\hat{\boldsymbol{\xi}}_{1,1|0,1} = \boldsymbol{\pi}^{\text{MSM}}$.

(FHMV) Set $\hat{\boldsymbol{\xi}}_{1,1|0,1} = \boldsymbol{\pi}^{\text{FHMV}}$ and $\hat{\boldsymbol{\xi}}^{C}_{1,1|0,1} = \boldsymbol{\pi}_C$.

2. Iterate for $i = 1, \ldots, N^{(t)}$ and $t = 1, \ldots, T$:

- Update step:

  Compute $\hat{\boldsymbol{\xi}}_{i,t|i,t} = \dfrac{\hat{\boldsymbol{\xi}}_{i,t|i-1,t} \odot \boldsymbol{\eta}_{i,t}}{\mathbf{1}'_{d_k} \cdot (\hat{\boldsymbol{\xi}}_{i,t|i-1,t} \odot \boldsymbol{\eta}_{i,t})}$ ($\odot$ denotes element-wise multiplication).

  The conditional densities of the return series $\{r_{i,t}\}$ for all states $h^{(1)}, \ldots, h^{(d_k)}$ ($k = 1, 2$) are summarized in the $d_k \times 1$ vector

$$\boldsymbol{\eta}_{i,t} = \begin{pmatrix} f(r_{i,t}|h^{(1)}, \mathcal{F}_{i-1,t}, \boldsymbol{\Theta}_k) \\ \vdots \\ f(r_{i,t}|h^{(d_k)}, \mathcal{F}_{i-1,t}, \boldsymbol{\Theta}_k) \end{pmatrix} \equiv \begin{pmatrix} f_{h^{(1)}} \\ \vdots \\ f_{h^{(d_k)}} \end{pmatrix},$$

  where the set $\mathcal{F}_{i-1,t}$ contains all past obervations up to date $(i-1, t)$, and $\boldsymbol{\Theta}_1 \equiv \boldsymbol{\Theta}^{\text{MSM}}, \boldsymbol{\Theta}_2 \equiv \boldsymbol{\Theta}^{\text{FHMV}}$.

(FHMV) For FHMV-MIDAS, additionally compute $\hat{\boldsymbol{\xi}}^{C}_{i,t|i,t} = \dfrac{\hat{\boldsymbol{\xi}}^{C}_{i,t|i-1,t} \odot (\boldsymbol{\zeta}_{i,t} \cdot \boldsymbol{\pi}_Z)}{\mathbf{1}'_{d_k} \cdot (\hat{\boldsymbol{\xi}}_{i,t|i-1,t} \odot \boldsymbol{\eta}_{i,t})}$, with

$$\boldsymbol{\zeta}_{i,t} \equiv \begin{pmatrix} f_{h^{(1)}} & f_{h^{(2)}} & \cdots & f_{h^{(N_Z)}} \\ f_{h^{(1+N_Z)}} & f_{h^{(2+N_Z)}} & \cdots & f_{h^{(2\cdot N_Z)}} \\ \vdots & \vdots & \ddots & \vdots \\ f_{h^{(d_2-[N_Z-1])}} & f_{h^{(d_2-[N_Z-2])}} & \cdots & f_{h^{(d_2)}} \end{pmatrix}.$$

- Prediction step:

(MSM) Compute $\hat{\boldsymbol{\xi}}_{i+1,t|i,t} = \mathbf{P}_M \cdot \hat{\boldsymbol{\xi}}_{i,t|i,t}$.

(FHMV) Compute $\hat{\boldsymbol{\xi}}^{C}_{i+1,t|i,t} = (\mathbf{P}_C \cdot \hat{\boldsymbol{\xi}}^{C}_{i,t|i,t})$, and $\hat{\boldsymbol{\xi}}_{i+1,t|i,t} = \hat{\boldsymbol{\xi}}^{C}_{i+1,t|i,t} \otimes \boldsymbol{\pi}_Z$.

3. Computation of log-likelihood (for $k = 1, 2$):

$$\ln(\mathcal{L}(\boldsymbol{\Theta}_k)) = \sum_{t=1}^{T} \sum_{i=1}^{N^{(t)}} \ln\left(\mathbf{1}'_{d_k}(\hat{\boldsymbol{\xi}}_{i,t|i-1,t} \odot \boldsymbol{\eta}_{i,t})\right).$$

Based on the MSM- and FHMV-MIDAS log-likelihood functions from Step 3, we obtain the ML estimates as

$$\hat{\boldsymbol{\Theta}}_k = \underset{\boldsymbol{\Theta}_k}{\operatorname{argmax}} \ln\left(\mathcal{L}(\boldsymbol{\Theta}_k)\right), \qquad k = 1, 2.$$

We used an adapted version of the 3-step algorithm to estimate our MSGARCH-MIDAS specification, while we applied standard ML techniques for the HYGARCH-MIDAS model.

3.2 DGP parameters

We generate our simulation data according to Eq. (1) with $\{\epsilon_{i,t}\} \overset{\text{i.i.d.}}{\sim} \mathbb{N}(0,1)$, and let the short-term component $\{g_{i,t}\}$ be driven by the respective processes (i) MSM, (ii) FHMV, (iii) MSGARCH, and (iv) HYGARCH. We combine each short-term process with the same long-term component $\{\tau_t\}$, whose dynamics—according to Eq. (2)—hinge on the explanatory variable X_t, for which we specify $X_t = 0.9 \cdot X_{t-1} + \nu_t$ with $\nu_t \overset{\text{i.i.d.}}{\sim} \mathbb{N}(0, 0.3^2)$. Our index-$t$ frequency is monthly, with each month consisting of $N^{(t)} = 22$ days (index-i frequency). In line with Conrad and Kleen (2020), we set the parameters of our beta-weighting scheme in Eqs. (2), (3) as $m = 0.1, \theta = 0.3, \omega_1 = 1, \omega_2 = 4$, and use the lag length $K = 36$ months (3 years). For the $\{g_{i,t}\}$-volatility components, we use the following parameter values in our data-generating process (DGP). MSM: $m_0 = 1.2, \gamma_{N_M} = 0.5, b = 2, N_M = 8$. FHMV: $c_1 = 2.5, \theta_C = 0.8, p = 0.995, N_C = 8$ (persistent components), $z_1 = 5, \theta_Z = 0.5, q = 0.1, N_Z = 8$ (jump components). MSGARCH: $\alpha_{11} = 0.09, \alpha_{12} = 0.05, \beta_1 = 0.55, \beta_2 = 0.7, p_{11} = p_{22} = 0.8 \equiv p$. HYGARCH: $\phi = 0.2, d = 0.4, \beta = 0.4, \kappa = 0.8$.

Besides the ML estimates under the above-described DGP, we also report the $\{g_{i,t}\}$-estimates obtained either under empirical misspecifications and/or under a (slightly) disturbed $\{\tau_t\}$-DGP. In particular, we consider the effects on the $\{g_{i,t}\}$-estimates (i) when using an incorrect lag-length in the empirical specification ($K = 12$ instead of the true $K = 36$), (ii) when using a noisy explanatory variable $\tilde{X}_t$ in the $\{\tau_t\}$-DGP (instead of X_t as described above), (iii) when using both (i) plus (ii), and (iv) when setting the $\{\tau_t\}$-component as constant in the empirical specification (i.e. when essentially ignoring the long-term variable).[6] For each setting, we conduct 1000 Monte-Carlo replications and use the 3 observation lengths $T = 2670, 5280, 10560$, which correspond to 10-, 20-, and 40-year simulation periods. Throughout this section, we waive the inclusion of the mean return μ, set the parameters of the declining beta-weighting scheme to $\omega_1 = 1, \omega_2 = 4$, and only estimate ω_2. Technically, we imposed the upper bound of 300 for the ML-estimates of ω_2 throughout our estimation routines (cf. Engle et al., 2013).

Tables 1–4 about here

[6] Explicitly, pertaining to (ii), we add noise to the explanatory variable in the form of $\tilde{X}_t = X_t + \upsilon_t$ with $\upsilon_t \overset{\text{i.i.d.}}{\sim} \mathbb{N}(0, 0.2 + 0.8|X_t|)$.

3.3 Simulation results

Tables 1–4 illustrate the estimation results for our four volatility MIDAS models, using simulated data. We report average deviations of the ML-estimates from their true DGP-values and standard errors (in parentheses), computed across the 1000 Monte-Carlo replications.

We first analyze the correctly specified empirical benchmark models 'MSM-MIDAS', 'FHMV-MIDAS', 'MSGARCH-MIDAS', 'HYGARCH-MIDAS' in the respective upper rows within each of the three blocks ($T = 2640, T = 5280, T = 10560$) of Tables 1–4. Except for ω_2, all average deviations are near zero and, for the vast majority, shrink towards zero in absolute value for increasing sample size T. For ω_2, we observe (as a trend) the same pattern of convergence towards zero, but substantial deviations appear to persist in large samples (most apparently in Tables 1, 2, 4). Two comments on the ω_2-estimates are in order at this point. (i) The associated standard errors in Tables 1, 2, 4 are large *vis-à-vis* the deviations. (ii) For beta-weights $\varphi_k(\omega_1, \omega_2)$ in Eq. (3), we find that (for fixed ω_1), the φ_k-weights are often not very sensitive to changes in ω_2.[7]

When analyzing the 4 (non-benchmark) specifications in Rows 2–5 of each block in Tables 1–4, we still (largely) find $\{g_{i,t}\}$-estimates close to their DGP-values (except for ω_2). Strikingly, under the MSM-MIDAS and FHMV-MIDAS DGPs in Tables 1 and 2, these deviations across all 4 non-benchmark misspecifications appear to be small *vis-à-vis* their standard errors. By contrast, in Tables 3 and 4, we often find the reverse (small standard errors *vis-à-vis* the deviations) for the HYGARCH-MIDAS and MSGARCH-MIDAS DGPs.

4 Empirical Application

4.1 Data

In this section, we analyze the forecasting performance of the four volatility MIDAS models and, for comparative purposes, include results for the GARCH-MIDAS (Engle et al., 2013) and GJR-GARCH-MIDAS (Conrad and Loch, 2015; Conrad and Kleen, 2020) specifications. We use daily observations on the S&P 500 and quarterly US macroeconomic data from 1980:Q1 until 2019:Q4. Our macroeconomic variables are real GDP (1) , industrial production (2), the unemployment rate (3), housing starts (4), nominal corporate profits after

[7] For example, for the correctly specified 'MSM-MIDAS' model in Table 1 with $T = 10560$, the first 36 φ_k-weights (averaged across the 1000 Monte-Carlo replications) virtually coincide with the $\varphi_k(\omega_1 = 1, \omega_2 = 4)$-weights.

tax (5), the inflaton rate (6), real personal consumption (10), and the term spread (11). As sentiment-based indicators of business-confidence and uncertainty-assessments, we use the Chicago National Activity Index 'NAI' (7), the new-order index of the Institute for Supply Management (8), and the consumer-sentiment index published by the University of Michigan (9).[8]

Following the lines of argument in Fang et al. (2020), we use real-time data. We apply the NAI and the new-order indices, the inflation rate and the term spread in levels, taking first differences for the unemployment rate and the consumer-sentiment index. For the remaining variables, we compute the annualized quarterly percentage changes as $100 \cdot (X_t/X_{t-1} - 1)^4$. We obtain the daily S&P500 returns as $r_{i,t} = 100 \cdot [\ln(p_{i,t}) - \ln(p_{i-1,t})]$, and compute quarterly realized volatility as $\mathrm{RV}_t = \sum_{i=1}^{N^{(t)}} r_{i,t}^2$. (We use $\{\mathrm{RV}_t\}$ as the 12th long-term variable in Tables 5–11.)

4.2 Forecasting strategy

We partition our data set into (i) an estimation period ranging from 1980:Q1 until 2003:Q4, and (ii) an out-of-sample period from 2004:Q1 until 2019:Q4. Our target variable to be forecasted is the cumulative S&P500-return variance, which we forecast for $1, 2, 3, 4$ quarters ahead, using a rolling-window scheme (with updated parameter estimates).

Formally, using all information available at day $N^{(t)}$ in quarter t, the optimal variance forecast for day k in quarter $t + s$, with forecast horizon $s \geq 1$, is given by

$$\hat{\sigma}^2_{k,t+s|N^{(t)},t} = \mathbb{E}_{N^{(t)},t}(\tau_{t+s} \cdot g_{k,t+s}) = \mathbb{E}_t(\tau_{t+s}) \cdot \mathbb{E}_{N^{(t)},t}(g_{k,t+s}) = \hat{\tau}_{t+s|t} \cdot \hat{g}_{k,t+s|N^{(t)},t}.$$

For the horizon $s = 1$, the forecast of the long-term component $\hat{\tau}_{t+1|t}$ is predetermined by Eq. (2). For $s > 1$, we need to know the distribution of the economic variable X_t in order to predict the long-term component. As in Conrad and Loch (2015), we assume smooth movements in the long-term component and set $\hat{\tau}_{t+s|t} = \hat{\tau}_{t+1|t}$ for $s > 1$.

For the MSM-MIDAS and FHMV-MIDAS models, we base the forecasts of the short-term components on the updated conditional probability $\hat{\boldsymbol{\xi}}_{N^{(t)},t|N^{(t)},t}$, as obtained for last day of the estimation period, via the Hamilton filter from Section 3.1. According to the prediction

[8] The numbers in parentheses refer to the enumeration of the long-term variables in Tables 5–11.

step, we respectively obtain the MSM and FHMV probability-vector forecasts as

$$\hat{\boldsymbol{\xi}}_{k,t+s|N^{(t)},t} = \mathbf{P}_M^{N^{(t+1)}+\ldots+N^{(t+s-1)}+k} \cdot \hat{\boldsymbol{\xi}}_{N^{(t)},t|N^{(t)},t},$$

and

$$\hat{\boldsymbol{\xi}}_{k,t+s|N^{(t)},t} = \mathbf{P}_g^{N^{(t+1)}+\ldots+N^{(t+s-1)}+k} \cdot \hat{\boldsymbol{\xi}}_{N^{(t)},t|N^{(t)},t},$$

from which we compute the forecasts of the respective short-term components as

$$\hat{g}_{k,t+s|N^{(t)},t} = \mathbf{h} \cdot \hat{\boldsymbol{\xi}}_{k,t+s|N^{(t)},t},$$

where $\mathbf{h} = \begin{pmatrix} h^1 & \ldots & h^{d_k} \end{pmatrix}$ denotes either the MSM- or the FHMV-specific row vector containing all elements of the state spaces $\mathcal{X}_M$ or $\mathcal{X}_g$. Pertaining to the computation of the GARCH-based model forecasts, we refer to Haas et al. (2004), Conrad (2010), and Conrad and Kleen (2020). Finally, aggregating the daily variance forecasts over the period $t+s$ yields the cumulative variance forecasts $\hat{\sigma}^2_{1:N^{(t+s)},t+s|N^{(t)},t} = \hat{\tau}_{t+s|t} \sum_{i=1}^{N^{(t+s)}} \hat{g}_{i,t+s|N^{(t)},t}$ (our target variable).

The first step of our out-of-sample analysis below, consists of investigating whether the inclusion of explanatory variables yields accuracy gains over the non-MIDAS models (without the long-term component). We conduct these accuracy comparisons via the mean-squared error (MSE) and the quasi-likelihood (QLike) loss functions, defined as

$$\begin{aligned}
\mathrm{MSE}(\sigma^2_{1:N^{(t+s)},t+s}, \hat{\sigma}^2_{1:N^{(t+s)},t+s|N^{(t)},t}) &= (\sigma^2_{1:N^{(t+s)},t+s} - \hat{\sigma}^2_{1:N^{(t+s)},t+s|N^{(t)},t})^2, \\
\mathrm{QLike}(\sigma^2_{1:N^{(t+s)},t+s}, \hat{\sigma}^2_{1:N^{(t+s)},t+s|N^{(t)},t}) &= \log\left(\hat{\sigma}^2_{1:N^{(t+s)},t+s|N^{(t)},t}\right) + \frac{\sigma^2_{1:N^{(t+s)},t+s}}{\hat{\sigma}^2_{1:N^{(t+s)},t+s|N^{(t)},t}},
\end{aligned}$$

where $\sigma^2_{1:N^{(t+s)},t+s}$ is a suitably chosen proxy for the unobservable variance. In our analysis below, we approximate this unobservable variance by the (aggregated) squared returns RV_t defined above, although this proxy may be noisy. However, Patton (2011) demonstrates that the forecast rankings induced by MSE and QLike loss functions are consistent, as long as the chosen proxy is conditionally unbiased. In order to test for equal (unconditional) predictive ability, we use Giacomini and White's (2006) test, which is applicable for nested models.

Based on the loss differential of two competing models i and j,

$$d_{ij}(t+s) = \mathrm{L}(\sigma^2_{1:N^{(t+s)},t+s}, \hat{\sigma}^{2,(i)}_{1:N^{(t+s)},t+s|N^{(t)},t}) - \mathrm{L}(\sigma^2_{1:N^{(t+s)},t+s}, \hat{\sigma}^{2,(j)}_{1:N^{(t+s)},t+s|N^{(t)},t})$$

($\mathrm{L}(\cdot,\cdot)$ is either the MSE or QLike loss), we test the null hypothesis

$$H_0 : \mathbb{E}\left(d_{ij}(t+s)\right) = 0 \text{ for all } t+s$$

via the test statistic

$$t_{ij} = \frac{\bar{d}_{ij}}{\sqrt{\widehat{\mathrm{Var}}(\bar{d}_{ij})}},$$

with $\bar{d}_{ij} = \frac{1}{T}\sum_{t=1}^{T} d_{ij}(t+s)$, T the number of out-of-sample forecast periods, and where we use an HAC-estimate of the variance in the test statistic. We consider Model j as the baseline model (non-MIDAS, i.e. no long-term component) and conduct all tests against the correponding MIDAS specifications with long-term component (Model i).

In a second step, we compare forecast accuracy among our model classes, by identifying the set of models with superior forecasting performance via Hansen et al.'s (2011) model confidence sets. We sequentially eliminate the 'worst-performing' model from the initial set $\mathcal{M}$ (encompassing all model specifications), if the null hypothesis

$$H_{0,\mathcal{M}} : \mathbb{E}(d_{ij}) = 0 \text{ for all } i,j \in \mathcal{M}, \tag{24}$$

is rejected at the significance level α. This iterative testing procedure is terminated once the null hypothesis cannot be rejected any further, and the set of remaining models constitutes the so-called model confidence set (MCS) $\widehat{\mathcal{M}}^*_{1-\alpha}$ at the confidence level $1-\alpha$. To approximate the nonstandard asymptotic distribution of the involved test statistic $T_{\mathcal{M}} = \max_{i,j\in\mathcal{M}} |t_{ij}|$, we apply a block-bootstrap.[9]

4.3 Forecasting results

In a preliminary model-selection analysis, we estimated the following two specifications per explanatory long-term variable and MIDAS model. (i) One variant with a restricted, declining beta-weighting scheme ($\omega_1 = 1$), and (ii) one with an unrestricted beta-weighting scheme.

[9] Specifically, we use Kevin Sheppard's Matlab toolbox 'MFE' (see https://www.kevinsheppard.com/code/matlab/mfe-toolbox).

Overall, the estimated variable-specific weighting-schemes appeared to be similar across most model specifications. We also conducted likelihood ratio tests for the hypotheses 'restricted' (null) versus 'unrestricted weighting scheme' (alternative), but only found unambiguous evidence in favor of an unrestricted weighting scheme at the 5% level (across all MIDAS models) for the variable 'housing starts', supporting the widespread view that housing starts may serve as a leading indicator (cf. Kydland et al., 2016). Ultimately, we decided to exclusively use the unrestricted beta-weighting scheme for the 'housing starts', and to apply the restricted variant ($\omega_1 = 1$) for all other long-term variables. Additionally, we opted to use (i) $N_M = 8$ short-term volatility multipliers in the MSM-MIDAS models, and (ii) $N_C = 6$ persistent volatility components plus $N_Z = 6$ distinct jump outcomes in the FHMV-MIDAS models. In line with the literature, we chose $K = 12$ lags in the long-term component (covering 3 years of macroeconomic data) for all specifications (cf.,*inter alia*, Conrad and Loch, 2016; Conrad and Kleen, 2020).

Tables 5 – 10 about here

Tables 5–10 display the forecasting performance of six volatility MIDAS models (MSM-, FHMV-, MSGARCH-, HYGARCH-, GJR-GARCH, GARCH-MIDAS). For each MIDAS model (i.e. in each table), we consider 12 variants, where each variant includes one single (out of 12) long-term variable. We measure forecasting performance in terms MSE and QLike losses relative to their counterpart-losses from the non-MIDAS benchmark model (MSE, QLike ratios), implying that ratios falling below 1 indicate superior forecasting accuracy of the MIDAS model compared with its non-MIDAS benchmark model.

For all volatility MIDAS models (except for HYGARCH in Table 8), the inclusion of (specific) long-term variables yields significant forecast-accuracy gains via the Giacomini-White test. For MSGARCH-MIDAS (Table 7), the results hinge on the loss type. Under MSE loss, adding long-term information leads to accuracy gains for forecast horizons longer than 2 quarters ahead. The (numerically) largest improvement occurs under the inclusion of 'housing starts' (4) with an accuracy improvement of about 65% for the 4-quarter forecast horizon. However, under QLike loss, MSGARCH-MIDAS exhibits (often significant) accuracy losses, except for the inclusion of 'housing starts' (4). For MSM-MIDAS (Table 5), FHMV-MIDAS (Table 5), GJR-GARCH-MIDAS (Tables 9), and GARCH-MIDAS (Table 10), the inclusion of the 'Term spread' (11) yields the (numerically) largest accuracy gains under both loss functions, a finding consistent with Estrella and Hardouvelis (1991), Estrella and Mishkin (1998) and Ang et al. (2006), who all classify the term spread as a powerful predictor of economic growth.

Table 11 about here

Up to now, our out-of-sample analysis focuses on pairwise comparisons between each volatility MIDAS model and its non-MIDAS counterpart. To assess the forecasting performance among the MIDAS models, we apply Hansen et al.'s (2011) MCS approach, as outlined in Section 4.2. Table 11 displays MCS p-values under MSE and QLike losses for 1 to 4 quarter-ahead cumulated variance forecasts.[10] A gray-shaded cell indicates that the associated variable-specific MIDAS variant belongs to the 90% MCS $\widehat{\mathcal{M}}^*_{0.9}$ (see Eq. (24) in Section 4.2).[11]

We start with the 1-quarter (1q) horizon under both losses. Across the 12 variable-specific MIDAS variants (Blocks (1)–(12) in Table 11), the confidence sets (most notably under the QLike loss) retain 'many' MIDAS models, and therefore, *prima facie*, appear to be rather uninformative. We recognize, however, that under the MSE loss, all 12 MSGARCH-MIDAS variants belong to the 90% MCSs, documenting some evidence in favor of the short-horizon predictor quality of MSGARCH-MIDAS. By contrast, for the 2, 3, 4 quarter horizons, MSGARCH-MIDAS constantly fails to attain the 90% MCSs (except for the 3q horizon in Block (2)). We also note that a qualitatively similar forecasting performance prevails for the MSM-MIDAS specifications.

Next, we address the contribution of the specific long-term variables to identifying volatility MIDAS models with superior forecasting performance. Two striking variables in Table 11 are (i) the 'Term spread' in Block (11), and (ii) 'Δ Hous.' (the housing starts) in Block (4). For the 'Term spread', the FHMV-MIDAS model belongs to the 90% MCSs across all forecast horizons under both losses (in all 8 cases). The same pattern emerges for GJR-GARCH-MIDAS and GARCH-MIDAS (in 7 out of 8 cases). A qualitatively similar finding holds for the housing-start variable in Block (4) under the QLike loss. These variable-specific find-

[10] For the computation of the p-values, we used 9,999 bootstrap replications.

[11] We included the non-MIDAS models in the MCS approach, but do not report results for these specifications here. Details are available upon request.

ings may appear to contradict the results in Conrad and Kleen (2020), who identify distinct long-term variables—entailing superior forecasts—for almost all of their forecast horizons.[12] However, our major finding is that the long-term variables 'term spread' and 'housing starts' can substantially improve the forecast accuracy of several volatility MIDAS models.[13] This result appears to apply particularly to the FHMV-MIDAS model across all forecast horizons.

5 Conclusion

We propose four new multiplicative-component volatility MIDAS models, three of which are based on Markov-switching short-term components. We establish covariance-stationarity properties of the models, and—for two of them (MSM-, FHMV-MIDAS)—we derive analytically closed-form formulae for the autocorrelation functions of the squared returns. Based on regime-switching filtering techniques (the Hamilton-filter), we set up a fully-fledged maximum-likelihood estimation approach and check for the unbiasedness of the ML estimates via Monte Carlo analyses. Our in-sample estimates turn out to be robust against (i) contaminated measurements of the long-term variable in the data-generating process, and (ii) misspecification of the lag-length in the estimated empirical specification.

In an out-of-sample analysis, we apply the volatility MIDAS models to forecast (cumulative) variances of S&P500 returns (daily observations), using quarterly US macroeconomic data (on 12 variables) from a 40-year time span. In a first step, we analyze whether the inclusion of specific long-term variables entails forecast accuracy gains for the volatility MIDAS models relative to their non-MIDAS counterparts which waive information from the long-term variables. We find that the inclusion of the variables 'Housing starts' and 'Term spread' can entail substantial forecast accuracy gains for several multiplicative volatility MIDAS models. In a second step, we construct model confidence sets to identify groups of volatility MIDAS models with superior forecasting performance. Again, the long-term variable 'Term spread' plays an accentuated role in that it robustly distils the FHMV-MIDAS model as a model-confidence-set member across (i) all forecast horizons, and (ii) the two loss functions used.

[12] This difference may be due to the differing time-dimensions of the forecast horizons used by Conrad and Kleen (2020) and those used in our study.

[13] The same long-term variables have also been identified as accuracy-improving by Conrad and Loch (2015) and Fang et al. (2020).

In this paper, we focus on modeling the short-term component of the volatility MIDAS framework as Markov-switching specifications. However, (Markovian) regime changes have generally turned out to constitute an important empirical feature, so that the inclusion of Markov-switching approaches in long-term MIDAS components in the spirit of Guérin and Marcellino (2013) could usefully be tackled in future research. Finally, we note that the current literature mostly uses long-term variables from macro-financial data. In view of the current global crises, it might be worth processing other conceivably non-economic data, such as variables representing geopolitical and/or pandemic risks.

Appendix: A. Proof(s) and remark(s)

Proof of Proposition 1. Let $\bar{r}_{i,t} \equiv r_{i,t} - \mu$ denote the demeaned return from Eq. (1). From Assumption 1, we have $\mathbb{E}(\bar{r}_{i,t}) = \mathbb{E}(\sqrt{\tau_t g_{i,t}}) \cdot \mathbb{E}(\varepsilon_{i,t}) = 0$, implying $\mathbb{E}(r_{i,t}) = \mu$ (= const.).

From Assumptions 1–3, we first have $\mathbb{V}\text{ar}(r_{i,t}) = \mathbb{V}\text{ar}(\bar{r}_{i,t}) = \mathbb{E}(\bar{r}_{i,t}^2) - [\mathbb{E}(\bar{r}_{i,t})]^2 = \mathbb{E}(g_{i,t})\mathbb{E}(\tau_t)\mathbb{E}(\varepsilon_{i,t}^2) = \mathbb{E}(g_{i,t})\mathbb{E}(\tau_t)$. Owing to Assumption 3(ii), the MSM and FHMV Markov-chains $\{g_{i,t}\}$ are (strictly) stationary, and—per construction—have expectation $\mathbb{E}(g_{i,t}) = 1$. Our HYGARCH-MIDAS short-term volatility process from Eqs. (18), (19) satisfies the covariance-stationarity condition $(1-\kappa)\Phi(1) = (1-\kappa)(1-\phi) > 0$ (cf. Conrad, 2010), and—per construction—$\mathbb{E}(g_{i,t}) = 1$. Overall, for the MSM-, FHMV- and HYGARCH-MIDAS models, Assumption 2 implies $\mathbb{V}\text{ar}(r_{i,t}) = \mathbb{E}(\tau_t) =$ const.

Finally, at the lags of u days and v periods ($\min\{u, v\} > 0$), the autocovariance function of $\{r_{i,t}\}$ is

$$
\begin{aligned}
\mathbb{C}\text{ov}(r_{i,t}, r_{i-u,t-v}) &= \mathbb{C}\text{ov}(\bar{r}_{i,t}, \bar{r}_{i-u,t-v}) \\
&= \mathbb{E}(\bar{r}_{i,t} \cdot \bar{r}_{i-u,t-v}) - \mathbb{E}(\bar{r}_{i,t}) \cdot \mathbb{E}(\bar{r}_{i-u,t-v}) \\
&= \mathbb{E}(\bar{r}_{i,t} \cdot \bar{r}_{i-u,t-v}) \\
&= \mathbb{E}\left(\sqrt{\tau_t g_{i,t}} \cdot \sqrt{\tau_{t-v} g_{i-u,t-v}}\right) \cdot \mathbb{E}(\varepsilon_{i,t}) \cdot \mathbb{E}(\varepsilon_{i-u,t-v}) \\
&= 0.
\end{aligned}
$$

$\square$

Proof of Proposition 2. Using the independence of $\{\Delta_{i,t}\}, \{\varepsilon_{i,t}\}$ and Assumptions 1–3, similar calculations to those in the proof of Proposition 1 yield

$$\begin{aligned}\mathbb{E}(\bar{r}_{i,t}) &= 0 \quad (\text{implying } \mathbb{E}(r_{i,t}) = \mu), \\ \mathbb{C}\text{ov}(r_{i,t}, r_{i-u,t-v}) &= 0.\end{aligned}$$

The variance of the (k-regime) MSGARCH-MIDAS(k) model is

$$\mathbb{V}\text{ar}(r_{i,t}) = \mathbb{E}(g_{i,t}^{(\Delta_{i,t})})\mathbb{E}(\tau_t),$$

which, in view of Assumption 2, is constant if the short-term component $\{g_{i,t}^{(\Delta_{i,t})}\}$ is covariance stationary. To prove the latter, we first consider the matrix

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{21} & \dots & \mathbf{M}_{k1} \\ \mathbf{M}_{12} & \mathbf{M}_{22} & \dots & \mathbf{M}_{k2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{M}_{1k} & \mathbf{M}_{2k} & \dots & \mathbf{M}_{kk} \end{bmatrix}, \text{ with } \mathbf{M}_{jl} = p_{jl}(\boldsymbol{\beta} + \boldsymbol{\alpha}_1 \mathbf{e}_l') \quad \text{for } j, l = 1, 2, \dots, k,$$

where $p_{jl}, \boldsymbol{\beta}, \boldsymbol{\alpha}_1$ as in Eqs. (16), (17), and $\mathbf{e}_l$ denotes the lth Euclidian standard basis vector of dimension $k \times 1$. Following Haas et al. (2004), the short-term process $\{g_{i,t}^{(\Delta_{i,t})}\}$ is covariance-stationary, if and only if the spectral radius $\rho(\cdot)$ of the matrix $\mathbf{M}$ is less than 1 (i.e. iff $\rho(\mathbf{M}) < 1$). For the two-regime case ($k = 2$), $\mathbf{M}$ reduces to the 4×4 matrix

$$\mathbf{M} = \begin{bmatrix} p_{11}(\alpha_{11} + \beta_1) & 0 & p_{21}(\alpha_{11} + \beta_1) & 0 \\ p_{11}\alpha_{12} & p_{11}\beta_2 & p_{21}\alpha_{12} & p_{21}\beta_2 \\ p_{12}\beta_1 & p_{12}\alpha_{11} & p_{22}\beta_1 & p_{22}\alpha_{11} \\ 0 & p_{12}(\alpha_{12} + \beta_2) & 0 & p_{22}(\alpha_{12} + \beta_2) \end{bmatrix},$$

for which we can establish an upper bound for the spectral radius $\rho(\mathbf{M})$. Defining the vector $\mathbf{x} = (p_{21}\ p_{21}\ p_{12}\ p_{12})'$, we apply Theorem 8.1.26 in Horn and Johnson (2012, p. 522) to obtain

$$\rho(\mathbf{M}) \leq \max\{\alpha_{11} + \beta_1, \alpha_{12} + \beta_2\}.$$

In view of the restriction $\boldsymbol{\alpha}_0 = \mathbf{1}_k - \boldsymbol{\alpha}_1 - \boldsymbol{\beta}\mathbf{1}_k > \mathbf{0}_k$ in Eq. (16), we have $\rho(\mathbf{M}) < 1$ for $k = 2$. □

Proof of Proposition 3. (i) To prove Eq. (21), we first show that $\mathbb{C}\text{ov}(r_{i,t}^2, r_{i-u,t-v}^2) =$

$\mathbb{C}\text{ov}(\bar{r}_{i,t}^2, \bar{r}_{i-u,t-v}^2)$, where $\bar{r}_{i,t}^2 = (r_{i,t} - \mu)^2 = g_{i,t}\tau_t\varepsilon_{i,t}^2$. For all i, t and $\min\{u, v\} > 0$, we have

$$\begin{aligned}
\mathbb{C}\text{ov}(\bar{r}_{i,t}^2, \bar{r}_{i-u,t-v}^2) &= \mathbb{C}\text{ov}(r_{i,t}^2 - 2\mu r_{i,t} + \mu^2, r_{i-u,t-v}^2 - 2\mu r_{i-u,t-v} + \mu^2) \\
&= \mathbb{C}\text{ov}(r_{i,t}^2, r_{i-u,t-v}^2) - 2\mu\mathbb{C}\text{ov}(r_{i,t}^2, r_{i-u,t-v}) \\
&\quad - 2\mu\mathbb{C}\text{ov}(r_{i,t}, r_{i-u,t-v}^2) + 4\mu^2\mathbb{C}\text{ov}(r_{i,t}, r_{i-u,t-v}).
\end{aligned} \tag{A.1}$$

Under Assumptions 1–3, the term $\mathbb{C}\text{ov}(r_{i,t}^2, r_{i-u,t-v})$ in Eq. (A.1) is

$$\begin{aligned}
\mathbb{C}\text{ov}(r_{i,t}^2, r_{i-u,t-v}) &= \mathbb{C}\text{ov}(\mu^2 + 2\mu\sqrt{g_{i,t}\tau_t}\varepsilon_{i,t} + g_{i,t}\tau_t\varepsilon_{i,t}^2, r_{i-u,t-v}) \\
&= 2\mu\mathbb{C}\text{ov}(\sqrt{g_{i,t}\tau_t}\varepsilon_{i,t}, r_{i-u,t-v}) + \mathbb{C}\text{ov}(g_{i,t}\tau_t\varepsilon_{i,t}^2, \mu + \sqrt{g_{i-u,t-v}\tau_{t-v}}\varepsilon_{i-u,t-v}) \\
&= 2\mu\left[\mathbb{E}(\sqrt{g_{i,t}\tau_t}\varepsilon_{i,t}r_{i-u,t-v}) - \mathbb{E}(\sqrt{g_{i,t}\tau_t}\varepsilon_{i,t})\mathbb{E}(r_{i-u,t-v})\right] \\
&\quad + \mathbb{C}\text{ov}(g_{i,t}\tau_t\varepsilon_{i,t}^2, \sqrt{g_{i-u,t-v}\tau_{t-v}}\varepsilon_{i-u,t-v}) \\
&= 2\mu\left[\mathbb{E}(\sqrt{g_{i,t}\tau_t}r_{i-u,t-v})\mathbb{E}(\varepsilon_{i,t}) - \mathbb{E}(\sqrt{g_{i,t}\tau_t})\mathbb{E}(\varepsilon_{i,t})\mathbb{E}(r_{i-u,t-v})\right] \\
&\quad + \mathbb{E}(g_{i,t}\tau_t\varepsilon_{i,t}^2\sqrt{g_{i-u,t-v}\tau_{t-v}}\varepsilon_{i-u,t-v}) \\
&\quad - \mathbb{E}(g_{i,t}\tau_t\varepsilon_{i,t}^2)\mathbb{E}(\sqrt{g_{i-u,t-v}\tau_{t-v}}\varepsilon_{i-u,t-v}) \\
&= 2\mu\left[\mathbb{E}(\sqrt{g_{i,t}\tau_t}r_{i-u,t-v}) \cdot 0 - \mathbb{E}(\sqrt{g_{i,t}\tau_t}) \cdot 0 \cdot \mathbb{E}(r_{i-u,t-v})\right] \\
&\quad + \mathbb{E}(g_{i,t}\tau_t\varepsilon_{i,t}^2\sqrt{g_{i-u,t-v}\tau_{t-v}})\mathbb{E}(\varepsilon_{i-u,t-v}) \\
&\quad - \mathbb{E}(g_{i,t}\tau_t\varepsilon_{i,t}^2)\mathbb{E}(\sqrt{g_{i-u,t-v}\tau_{t-v}})\mathbb{E}(\varepsilon_{i-u,t-v}) \\
&= 2\mu \cdot 0 + \mathbb{E}(g_{i,t}\tau_t\varepsilon_{i,t}^2\sqrt{g_{i-u,t-v}\tau_{t-v}}) \cdot 0 - \mathbb{E}(g_{i,t}\tau_t\varepsilon_{i,t}^2)E(\sqrt{g_{i-u,t-v}\tau_{t-v}}) \cdot 0 \\
&= 0.
\end{aligned} \tag{A.2}$$

Similarly,

$$\mathbb{C}\text{ov}(r_{i,t}, r_{i-u,t-v}^2) = 0, \tag{A.3}$$

and, from the proof of Proposition 1,

$$\mathbb{C}\text{ov}(r_{i,t}, r_{i-u,t-v}) = 0. \tag{A.4}$$

Inserting (A.2)–(A.4) into (A.1) yields

$$\mathbb{C}\text{ov}(\bar{r}_{i,t}^2, \bar{r}_{i-u,t-v}^2) = \mathbb{C}\text{ov}(r_{i,t}^2, r_{i-u,t-v}^2). \tag{A.5}$$

Next, the autocovariance function of the squared demeaned returns can be written as

$$
\begin{aligned}
\mathbb{C}\text{ov}(\bar{r}_{i,t}^2, \bar{r}_{i-u,t-v}^2) &= \mathbb{E}\left(\tau_t \tau_{t-v} g_{i,t} g_{i-u,t-v} \varepsilon_{i,t}^2 \varepsilon_{i-u,t-v}^2\right) - \mathbb{E}\left(\tau_t g_{i,t} \varepsilon_{i,t}^2\right) \mathbb{E}\left(\tau_{t-v} g_{i-u,t-v} \varepsilon_{i-u,t-v}^2\right) \\
&= \mathbb{E}\left(\tau_t \tau_{t-v}\right) \mathbb{E}\left(g_{i,t} g_{i-u,t-v}\right) \\
&\quad - \mathbb{E}\left(\tau_t\right) \mathbb{E}\left(\tau_{t-v}\right) \mathbb{E}\left(g_{i,t}\right) \mathbb{E}\left(g_{i-u,t-v}\right) \\
&= \left[\mathbb{E}(\tau_t \tau_{t-v}) - \mathbb{E}(\tau_t)\mathbb{E}(\tau_{t-v})\right] \mathbb{E}\left(g_{i,t} g_{i-u,t-v}\right) \\
&\quad + \mathbb{E}(\tau_t)\mathbb{E}(\tau_{t-v}) \big[\mathbb{E}(g_{i,t} g_{i-u,t-v}) \\
&\qquad\qquad - \mathbb{E}\left(g_{i,t}\right) \mathbb{E}\left(g_{i-u,t-v}\right) \big] \\
&= \mathbb{C}\text{ov}\left(\tau_t, \tau_{t-v}\right) \big[\mathbb{E}\left(g_{i,t} g_{i-u,t-v}\right) \\
&\qquad\qquad - \mathbb{E}\left(g_{i,t}\right) \mathbb{E}\left(g_{i-u,t-v}\right) \big] \\
&\quad + \mathbb{C}\text{ov}\left(\tau_t, \tau_{t-v}\right) + \mathbb{E}(\tau_t)^2 \mathbb{C}\text{ov}\left(g_{i,t}, g_{i-u,t-v}\right) \\
&= \mathbb{C}\text{ov}\left(\tau_t, \tau_{t-v}\right) \\
&\quad + \mathbb{C}\text{ov}\left(g_{i,t}, g_{i-u,t-v}\right) \left[\mathbb{E}(\tau_t)^2 + \mathbb{C}\text{ov}\left(\tau_t, \tau_{t-v}\right)\right]. \qquad \text{(A.6)}
\end{aligned}
$$

Using Eqs. (A.5) and (A.6), we write the ACF of the squared returns as

$$
\begin{aligned}
\rho_{r^2}(u,v) &= \frac{\mathbb{C}\text{ov}(r_{i,t}^2, r_{i-u,t-v}^2)}{\mathbb{V}\text{ar}(r_{i,t}^2)} \\
&= \frac{\mathbb{C}\text{ov}(\tau_t, \tau_{t-v})}{\mathbb{V}\text{ar}(\tau_t)} \cdot \frac{\mathbb{V}\text{ar}(\tau_t)}{\mathbb{V}\text{ar}(r_{i,t}^2)} \\
&\quad + \frac{\mathbb{C}\text{ov}(g_{i,t}, g_{i-u,t-v})}{\mathbb{V}\text{ar}(g_{i,t})} \cdot \frac{\mathbb{V}\text{ar}(g_{i,t})}{\mathbb{V}\text{ar}(r_{i,t}^2)} \left[\mathbb{E}(\tau_t)^2 + \frac{\mathbb{C}\text{ov}(\tau_t, \tau_{t-v})}{\mathbb{V}\text{ar}(\tau_t)} \mathbb{V}\text{ar}(\tau_t)\right] \\
&= \rho_\tau(v) \frac{\mathbb{V}\text{ar}(\tau_t)}{\mathbb{V}\text{ar}(r_{i,t}^2)} + \rho_g(u,v) \frac{\mathbb{V}\text{ar}(g_{i,t})}{\mathbb{V}\text{ar}(r_{i,t}^2)} \left(\mathbb{E}(\tau_t)^2 + \rho_\tau(v)\mathbb{V}\text{ar}(\tau_t)\right),
\end{aligned}
$$

as claimed in Eq. (21).

To prove Eq. (22), we first recall from Section 2.1 that the N_M multipliers $M_{i,t}^{(1)}, \ldots, M_{i,t}^{(N_M)}$ are independent, each with expectation $\mathbb{E}(M_{i,t}^{(j)}) = 1$ for $j = 1, \ldots, N_M$. Thus, the autocovariance function of $\{g_{i,t}\}$ from Eq. (4) is

$$
\mathbb{C}\text{ov}(g_{i,t}, g_{i-u,t-v}) = \prod_{j=1}^{N_M} \mathbb{E}\left(M_{i,t}^{(j)} M_{i-u,t-v}^{(j)}\right) - 1. \qquad \text{(A.7)}
$$

To compute the product in Eq. (A.7), we first apply the law of iterated expectation and the pull-through property to obtain

$$\begin{aligned}
\mathbb{E}\left(M_{i,t}^{(j)} M_{i-u,t-v}^{(j)}\right) &= \mathbb{E}\left(M_{i,t}^{(j)} M_{i-u,t-v}^{(j)} | M_{i-u,t-v}^{(j)} = m_0\right) \mathbb{P}\mathrm{rob}\left\{M_{i-u,t-v}^{(j)} = m_0\right\} \\
&\quad + \mathbb{E}\left(M_{i,t}^{(j)} M_{i-u,t-v}^{(j)} | M_{i-u,t-v}^{(j)} = 2 - m_0\right) \mathbb{P}\mathrm{rob}\left\{M_{i-u,t-v}^{(j)} = 2 - m_0\right\} \\
&= 0.5 \cdot m_0 \cdot \mathbb{E}\left(M_{i,t}^{(j)} | M_{i-u,t-v}^{(j)} = m_0\right) \\
&\quad + 0.5 \cdot (2 - m_0) \cdot \mathbb{E}\left(M_{i,t}^{(j)} | M_{i-u,t-v}^{(j)} = 2 - m_0\right). \qquad \text{(A.8)}
\end{aligned}$$

Next, we decompose the multiplier transition probability matrix from Eq. (6):

$$\begin{aligned}
\mathbf{P}^{(j)} &= \begin{pmatrix} 1 - 0.5\gamma_j & 0.5\gamma_j \\ 0.5\gamma_j & 1 - 0.5\gamma_j \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 - \gamma_j \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \\
&= \mathbf{E}\mathbf{A}^{(j)}\mathbf{E}', \qquad \text{(A.9)}
\end{aligned}$$

where the columns of $\mathbf{E}$ are two eigenvectors of $\mathbf{P}^{(j)}$, and the elements on the main diagonal of $\mathbf{A}^{(j)}$ are the corresponding eigenvalues. For ease of notation, we stack the two mass points of the multipliers' base distribution in the vector $\mathbf{m} = (m_0, 2 - m_0)'$, and define the lag $h = N^{(t-v)} + \cdots + N^{(t-1)} + u$. Then, using (A.9), we write the right-hand side of (A.8) as

$$\begin{aligned}
\mathbb{E}\left(M_{i,t}^{(j)} M_{i-u,t-v}^{(j)}\right) &= 0.5\mathbf{m}'\left(\mathbf{P}^{(j)}\right)^h \mathbf{m} \\
&= 0.5\mathbf{m}'\mathbf{E}\left(\mathbf{A}^{(j)}\right)^h \mathbf{E}'\mathbf{m} \\
&= 1 + (1 - \gamma_j)^h (m_0 - 1)^2. \qquad \text{(A.10)}
\end{aligned}$$

Inserting (A.10) into (A.7) yields the autocovariance function

$$\mathbb{C}\mathrm{ov}(g_{i,t}, g_{i-u,t-v}) = \prod_{j=1}^{N_M} \left(1 + (1 - \gamma_j)^h (m_0 - 1)^2\right) - 1. \qquad \text{(A.11)}$$

Along similar lines, we obtain

$$\mathbb{V}\mathrm{ar}(g_{i,t}) = \left(1 + (m_0 - 1)^2\right)^{N_m} - 1, \qquad \text{(A.12)}$$

and taking the ratio of (A.11) and (A.12) yields Eq. (22).

We prove Eq. (23) for FHMV-MIDAS with the same technique. First,

$$\begin{aligned}
\mathbb{C}\mathrm{ov}(g_{i,t}, g_{i-u,t-v}) &= \mathbb{E}\left[\left(C_{i,t}Z_{i,t}\right)\left(C_{i-u,t-v}Z_{i-u,t-v}\right)\right] - \mathbb{E}\left(C_{i,t}Z_{i,t}\right)\cdot\mathbb{E}\left(C_{i-u,t-v}Z_{i-u,t-v}\right) \\
&= \mathbb{E}\left(C_{i,t}C_{i-u,t-v}\right)\mathbb{E}\left(Z_{i,t}\right)\mathbb{E}\left(Z_{i-u,t-v}\right) \qquad \text{(A.13)} \\
&\quad - \mathbb{E}\left(C_{i,t}\right)\mathbb{E}\left(Z_{i,t}\right)\mathbb{E}\left(C_{i-u,t-v}\right)\mathbb{E}\left(Z_{i-u,t-v}\right) \\
&= \mathbb{E}\left(C_{i,t}C_{i-u,t-v}\right) - 1 \\
&= \prod_{j=1}^{N_C} \frac{\mathbb{E}\left(C_{i,t}^{(j)}C_{i-u,t-v}^{(j)}\right)}{\mathbb{E}\left(C_{i,t}^{(j)}\right)\mathbb{E}\left(C_{i-u,t-v}^{(j)}\right)} - 1, \qquad \text{(A.14)}
\end{aligned}$$

where we used in (A.13) the jump component $\{Z_{i.t}\}$, per construction, being independent of its own past (see Section 2.2). We decompose the transition probability matrix $\mathbf{P}$ of the $C_{i,t}^{(j)}$-multipliers in Eq. (9) as

$$\begin{aligned}
\mathbf{P} &= \begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2p-1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \\
&= \mathbf{E}\mathbf{A}\mathbf{E}'. \qquad \text{(A.15)}
\end{aligned}$$

We collect the two elements of the $C_{i,t}^{(j)}$-support in the 2×1 vector $\mathbf{c} = (c_j, 1)'$ and obtain (for $h = N^{(t-v)} + \cdots + N^{(t-1)} + u$)

$$\begin{aligned}
\mathbb{E}\left(C_{i,t}^{(j)}C_{i-u,t-v}^{(j)}\right) &= \mathbb{E}\left(C_{i,t}^{(j)}C_{i-u,t-v}^{(j)}|C_{i-u,t-v}^{(j)} = c_j\right)\mathbb{P}\mathrm{rob}\left\{C_{i-u,t-v}^{(j)} = c_j\right\} \\
&\quad + \mathbb{E}\left(C_{i,t}^{(j)}C_{i-u,t-v}^{(j)}|C_{i-u,t-v}^{(j)} = 1\right)\mathbb{P}\mathrm{rob}\left\{C_{i-u,t-v}^{(j)} = 1\right\} \\
&= 0.5\cdot c_j\cdot\mathbb{E}\left(C_{i,t}^{(j)}|C_{i-u,t-v}^{(j)} = c_j\right) \\
&\quad + 0.5\cdot\mathbb{E}\left(C_{i,t}^{(j)}|C_{i-u,t-v}^{(j)} = 1\right) \qquad \text{(A.16)} \\
&= 0.5\cdot\mathbf{c}'\mathbf{P}^h\mathbf{c} = 0.5\cdot\mathbf{c}'\mathbf{E}\mathbf{A}^h\mathbf{E}'\mathbf{c} \\
&= 0.25\left(c_j+1\right)^2 + 0.25(2p-1)^h\left(c_j-1\right)^2, \qquad \text{(A.17)}
\end{aligned}$$

where we used in (A.16) the unconditional distribution on the FHMV multiplier $C_{i,t}^{(j)}$ being given by $\mathbb{P}\mathrm{rob}\left\{C^{(j)}_{i-u,t-v}=c_j\right\}=\mathbb{P}\mathrm{rob}\left\{C^{(j)}_{i-u,t-v}=1\right\}=0.5.$[14] With these point probabilities, it follows that

$$\mathbb{E}\left(C^{(j)}_{i,t}\right)=0.5\cdot(c_j+1)\quad\text{and}\quad\mathbb{V}\mathrm{ar}\left(C^{(j)}_{i,t}\right)=0.25\cdot(c_j-1)^2,\tag{A.18}$$

so that Eq. (A.17) can be written as

$$\mathbb{E}\left(C^{(j)}_{i,t}C^{(j)}_{i-u,t-v}\right)=\left[\mathbb{E}\left(C^{(j)}_{i,t}\right)\right]^2+\mathbb{V}\mathrm{ar}\left(C^{(j)}_{i,t}\right)\cdot(2p-1)^h.\tag{A.19}$$

Inserting (A.18), (A.19) into Eq. (A.14) yields the covariance function

$$\begin{aligned}\mathbb{C}\mathrm{ov}(g_{i,t},g_{i-u,t-v}) &= \prod_{j=1}^{N_C}\frac{\left[\mathbb{E}\left(C^{(j)}_{i,t}\right)\right]^2+\mathbb{V}\mathrm{ar}\left(C^{(j)}_{i,t}\right)(2p-1)^h}{\left[\mathbb{E}\left(C^{(j)}_{i,t}\right)\right]^2}-1\\ &= \prod_{j=1}^{N_C}\left[1+\left(\frac{c_j-1}{c_j+1}\right)^2(2p-1)^h\right]-1.\end{aligned}\tag{A.20}$$

Similarly, we obtain

$$\mathbb{V}\mathrm{ar}\left(g_{i,t}\right)=\mathbb{E}\left(C^2_{i,t}\right)\cdot\mathbb{E}\left(Z^2_{i,t}\right)-1,$$

where $\mathbb{E}\left(Z^2_{i,t}\right)=z_0^2\left(\frac{q}{N_Z-1}\sum_{j=1}^{N_Z-1}z_j^2+(1-q)\right)$, and

$$\mathbb{E}\left[C^2_{i,t}\right]=\prod_{j=1}^{N_C}\left\{\frac{\mathbb{E}\left[(C^{(j)}_{i,t})^2\right]}{\left[\mathbb{E}(C^{(j)}_{i,t})\right]^2}\right\}=\prod_{j=1}^{N_C}\left(\frac{0.5+0.5c_j^2}{0.25(c_j+1)^2}\right)=\prod_{j=1}^{N_C}\left(1+\left(\frac{c_j-1}{c_j+1}\right)^2\right),$$

and thus,

$$\mathbb{V}\mathrm{ar}\left(g_{i,t}\right)=\prod_{j=1}^{N_C}\left(1+\left(\frac{c_j-1}{c_j+1}\right)^2\right)\cdot z_0^2\left(\frac{q}{N_Z-1}\sum_{j=1}^{N_Z-1}z_j^2+(1-q)\right)-1.\tag{A.21}$$

Taking the ratio of (A.20) and (A.21) yields Eq. (23). □

[14] The explicit form of the unconditional distribution follows from the general FHMV framework in Section 2.2. A proof is given in the supplement of Augustyniak et al. (2019) on pp. 13, 14.

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Tables and Figures

Table 1: Deviations of ML-estimates from the true parameters; DGP: MSM-MIDAS

	Empirical specification	m_0	γ_{N_M}	b	m	θ	ω_2
$T = 2640$	MSM-MIDAS	0.010	-0.006	0.175	0.018	-0.065	-29.688
		(0.026)	(0.295)	(0.477)	(0.143)	(0.317)	(82.811)
	MSM-MIDAS($K = 12$)	0.007	-0.016	0.102	0.010	0.052	-27.080
		(0.020)	(0.277)	(0.450)	(0.109)	(0.192)	(62.792)
	MSM-MIDAS($\tilde{X}_t$)	0.010	0.005	0.173	0.017	-0.058	-15.484
		(0.029)	(0.300)	(0.496)	(0.130)	(0.299)	(57.547)
	MSM-MIDAS($K = 12, \tilde{X}_t$)	0.007	-0.013	0.079	0.013	0.059	-16.226
		(0.028)	(0.280)	(0.525)	(0.105)	(0.177)	(50.430)
	MSM	0.003	-0.021	-0.314	0.043	–	–
		(0.068)	(0.262)	(1.654)	(0.074)	–	–
$T = 5280$	MSM-MIDAS	0.004	-0.012	0.072	0.009	-0.036	-17.120
		(0.013)	(0.241)	(0.372)	(0.083)	(0.184)	(63.102)
	MSM-MIDAS($K = 12$)	0.003	-0.022	0.019	0.005	0.040	-14.365
		(0.013)	(0.233)	(0.435)	(0.078)	(0.127)	(48.047)
	MSM-MIDAS($\tilde{X}_t$)	0.005	-0.011	0.071	0.008	-0.023	-9.320
		(0.019)	(0.237)	(0.385)	(0.084)	(0.175)	(44.890)
	MSM-MIDAS($K = 12, \tilde{X}_t$)	0.003	-0.018	0.033	0.007	0.063	-8.457
		(0.013)	(0.227)	(0.379)	(0.077)	(0.123)	(39.341)
	MSM	-0.002	-0.023	-0.295	0.043	–	–
		(0.059)	(0.225)	(0.968)	(0.067)	–	–
$T = 10560$	MSM-MIDAS	0.003	-0.011	0.039	0.005	-0.026	-4.131
		(0.009)	(0.175)	(0.228)	(0.057)	(0.112)	(25.884)
	MSM-MIDAS($K = 12$)	0.002	-0.012	0.023	0.004	0.037	-1.908
		(0.009)	(0.169)	(0.221)	(0.055)	(0.088)	(23.840)
	MSM-MIDAS($\tilde{X}_t$)	0.003	-0.011	0.040	0.005	-0.019	-3.412
		(0.009)	(0.175)	(0.231)	(0.055)	(0.110)	(24.925)
	MSM-MIDAS($K = 12, \tilde{X}_t$)	0.002	-0.012	0.021	0.004	0.049	0.112
		(0.009)	(0.169)	(0.226)	(0.055)	(0.082)	(16.220)
	MSM	-0.010	-0.010	-0.458	0.049	–	–
		(0.049)	(0.163)	(1.125)	(0.064)	–	–

Note: Standard errors of the ML estimators are in parentheses. For the innovation process in Eq. (1), we assume $\{\varepsilon_t\} \overset{\text{i.i.d.}}{\sim} \mathbb{N}(0,1)$. The parameters of the $\{\tau_t\}$-DGP from Eqs. (2)–(3) are $K = 36, m = 0.1, \theta = 0.3, \omega_1 = 1$ (not estimated), $\omega_2 = 4$. The explanatory variable is specified as $X_t = 0.9 \cdot X_{t-1} + \nu_t$ with $\nu_t \overset{\text{i.i.d.}}{\sim} \mathbb{N}(0, 0.3^2)$. The DGP parameters of the MSM short-term volatility component $\{g_{i,t}\}$ from Eqs. (4)–(6) are $m_0 = 1.2, \gamma_{N_M} = 0.5, b = 2, N_M = 8$. 'MSM-MIDAS' denotes the benchmark specification estimated under all the above-mentioned DGP parameters. 'MSM-MIDAS($K = 12$)' denotes the specification, estimated with $K = 12$ (instead of $K = 36$). 'MSM-MIDAS($\tilde{X}_t$)' denotes the estimated specification when the $\{\tau_t\}$-DGP is based on the noisy explanatory variable $\tilde{X}_t = X_t + \upsilon_t$ with $\upsilon_t \overset{\text{i.i.d.}}{\sim} \mathbb{N}(0, 0.2 + 0.8|X_t|)$ (*ceteris paribus*). 'MSM-MIDAS($K = 12, \tilde{X}_t$)' combines 'MSM-MIDAS($K = 12$)' and 'MSM-MIDAS($\tilde{X}_t$)'. In the empirical specification 'MSM', we only estimated the MSM short-term volatility $\{g_{i,t}\}$ (no long-term component) under the fully-specified DGP.

Table 2: Deviations of ML-estimates from the true parameters; DGP: FHMV-MIDAS

	Empirical specification	c_1	θ_C	p	z_1	θ_Z	q	m	θ	ω_2
$T = 2640$	FHMV-MIDAS	-0.262	0.109	0.002	-0.988	0.120	-0.156	0.027	-0.052	-14.549
		(0.861)	(0.234)	(0.003)	(14.646)	(0.406)	(0.265)	(0.324)	(0.725)	(48.332)
	FHMV-MIDAS($K = 12$)	-0.209	0.076	0.001	-0.361	0.103	-0.130	0.022	0.053	-17.218
		(0.773)	(0.212)	(0.002)	(4.309)	(0.405)	(0.243)	(0.255)	(0.388)	(44.759)
	FHMV-MIDAS($\tilde{X}_t$)	-0.287	0.115	0.001	-0.214	0.101	-0.148	0.042	-0.110	-9.735
		(0.841)	(0.227)	(0.003)	(3.964)	(0.411)	(0.255)	(0.308)	(0.657)	(37.769)
	FHMV-MIDAS($K = 12, \tilde{X}_t$)	-0.220	0.078	0.001	-0.058	0.096	-0.126	0.037	0.080	-13.710
		(0.781)	(0.205)	(0.002)	(3.660)	(0.406)	(0.232)	(0.244)	(0.299)	(41.574)
	FHMV	-0.115	0.032	0.001	-0.605	0.131	-0.129	0.045	–	–
		(0.683)	(0.181)	(0.002)	(7.031)	(0.403)	(0.237)	(0.114)	–	–
$T = 5280$	FHMV-MIDAS	-0.072	0.034	0.001	0.085	0.104	-0.091	0.036	-0.049	-11.442
		(0.627)	(0.171)	(0.001)	(2.450)	(0.387)	(0.196)	(0.191)	(0.417)	(43.991)
	FHMV-MIDAS($K = 12$)	-0.059	0.025	0.001	0.101	0.091	-0.078	0.028	0.053	-13.664
		(0.598)	(0.161)	(0.001)	(2.604)	(0.383)	(0.175)	(0.175)	(0.279)	(41.429)
	FHMV-MIDAS($\tilde{X}$)	-0.084	0.041	0.001	0.151	0.107	-0.079	0.026	-0.046	-10.223
		(0.615)	(0.169)	(0.002)	(2.586)	(0.378)	(0.180)	(0.195)	(0.379)	(40.916)
	FHMV-MIDAS($K = 12, \tilde{X}_t$)	-0.069	0.029	0.001	0.227	0.113	-0.071	0.017	0.081	-9.647
		(0.587)	(0.161)	(0.001)	(2.156)	(0.378)	(0.158)	(0.174)	(0.215)	(34.771)
	FHMV	-0.042	0.005	0.000	-0.679	0.111	-0.078	0.049	–	–
		(0.557)	(0.144)	(0.001)	(19.966)	(0.376)	(0.166)	(0.089)	–	–
$T = 10560$	FHMV-MIDAS	-0.010	0.008	0.000	0.276	0.084	-0.041	0.013	-0.048	-10.558
		(0.487)	(0.135)	(0.001)	(1.628)	(0.361)	(0.114)	(0.125)	(0.245)	(40.268)
	FHMV-MIDAS($K = 12$)	-0.003	0.003	0.000	0.292	0.091	-0.042	0.008	0.049	-9.876
		(0.480)	(0.131)	(0.001)	(1.570)	(0.358)	(0.112)	(0.120)	(0.178)	(36.510)
	FHMV-MIDAS($\tilde{X}_t$)	-0.013	0.010	0.000	0.284	0.092	-0.043	0.015	-0.031	-8.088
		(0.471)	(0.131)	(0.001)	(1.655)	(0.367)	(0.107)	(0.125)	(0.235)	(37.008)
	FHMV-MIDAS($K = 12, \tilde{X}$)	0.001	0.004	0.000	0.335	0.083	-0.043	0.012	0.072	-8.201
		(0.457)	(0.126)	(0.001)	(1.600)	(0.370)	(0.113)	(0.120)	(0.157)	(34.296)
	FHMV	0.009	-0.010	0.000	0.264	0.106	-0.044	0.046	–	–
		(0.452)	(0.121)	(0.001)	(2.043)	(0.354)	(0.108)	(0.063)	–	–

Note: Analogous to the notes for Table 1. The DGP parameters of the FHMV short-term volatility component $\{g_{i,t}\}$ from Eqs. (7)–(14) are $c_1 = 2.5, \theta_C = 0.8, p = 0.995, N_C = 8$ (persistent components), $z_1 = 5, \theta_Z = 0.5, q = 0.1, N_Z = 8$ (jump components).

Table 3: Deviations of ML-estimates from the true parameters; DGP: MSGARCH-MIDAS

	Empirical specification	α_{11}	α_{12}	β_1	β_2	p	m	θ	ω_2
$T = 2640$	MSGARCH-MIDAS	-0.002	0.036	-0.099	-0.238	-0.299	-0.038	-0.114	-1.005
		(0.006)	(0.007)	(0.083)	(0.130)	(0.174)	(0.002)	(0.020)	(1.347)
	MSGARCH-MIDAS($K = 12$)	-0.002	0.034	-0.092	-0.249	-0.297	-0.039	-0.122	-1.047
		(0.007)	(0.007)	(0.087)	(0.138)	(0.176)	(0.002)	(0.022)	(1.427)
	MSGARCH-MIDAS($\tilde{X}_t$)	-0.007	0.037	-0.101	-0.232	-0.318	-0.043	-0.053	-0.804
		(0.005)	(0.009)	(0.077)	(0.118)	(0.179)	(0.003)	(0.005)	(0.939)
	MSGARCH-MIDAS($K = 12, \tilde{X}$)	-0.003	0.036	-0.099	-0.218	-0.328	-0.044	-0.087	-1.173
		(0.006)	(0.008)	(0.087)	(0.122)	(0.185)	(0.003)	(0.011)	(1.680)
	MSGARCH	-0.008	0.036	-0.083	-0.216	-0.300	0.005	–	–
		(0.005)	(0.006)	(0.096)	(0.133)	(0.165)	(0.015)	–	–
$T = 5280$	MSGARCH-MIDAS	-0.009	0.032	-0.076	-0.215	-0.297	-0.035	-0.108	-0.973
		(0.004)	(0.005)	(0.063)	(0.102)	(0.151)	(0.002)	(0.017)	(1.286)
	MSGARCH-MIDAS($K = 12$)	-0.011	0.029	-0.064	-0.220	-0.309	-0.035	-0.116	-1.026
		(0.003)	(0.004)	(0.070)	(0.112)	(0.163)	(0.002)	(0.019)	(1.397)
	MSGARCH-MIDAS($\tilde{X}_t$)	-0.012	0.029	-0.060	-0.219	-0.304	-0.044	-0.048	-0.805
		(0.002)	(0.004)	(0.057)	(0.102)	(0.152)	(0.003)	(0.004)	(0.917)
	MSGARCH-MIDAS($K = 12, \tilde{X}_t$)	-0.011	0.030	-0.037	-0.216	-0.311	-0.041	-0.075	-1.281
		(0.003)	(0.004)	(0.074)	(0.119)	(0.162)	(0.003)	(0.008)	(1.926)
	MSGARCH	-0.012	0.029	-0.037	-0.212	-0.279	0.010	–	–
		(0.003)	(0.003)	(0.091)	(0.139)	(0.146)	(0.009)	–	–
$T = 10560$	MSGARCH-MIDAS	-0.016	0.025	-0.054	-0.195	-0.289	-0.034	-0.097	-0.889
		(0.002)	(0.002)	(0.050)	(0.084)	(0.135)	(0.002)	(0.014)	(1.128)
	MSGARCH-MIDAS($K = 12$)	-0.016	0.025	-0.047	-0.189	-0.283	-0.033	-0.108	-1.012
		(0.002)	(0.003)	(0.054)	(0.087)	(0.141)	(0.002)	(0.016)	(1.344)
	MSGARCH-MIDAS($\tilde{X}_t$)	-0.014	0.023	-0.042	-0.212	-0.295	-0.047	-0.046	-0.754
		(0.002)	(0.002)	(0.046)	(0.091)	(0.133)	(0.003)	(0.003)	(0.821)
	MSGARCH-MIDAS($K = 12, \tilde{X}_t$)	-0.016	0.027	-0.021	-0.188	-0.313	-0.044	-0.070	-1.253
		(0.002)	(0.003)	(0.064)	(0.099)	(0.150)	(0.003)	(0.007)	(1.842)
	MSGARCH	-0.013	0.027	-0.033	-0.191	-0.231	0.011	–	–
		(0.002)	(0.003)	(0.106)	(0.141)	(0.127)	(0.005)	–	–

Note: Analogous to the notes for Table 1. The DGP parameters of the MSGARCH short-term volatility component $\{g_{i,t}\}$ from Eqs. (16)–(17) are $\alpha_{11} = 0.09, \alpha_{12} = 0.05, \beta_1 = 0.55, \beta_2 = 0.7, p_{11} = p_{22} = 0.8 \equiv p$.

Table 4: Deviations of ML-estimates from the true parameters; DGP: HYGARCH-MIDAS

	Empirical specification	ϕ	d	β	κ	m	θ	ω_2
$T = 2640$	HYGARCH-MIDAS	-0.063	0.023	-0.025	0.075	0.008	-0.078	3.745
		(0.405)	(0.619)	(0.727)	(0.765)	(0.195)	(0.411)	(48.474)
	HYGARCH-MIDAS($K = 12$)	-0.060	0.032	-0.013	0.075	0.010	-0.031	6.915
		(0.213)	(0.440)	(0.479)	(0.427)	(0.149)	(0.226)	(51.423)
	HYGARCH-MIDAS($\tilde{X}_t$)	-0.106	0.227	0.116	0.043	0.006	-0.081	0.806
		(0.170)	(0.354)	(0.380)	(0.213)	(0.096)	(0.032)	(43.979)
	HYGARCH-MIDAS($K = 12, \tilde{X}_t$)	-0.105	0.228	0.118	0.044	0.008	-0.087	2.101
		(0.723)	(2.210)	(1.933)	(1.142)	(0.105)	(0.094)	(141.179)
	HYGARCH	0.004	-0.049	0.010	0.147	0.019	–	–
		(0.014)	(0.021)	(0.022)	(0.028)	(0.174)	–	–
$T = 5280$	HYGARCH-MIDAS	0.047	0.012	0.037	0.081	-0.003	-0.067	2.102
		(0.227)	(0.425)	(0.416)	(0.549)	(0.187)	(0.204)	(33.681)
	HYGARCH-MIDAS($K = 12$)	-0.049	-0.004	-0.032	0.079	0.000	-0.051	5.104
		(0.287)	(0.586)	(0.392)	(1.076)	(0.181)	(0.129)	(25.840)
	HYGARCH-MIDAS($\tilde{X}_t$)	-0.098	0.223	0.133	0.050	-0.000	-0.081	0.688
		(0.101)	(0.441)	(0.545)	(0.290)	(0.102)	(0.041)	(108.105)
	HYGARCH-MIDAS($K = 12, \tilde{X}_t$)	-0.094	0.223	0.129	0.048	0.001	-0.092	1.420
		(0.141)	(0.262)	(0.319)	(0.167)	(0.109)	(0.025)	(42.449)
	HYGARCH	0.039	-0.089	0.024	0.186	0.003	–	–
		(0.010)	(0.011)	(0.013)	(0.036)	(0.115)	–	–
$T = 10560$	HYGARCH-MIDAS	-0.031	-0.036	-0.041	0.080	0.002	-0.055	2.391
		(0.137)	(0.274)	(0.245)	(0.333)	(0.098)	(0.121)	(13.134)
	HYGARCH-MIDAS($K = 12$)	-0.035	-0.030	-0.040	0.079	0.007	-0.063	3.521
		(0.166)	(0.282)	(0.261)	(0.376)	(0.155)	(0.017)	(20.476)
	HYGARCH-MIDAS($\tilde{X}_t$)	-0.103	0.264	0.167	0.056	-0.009	-0.080	-0.108
		(0.076)	(0.237)	(0.300)	(0.148)	(0.069)	(0.017)	(22.049)
	HYGARCH-MIDAS($K = 12, \tilde{X}_t$)	-0.101	0.262	0.166	0.056	-0.006	-0.095	0.323
		(0.118)	(0.272)	(0.339)	(0.124)	(0.081)	(0.022)	(54.470)
	HYGARCH	0.060	-0.088	0.050	0.199	-0.023	–	–
		(0.008)	(0.009)	(0.009)	(0.040)	(0.070)	–	–

Note: Analogous to the notes for Table 1. The DGP parameters of the HYGARCH short-term volatility component $\{g_{i,t}\}$ from Eqs. (18)–(19) are $\phi = 0.2, d = 0.4, \beta = 0.4, \kappa = 0.8$.

Table 5: MSE and QLike ratios for MSM-MIDAS specifications, benchmark: (non-MIDAS) MSM($N_M = 8$)

	MSE				QLike			
	Forecast horizon							
Long-term variable	1q	2q	3q	4q	1q	2q	3q	4q
(1) Δ Real GDP	0.992	0.997	0.990	0.991	0.996*	0.997	0.994*	0.994*
	(0.399)	0.637)	(0.123)	(0.120)	(0.099)	(0.162)	(0.078)	0.088
(2) Δ Ind. prod.	1.049	1.072	1.037	1.047	1.000	1.001	0.999	1.000
	(0.570)	(0.381)	(0.414)	(0.384)	(0.986)	(0.507)	(0.499)	(0.844)
(3) Δ Unemp.	0.984	0.988	0.980	0.985	0.997**	0.995*	0.988	0.991
	(0.409)	(0.405)	(0.118)	(0.255)	(0.023)	(0.057)	(0.137)	(0.231)
(4) Δ Hous.	1.094	1.139	1.120	1.142	0.972*	0.966	0.952	0.954
(unrestricted ω_1)	(0.328)	(0.365)	(0.449)	(0.423)	(0.060)	(0.240)	(0.321)	(0.314)
(5) Δ Corp. prof.	0.967	0.994	0.987	0.999	0.974**	0.982	0.981	0.986
	(0.575)	(0.859)	(0.504)	(0.954)	(0.035)	(0.151)	(0.215)	(0.337)
(6) Inflation	0.977	0.989	0.975	0.984	0.997	0.993	0.995**	1.005
	(0.607)	(0.444)	(0.242)	(0.462)	(0.434)	(0.148)	(0.041)	(0.532)
(7) NAI	0.983	0.998	0.986	0.981*	1.001	1.002	1.002	0.999
	(0.398)	(0.864)	(0.273)	(0.083)	(0.489)	(0.478)	(0.529)	(0.599)
(8) New orders	1.098	1.099	1.082	1.097	0.995	0.996	0.994	1.000
	(0.186)	(0.283)	(0.285)	(0.250)	(0.122)	(0.272)	(0.209)	(0.926)
(9) Δ Cons. sent.	0.985	0.989	0.987	0.993	0.989	0.987*	0.982	0.985
	(0.296)	(0.337)	(0.140)	(0.396)	(0.102)	(0.099)	(0.115)	(0.142)
(10) Δ Real cons.	1.004	0.999	0.998	1.002	1.000	0.997	0.998	1.002
	(0.691)	(0.903)	(0.635)	(0.439)	(0.932)	(0.311)	(0.437)	(0.529)
(11) Term spread	0.969	0.981	0.958	0.960	0.973**	0.960	0.941	0.933
	(0.456)	(0.709)	(0.376)	(0.445)	(0.029)	(0.110)	(0.192)	(0.142)
(12) RV	1.633	1.656	1.492	1.507	1.002	1.002	1.000	1.006
	(0.277)	(0.287)	(0.277)	(0.265)	(0.675)	(0.671)	(0.937)	(0.167)

Note: We report ratios of the average MSE and QLike losses relative to their counterparts from the benchmark models. Δ denotes either the first-difference operator or the (annualized quarterly) percentage change. The long-term variables are: (1) real GDP, (2) industrial production, (3) unemployment rate, (4) housing starts, (5) nominal corporate profits after tax, (6) inflation rate, (7) Chicago National Activity Index (NAI), (8) new-order index of the Institute for Supply Management, (9) consumer-sentiment index University of Michigan, (10) real personal consumption, (11) term spread, (12) realized volatility (cf. Section 4.1). We test for significant forecasting improvements via the Giacomoni-White (2006) test. p-values are in parentheses. *, **, *** denote significant accuracy gains at the 10, 5, and 1% levels, respectively.

Table 6: MSE and QLike ratios for FHMV-MIDAS specifications, benchmark: (non-MIDAS) FHMV($N_C = 6, N_Z = 6$)

	MSE				QLike			
	Forecast horizon							
Long-term variable	1q	2q	3q	4q	1q	2q	3q	4q
(1) Δ Real GDP	1.013	1.013	1.009	1.011	0.999	1.000	1.001	1.002
	(0.239)	(0.206)	(0.221)	(0.136)	(0.619)	(0.746)	(0.340)	(0.173)
(2) Δ Ind. prod.	1.011	1.011	1.008	1.012	1.002	1.001	1.002	1.005
	(0.196)	(0.195)	(0.242)	(0.170)	(0.214)	(0.485)	(0.263)	(0.235)
(3) Δ Unemp.	0.994**	1.003	0.995	0.998	0.999	1.000	1.000	1.001
	(0.020)	(0.804)	(0.372)	(0.848)	(0.469)	(0.979)	(0.959)	(0.672)
(4) Δ Hous.	1.009	1.039	0.979	0.952	0.995	0.995	0.984	0.975
(unrestricted ω_1)	(0.747)	(0.438)	(0.609)	(0.396)	(0.331)	(0.644)	(0.339)	(0.220)
(5) Δ Corp. prof.	1.001	1.002	1.005	1.010	0.997*	0.999	1.002	1.004
	(0.878)	(0.624)	(0.392)	(0.276)	(0.099)	(0.282)	(0.566)	(0.420)
(6) Inflation	1.005	1.011	1.041**	1.072***	1.004	1.000	1.012**	1.017**
	(0.932)	(0.781)	(0.018)	(0.000)	(0.528)	(0.986)	(0.003)	(0.025)
(7) NAI	0.987	0.998	0.983	0.982	1.000	0.997	0.997	0.996
	(0.111)	(0.947)	(0.227)	(0.297)	(0.797)	(0.466)	(0.558)	(0.493)
(8) New orders	0.994	1.030	1.004	1.014	1.008*	1.008	1.005	1.003
	(0.923)	(0.525)	(0.885)	(0.499)	(0.057)	(0.284)	(0.494)	(0.615)
(9) Δ Cons. sent.	1.037	1.140	1.163	1.185	0.989	0.996	1.005	1.007
	(0.480)	(0.285)	(0.220)	(0.200)	(0.123)	(0.618)	(0.476)	(0.454)
(10) Δ Real cons.	0.983	0.987	0.979	0.981	0.999	0.998	0.995	0.993
	(0.494)	(0.525)	(0.173)	(0.223)	(0.556)	(0.466)	(0.329)	(0.305)
(11) Term spread	0.922**	0.928**	0.898***	0.890***	0.986	0.979***	0.965***	0.959***
	(0.021)	(0.013)	(0.001)	(0.002)	(0.135)	(0.000)	(0.004)	(0.007)
(12) RV	1.029	1.036	1.009	1.015	1.003*	1.007	1.005	1.005
	(0.367)	(0.276)	(0.663)	(0.442)	(0.097)	(0.225)	(0.382)	(0.447)

Note: Analogous to the notes to Table 5.

Table 7: MSE and QLike ratios for MSGARCH-MIDAS specifications, benchmark: (non-MIDAS) MSGARCH

	MSE				QLike			
	Forecast horizon							
Long-term variable	1q	2q	3q	4q	1q	2q	3q	4q
(1) Δ Real GDP	1.380	0.922	0.595*	0.380***	1.629***	1.684***	1.474***	1.343**
	(0.213)	(0.745)	(0.078)	(0.001)	(0.000)	(0.006)	(0.007)	(0.025)
(2) Δ Ind. prod.	1.603	0.875	0.564**	0.376***	1.530***	1.492***	1.410**	1.299**
	(0.251)	(0.523)	(0.039)	(0.001)	(0.000)	(0.004)	(0.017)	(0.045)
(3) Δ Unemp.	1.865	0.987	0.630	0.405***	2.003***	2.110***	1.820***	1.783**
	(0.242)	(0.964)	(0.141)	(0.005)	(0.000)	(0.006)	(0.006)	(0.040)
(4) Δ Hous.	0.939	0.849	0.571*	0.357***	0.988	0.992	0.974	0.951
(unrestricted ω_1)	(0.430)	(0.442)	(0.062)	(0.001)	(0.239)	(0.857)	(0.649)	(0.469)
(5) Δ Corp. prof.	1.781	0.973	0.642	0.374***	1.385***	1.536**	1.503*	1.309*
	(0.261)	(0.922)	(0.184)	(0.001)	(0.000)	(0.047)	(0.088)	(0.080)
(6) Inflation	1.863	0.972	0.648	0.400***	1.728***	1.665***	1.705**	1.539**
	(0.227)	(0.918)	(0.198)	(0.004)	(0.000)	(0.002)	(0.017)	(0.026)
(7) NAI	1.562	0.909	0.594*	0.383***	1.746***	1.631***	1.559***	1.461***
	(0.219)	(0.686)	(0.081)	(0.002)	(0.000)	(0.001)	(0.007)	(0.010)
(8) New orders	1.828	0.950	0.598	0.373***	1.528***	1.439*	1.222*	1.107
	(0.238)	(0.852)	(0.101)	(0.001)	(0.004)	(0.073)	(0.088)	(0.366)
(9) Δ Cons. sent.	1.870	0.996	0.655	0.394***	2.074***	2.210***	2.011***	1.756**
	(0.226)	(0.989)	(0.212)	(0.004)	(0.000)	(0.005)	(0.009)	(0.037)
(10) Δ Real cons.	1.676	0.955	0.647	0.395***	1.724***	1.630***	1.558**	1.526*
	(0.116)	(0.843)	(0.185)	(0.004)	(0.000)	(0.001)	(0.021)	(0.093)
(11) Term spread	1.636	0.947	0.646	0.402***	1.647***	1.602***	1.637**	1.568*
	(0.240)	(0.832)	(0.194)	(0.005)	(0.000)	(0.001)	(0.026)	(0.096)
(12) RV	1.820	1.160	0.812	0.546**	1.203**	1.220*	1.262*	1.243
	(0.245)	(0.585)	(0.474)	(0.032)	(0.024)	(0.095)	(0.096)	(0.204)

Note: Analogous to the notes to Table 5.

Table 8: MSE and QLike ratios for HYGARCH-MIDAS specifications, benchmark: (non-MIDAS) HYGARCH

	MSE				QLike			
	Forecast horizon							
Long-term variable	1q	2q	3q	4q	1q	2q	3q	4q
(1) Δ Real GDP	1.122	0.972	0.959	0.953	1.002	1.006	1.019	1.009
	(0.499)	(0.515)	(0.385)	(0.259)	(0.604)	(0.338)	(0.375)	(0.456)
(2) Δ Ind. prod.	1.123	0.986	0.965	0.958	1.004	1.013	1.008	1.011
	(0.495)	(0.805)	(0.515)	(0.373)	(0.537)	(0.365)	(0.487)	(0.404)
(3) Δ Unemp.	1.007	1.009	1.008	1.007	1.002	1.006	1.005	1.008
	(0.480)	(0.367)	(0.522)	(0.566)	(0.404)	(0.241)	(0.363)	(0.253)
(4) Δ Hous.	1.012	1.006	1.005	1.002	1.000	1.005	1.006	1.011
(unrrestricted ω_1)	(0.597)	(0.715)	(0.809)	(0.891)	(0.946)	(0.429)	(0.463)	(0.348)
(5) Δ Corp. prof.	1.013	1.008	1.007	1.002	1.001	1.007	1.008	1.010
	(0.578)	(0.617)	(0.728)	(0.899)	(0.566)	(0.230)	(0.273)	(0.219)
(6) Inflation	1.003	1.009	1.014	1.006	1.003	1.014	1.020	1.020
	(0.893)	(0.630)	(0.553)	(0.760)	(0.611)	(0.301)	(0.273)	(0.207)
(7) NAI	1.117	0.995	0.985	0.970	1.008	1.023	1.025	1.026
	(0.517)	(0.942)	(0.823)	(0.572)	(0.398)	(0.248)	(0.285)	(0.208)
(8) New orders	0.907	1.012	1.011	0.968	1.002	1.011	1.013	1.003
	(0.203)	(0.724)	(0.757)	(0.138)	(0.662)	(0.377)	(0.348)	(0.598)
(9) Δ Cons. sent.	1.010	0.980	0.969	0.972	1.001	1.002	1.000	1.004
	(0.862)	(0.539)	(0.386)	(0.369)	(0.541)	(0.770)	(0.954)	(0.617)
(10) Δ Real cons.	1.138	1.091	1.131	1.124	1.001	1.008	1.015	1.000
	(0.343)	(0.400)	(0.318)	(0.375)	(0.776)	(0.470)	(0.442)	(0.989)
(11) Term spread	1.092	0.989	0.981	0.977	1.003	1.013	1.027	1.033
	(0.542)	(0.848)	(0.750)	(0.666)	(0.316)	(0.352)	(0.213)	(0.246)
(12) RV	0.698	0.962	0.975	0.965	0.998	0.999	0.997	0.988
	(0.268)	(0.208)	(0.152)	(0.287)	(0.242)	(0.867)	(0.606)	(0.426)

Note: Analogous to the notes to Table 5.

Table 9: MSE and QLike ratios for GJR-GARCH-MIDAS specifications, benchmark: (non-MIDAS) GJR-GARCH

	MSE				QLike			
	Forecast horizon							
Long-term variable	1q	2q	3q	4q	1q	2q	3q	4q
(1) Δ Real GDP	1.015	1.003	1.000	0.996**	0.999	1.001	1.000	0.997**
	(0.309)	(0.331)	(0.823)	(0.034)	(0.460)	(0.659)	(0.826)	(0.028)
(2) Δ Ind. prod.	1.065	1.009	1.005*	1.001	0.999	1.003	1.003	1.000
	(0.172)	(0.125)	(0.051)	(0.740)	(0.652)	(0.242)	(0.104)	(0.825)
(3) Δ Unemp.	1.244	1.032	1.014	1.022	0.992	0.999	1.002	1.009*
	(0.182)	(0.210)	(0.262)	(0.128)	(0.244)	(0.845)	(0.567)	(0.090)
(4) Δ Hous.	3.532	1.164	1.017	1.014	0.967	0.966	0.966	0.970
(unrestricted ω_1)	(0.215)	(0.437)	(0.872)	(0.882)	(0.208)	(0.341)	(0.283)	(0.304)
(5) Δ Corp. prof.	1.297	1.031	1.051	1.063	1.017*	1.052*	1.075	1.094
	(0.174)	(0.268)	(0.173)	(0.152)	(0.093)	(0.077)	(0.131)	(0.151)
(6) Inflation	1.062	1.021	1.011	1.013	0.998	1.003	1.006	1.011
	(0.338)	(0.140)	(0.230)	(0.269)	(0.748)	(0.440)	(0.379)	(0.243)
(7) NAI	1.818	1.128	1.051	1.049	0.997	1.001	1.013	1.028
	(0.145)	(0.184)	(0.165)	(0.178)	(0.740)	(0.914)	(0.239)	(0.205)
(8) New orders	1.758	1.108*	1.046	1.040	1.014	1.021	1.032	1.026
	(0.144)	(0.097)	(0.117)	(0.145)	(0.451)	(0.264)	(0.246)	(0.263)
(9) Δ Cons. sent.	1.177	0.997	0.995*	1.005	0.994	0.996	0.996	1.003
	(0.301)	(0.781)	(0.096)	(0.243)	(0.480)	(0.407)	(0.104)	(0.370)
(10) Δ Real cons.	0.982	1.004	0.999	0.998	1.006	1.003	1.000	0.999
	(0.331)	(0.491)	(0.706)	(0.417)	(0.325)	(0.202)	(0.785)	(0.500)
(11) Term spread	1.494	0.907	0.924	0.940	0.960	0.964	0.962	0.965
	(0.387)	(0.141)	(0.146)	(0.152)	(0.137)	(0.247)	(0.163)	(0.112)
(12) RV	1.625	1.162	1.151	1.177	1.018	1.032*	1.043	1.049
	(0.132)	(0.144)	(0.146)	(0.144)	(0.172)	(0.096)	(0.148)	(0.143)

Note: Analogous to the notes to Table 5.

Table 10: MSE and QLike ratios for GARCH-MIDAS specifications, benchmark: (non-MIDAS) GARCH

	MSE				QLike			
	Forecast horizon							
Long-term variable	1q	2q	3q	4q	1q	2q	3q	4q
(1) Δ Real GDP	0.986*	1.002	1.000	1.001	1.002	1.000	1.001	1.001
	(0.082)	(0.843)	(0.962)	(0.846)	(0.345)	(0.923)	(0.802)	(0.868)
(2) Δ Ind. prod.	1.053	1.023	1.012	1.005	1.000	1.001	1.001	0.999
	(0.117)	(0.153)	(0.159)	(0.432)	(0.761)	(0.275)	(0.202)	(0.652)
(3) Δ Unemp.	1.618	1.251	1.122	1.091	0.995	1.002	1.005	1.010
	(0.115)	(0.163)	(0.216)	(0.194)	(0.516)	(0.674)	(0.290)	(0.108)
(4) Δ Hous.	7.038	2.256	1.379	1.219	0.974	0.978	0.975	0.979
(unrestricted ω_1)	(0.132)	(0.186)	(0.309)	(0.380)	(0.327)	(0.480)	(0.355)	(0.337)
(5) Δ Corp. prof.	1.613	1.033	1.024	1.063	1.008	1.037*	1.054	1.071
	(0.126)	(0.342)	(0.667)	(0.284)	(0.411)	(0.068)	(0.173)	(0.191)
(6) Inflation	1.086	1.045	1.019	1.013	0.992	0.998	0.998	1.005
	(0.319)	(0.257)	(0.382)	(0.283)	(0.113)	(0.355)	(0.600)	(0.372)
(7) NAI	0.915	0.954	0.977	0.997	1.000	0.997	1.006	1.017
	(0.112)	(0.234)	(0.372)	(0.855)	(0.988)	(0.201)	(0.509)	(0.314)
(8) New orders	2.642	1.522	1.185	1.101	1.007	1.010	1.019	1.015
	(0.102)	(0.153)	(0.181)	(0.195)	(0.617)	(0.347)	(0.296)	(0.379)
(9) Δ Cons. sent.	1.235	1.048	0.999	1.000	0.990	0.996	0.997	0.999
	(0.223)	(0.269)	(0.958)	(0.968)	(0.120)	(0.428)	(0.504)	(0.785)
(10) Δ Real cons.	0.887	0.961	0.984	0.985***	1.009	1.003	1.001	0.995**
	(0.228)	(0.375)	(0.362)	(0.005)	(0.185)	(0.491)	(0.911)	(0.012)
(11) Term spread	2.025	0.837*	0.855**	0.900**	0.958	0.964	0.959*	0.961**
	(0.295)	(0.075)	(0.046)	(0.042)	(0.148)	(0.204)	(0.091)	(0.023)
(12) RV	1.649	1.194	1.137	1.170*	1.028	1.040	1.045	1.049
	(0.189)	(0.219)	(0.172)	(0.093)	(0.101)	(0.122)	(0.180)	(0.197)

Note: Analogous to the notes to Table 5.

Table 11: MCS p-values for MSE and QLike losses and different forecast horizons

Long-term variable / MIDAS model	MSE				QLike			
	Forecast horizon							
	1q	2q	3q	4q	1q	2q	3q	4q
(1) Δ Real GDP								
MSM	0.322	0.080	0.001	0.002	0.335	0.011	0.006	0.000
FHMV	0.083	0.080	0.001	0.002	0.322	0.011	0.006	0.000
MSGARCH	0.322	0.080	0.001	0.002	0.169	0.011	0.006	0.000
HYGARCH	0.322	0.634	0.372	0.002	0.763	0.011	0.006	0.000
GJR-GARCH	0.083	0.080	0.336	0.058	0.347	0.011	0.006	0.029
GARCH	0.083	0.080	0.038	0.002	0.347	0.011	0.136	0.003
(2) Δ Ind. prod.								
MSM	0.083	0.080	0.001	0.002	0.253	0.011	0.006	0.000
FHMV	0.083	0.080	0.001	0.002	0.253	0.011	0.006	0.000
MSGARCH	0.322	0.080	0.336	0.002	0.083	0.011	0.006	0.000
HYGARCH	0.322	0.080	0.372	0.012	0.763	0.011	0.163	0.000
GJR-GARCH	0.083	0.080	0.038	0.058	0.347	0.011	0.006	0.029
GARCH	0.083	0.080	0.001	0.005	0.347	0.011	0.136	0.003
(3) Δ Unemp.								
MSM	0.322	0.080	0.001	0.002	0.347	0.011	0.006	0.000
FHMV	0.083	0.080	0.001	0.002	0.322	0.011	0.006	0.000
MSGARCH	0.322	0.080	0.001	0.002	0.076	0.011	0.006	0.000
HYGARCH	0.322	0.080	0.001	0.002	0.763	0.011	0.730	0.000
GJR-GARCH	0.083	0.080	0.038	0.002	0.347	0.011	0.006	0.000
GARCH	0.083	0.080	0.001	0.002	0.347	0.011	0.006	0.000
(4) Δ Hous. (unrestricted ω_1)								
MSM	0.083	0.080	0.001	0.002	0.347	0.011	0.006	0.000
FHMV	0.083	0.080	0.001	0.058	0.347	0.131	0.163	0.029
MSGARCH	0.322	0.080	0.038	0.002	0.763	0.011	0.006	0.000
HYGARCH	0.322	0.080	0.001	0.002	0.764	0.011	0.163	0.000
GJR-GARCH	0.083	0.080	0.038	0.058	0.347	0.581	0.730	0.250
GARCH	0.083	0.080	0.001	0.002	0.347	0.581	0.730	0.250
(5) Δ Corp. prof.								
MSM	0.322	0.080	0.001	0.002	0.347	0.011	0.006	0.000
FHMV	0.083	0.080	0.001	0.002	0.322	0.011	0.006	0.000
MSGARCH	0.322	0.080	0.001	0.002	0.221	0.011	0.006	0.000
HYGARCH	0.322	0.080	0.001	0.002	0.764	0.011	0.163	0.000
GJR-GARCH	0.083	0.080	0.001	0.002	0.347	0.011	0.006	0.000
GARCH	0.083	0.080	0.001	0.002	0.347	0.011	0.006	0.000
(6) Inflation								
MSM	0.322	0.080	0.001	0.002	0.347	0.011	0.006	0.000
FHMV	0.083	0.080	0.001	0.002	0.221	0.011	0.006	0.000
MSGARCH	0.322	0.080	0.001	0.002	0.198	0.011	0.006	0.000
HYGARCH	0.663	0.080	0.001	0.002	0.763	0.011	0.006	0.000
GJR-GARCH	0.083	0.080	0.038	0.005	0.347	0.011	0.006	0.000
GARCH	0.083	0.080	0.001	0.002	0.347	0.011	0.136	0.000
(7) NAI								
MSM	0.322	0.080	0.001	0.002	0.347	0.011	0.006	0.000
FHMV	0.083	0.080	0.001	0.002	0.347	0.011	0.006	0.000
MSGARCH	0.322	0.080	0.001	0.002	0.076	0.011	0.006	0.000
HYGARCH	0.663	0.080	0.038	0.002	0.763	0.011	0.006	0.000
GJR-GARCH	0.083	0.080	0.001	0.002	0.347	0.011	0.006	0.000
GARCH	0.322	0.080	0.038	0.012	0.347	0.011	0.006	0.000

Continued on next page.

Table 11: Continued.

Long-term variable / MIDAS model	MSE				QLike			
	Forecast horizon							
	1q	2q	3q	4q	1q	2q	3q	4q
(8) New orders								
MSM	0.083	0.080	0.001	0.002	0.322	0.011	0.006	0.000
FHMV	0.083	0.080	0.001	0.002	0.221	0.011	0.006	0.000
MSGARCH	0.322	0.080	0.001	0.002	0.221	0.011	0.006	0.000
HYGARCH	0.663	0.080	0.001	0.002	0.763	0.011	0.136	0.000
GJR-GARCH	0.083	0.080	0.001	0.002	0.347	0.011	0.006	0.000
GARCH	0.083	0.080	0.001	0.002	0.347	0.011	0.006	0.000
(9) Δ Cons. sent.								
MSM	0.322	0.080	0.001	0.002	0.335	0.011	0.006	0.000
FHMV	0.083	0.080	0.001	0.002	0.347	0.011	0.006	0.000
MSGARCH	0.322	0.080	0.001	0.002	0.043	0.011	0.006	0.000
HYGARCH	0.663	0.080	0.038	0.002	0.763	0.581	0.730	0.003
GJR-GARCH	0.083	0.080	0.372	0.058	0.347	0.011	0.136	0.003
GARCH	0.083	0.080	0.038	0.002	0.347	0.011	0.163	0.003
(10) Δ Real cons.								
MSM	0.322	0.080	0.001	0.002	0.322	0.011	0.006	0.000
FHMV	0.083	0.080	0.001	0.002	0.253	0.011	0.006	0.003
MSGARCH	0.322	0.080	0.001	0.002	0.198	0.011	0.006	0.000
HYGARCH	0.322	0.080	0.001	0.002	0.764	0.011	0.136	0.003
GJR-GARCH	0.322	0.080	0.372	0.247	0.347	0.011	0.006	0.029
GARCH	0.322	0.080	0.038	0.058	0.347	0.011	0.163	0.029
(11) Term spread								
MSM	0.322	0.080	0.001	0.002	0.347	0.011	0.006	0.000
FHMV	0.322	0.634	0.372	0.247	0.347	0.599	0.730	0.319
MSGARCH	0.322	0.080	0.001	0.002	0.043	0.011	0.006	0.000
HYGARCH	0.663	0.080	0.038	0.002	0.763	0.011	0.006	0.000
GJR-GARCH	0.083	0.634	0.372	0.247	0.347	0.581	0.730	0.319
GARCH	0.083	1.000	1.000	1.000	0.347	1.000	1.000	1.000
(12) RV								
MSM	0.083	0.080	0.001	0.002	0.253	0.011	0.006	0.000
FHMV	0.083	0.080	0.001	0.002	0.253	0.011	0.006	0.000
MSGARCH	0.322	0.080	0.001	0.002	0.347	0.011	0.006	0.000
HYGARCH	1.000	0.634	0.038	0.002	1.000	0.581	0.730	0.029
GJR-GARCH	0.083	0.080	0.001	0.002	0.347	0.011	0.006	0.000
GARCH	0.083	0.080	0.001	0.002	0.347	0.011	0.006	0.000

Note: We report the MCS p-values under MSE and QLike losses for 1 to 4 quarter-ahead (1q, 2q, 3q, 4q) cumulative variance forecasts. For the significance level α, model i with MCS p-value p_i belongs to the $(1-\alpha)$ MCS $\widehat{\mathcal{M}}^*_{1-\alpha}$, if $p_i \geq \alpha$; see Eq. (24). Gray-shaded cells indicate that the associated MIDAS model belongs to the 90% MCS ($\alpha = 0.1$).