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The Case of Herding is Stronger than You Think

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Abstract

In case of herding, investors follow each other, prices move together more than they normally do, and the cross-sectional dispersion of returns decreases. Chang, Cheng, and Khorana (2000) suggest to test for herding by regressing the cross-sectional absolute deviation on the absolute and squared excess market return. They argue that there is evidence for herding in case of large market movements when the coefficient of the squared excess market return is significantly smaller than zero. We show that the true coefficient of the squared excess market return is positive under the null hypothesis of no herding. The test of Chang, Cheng and Khorana is thus biased against finding evidence for herding. We find that this bias matters. For the S&P 500, the test of Chang, Cheng and Khorana signals that there is no herding over the period from 2008 to 2013, while the modified test based on the correct null hypothesis provides clear evidence for herding.

Keywords: herding, cross-sectional deviation

JEL: G12

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1 Introduction

In case of herd behaviour individual investors suppress their own beliefs and base their investment decision on the collective actions of the market. As a result, herd behaviour leads a group of investors to move in the same direction, pushing stock prices further away from their economic fundamentals, causing momentum and excess volatility (Bikhchandani, Hirshleifer, and Welch (1992), Nofsinger and Sias (1999)).[1] In their classical study Christie and Huang (1995) put forward the cross-sectional standard deviation of stock returns to measure the presence of herd behaviour among investors. Christie and Huang's empirical approach relies on conflicting predictions of rational asset pricing models and herd behaviour during periods of market stress. While rational asset pricing models predict an increase in the level of dispersion, herd behaviour translates into a reduced level of dispersion of individual stock returns around the market. To differentiate empirically between the two hypotheses Christie and Huang isolate the level of dispersion into the lower and upper tail of the returns distribution and test whether these differ from the average level of dispersion.

In a follow up paper Chang, Cheng, and Khorana (2000) argue that under the assumption of rational asset pricing the dispersion measure is linear and strictly monotonically increasing in the expected value of the absolute market return. By contrast, herd behaviour is captured by a function of the dispersion measure that is either non-linear decreasing or reaches a maximum at a certain threshold value of the expected absolute market return and declining thereafter. To test for herd behaviour the cross-sectional absolute deviation of returns is regressed on a constant, the absolute market return and its squared value. A negative parameter on the squared value of market return is an indication of herd behaviour, since it reflects that in periods of market stress the return dispersions decline. The null hypothesis of no herding refers to a coefficient of zero.

While the two methods are similar in spirit, the empirical literature on herd behaviour relies mostly on the regression approach suggested by Chang, Cheng, and Khorana (2000). Their own empirical findings support an increased tendency to herd in South Korea and Taiwan, partial evidence of herding in Japan, but reveal no evidence for herd behaviour in the US and Hong Kong. Tan, Chiang, Mason, and Nelling (2008) find herding in Chinese A and B stocks. Furthermore, herding occurs under both rising and falling market conditions

[1] This type of herding refers to intentional herding. In contrast, unintentional herding is mainly driven by fundamentals. If investors receive correlated private information, share a similar educational background and have equivalent analytical skills, they make similar investment decisions (Hirshleifer, Subrahmanyam, and Titman (1994)).

and is especially present in A-share markets that are dominated by domestic individual investors. Analysing the Polish stock market Bohl, Gebka, and Goodfellow (2009) highlight differences in trading patterns between individual and institutional investors. While the former engage in herding, particularly during markets downturns, the latter are unlikely to be driven by herd behaviour.

The papers cited above deal with herd behaviour within a given stock market. Chiang and Zheng (2010) take into account potential international linkages by investigating the impact of the US market on herding in 17 other stock markets around the world. Their evidence is in favour of the existence of herding in advanced stock markets except the US and in Asian markets, but not in Latin American markets. Moreover, stock market dispersions in the US play a significant role in explaining herd behaviour in non-US stock markets. More recently, Chiang, Li, Tan, and Nelling (2013) examine herding activity in ten Pacific-Basin markets and the US stock market. From the methodological point of view they apply Chang, Chen and Khorana's constant coefficient model as well as a time-varying approach by using a Kalman filter based model to estimate dynamic herd behaviour. The evidence for the constant regression model is in favour of herding behaviour in each market including the US one. Dynamic herding activities can be observed in each market except the US.

The available evidence allows us to draw at least three conclusions. First, herd behaviour is often found in emerging markets and to a lesser extent in developed stock markets although the evidence is mixed. Second, herding is more pronounced among individual investors compared to institutions. Third and more important for our purpose, the approach by Chang, Cheng, and Khorana (2000) is mainly applied without questioning its statistical properties. In this paper, we challenge their test approach and show that the coefficient of the null hypothesis of no herding is positive, but not zero as assumed in the literature. Consequently, the case of no herd behaviour is too often accepted in studies using Chang, Chen and Khorana's approach.

The paper proceeds as follows. Section 2 investigates the statistical properties of the dispersion measures used in the literature (i.e. the cross-sectional absolute deviation and the cross-sectional standard deviation), and then outlines the test approach. Section 3 presents an empirical example to demonstrate that the conventional testing approach is prone to overlook herding. Section 4 concludes.

2 Herding and Cross-Sectional Deviation of Returns

2.1 Tests for Herding Based on the Cross-Sectional Deviation

When there is herding, investors move together more than during normal times. As a consequence, prices move more in line with each other and the market than they normally do, and the cross-sectional deviation of returns drops below the level it would have without herding. Analogously, anti-herding describes a situation in which returns move less in line with the market than they normally do. The cross-sectional deviation of returns is then above its normal level.

Based on this argument, Christie and Huang (1995) and Chang, Cheng, and Khorana (2000) suggest to test for herding by comparing the cross-sectional dispersion of returns in situations in which herding is supposed to occur to the level it should have in a rational asset pricing model without herding. Too small a deviation of returns is interpreted as herding, while too large a dispersion is seen as evidence for anti-herding.

Both papers focus on periods of large market movements, which are particulary prone for herding. They test whether the cross-sectional deviation of returns is lower than predicted by a rational asset pricing model if there are large upward or downward movements of the market. Extensions of this basic approach also test for herding conditional on, e.g., high or low volatility of the market, high or low trading volume, or for herding triggered by US returns.

The cross-sectional deviation of returns can be measured by the cross-sectional standard deviation or by the cross-sectional absolute deviation. The cross-sectional standard deviation of returns at time t is

$$CSSD_t = \left(\frac{1}{N} \sum_{i=1}^{N} (R_{it} - R_{mt})^2 \right)^{0.5},$$

where R_{it} is the return on asset i and R_{mt} is the return on the market. The cross-sectional absolute deviation of returns at time t is

$$CSAD_t = \frac{1}{N} \sum_{i=1}^{N} |R_{it} - R_{mt}| .$$

It is less sensitive to outliers than $CSSD$, which is one reason why Chang, Cheng, and Khorana (2000) advocate the use of $CSAD$ over $CSSD$.

Christie and Huang (1995) test whether the cross-sectional deviation is larger for extreme market movements than during normal times. They regress $CSSD$ (or $CSAD$) on dummies for extreme returns, i.e. they estimate

$$CSAD_t = \beta_0 + \beta_L D_t^L + \beta_U D_t^U + v_t$$

or

$$CSSD_t = \beta_0 + \beta_L D_t^L + \beta_U D_t^U + v_t,$$

where D^L (D^U) equals one if the return is in the lower (upper) α-quantile of the distribution, with typical values of $\alpha = 0.01, 0.02, 0.05$. They argue that there is evidence for herding conditional on large downward (upward) movements of the market if β_L (β_U) is significantly negative.

As Chang, Cheng, and Khorana (2000) point out, this test is overly restrictive. Stocks differ in their sensitivities with respect to the market. They thus react differently to a given market return, and the cross-sectional deviation of betas translates into a cross-sectional deviation of returns. The larger the return on the market, the larger this induced cross-sectional deviation. Hence, the true value of β_L and β_U under the null hypothesis of no herding is not zero, but equal to some positive number. A test that is based on a true value of zero is biased against finding evidence for herding.

To take the impact of the market return on the cross-sectional deviation into account, Chang, Cheng, and Khorana (2000) suggest to regress $CSAD$ (or $CSSD$) on the absolute and squared excess market return

$$CSAD_t \quad = \quad \gamma_0 + \gamma_1 |R_{mt} - R_{ft}| + \gamma_2 (R_{mt} - R_{ft})^2 + v_t \tag{1}$$

or

$$CSSD_t = \gamma_0 + \gamma_1 |R_{mt} - R_{ft}| + \gamma_2 (R_{mt} - R_{ft})^2 + v_t. \tag{2}$$

As argued above, the cross-sectional deviation of returns is increasing ing the absolute excess market return. The linear part of this dependence is picked up by the (positive) coefficient γ_1, while the coefficient γ_2 picks up the non-linear part. Chang, Cheng, and Khorana (2000) analyze γ_2 for $CSAD$ and argue that its true value under the null hypothesis of no herding is equal to zero.

Following this argument, the resulting test for herding is based on the coefficient γ_2 in Equation (1). If γ_2 is negative, the cross-sectional deviation of returns increases less than linearly (or even decreases) in the market return when the latter becomes large in absolute terms. This is interpreted as evidence for herding in case of large market movements. Analogously, a positive value of γ_2 is interpreted as anti-herding for large market movements.

As we show in the following, this test is again biased against finding evidence for herding. Under the null hypotheses of no herding, the true value of γ_2 is not given by zero, but equal to some positive number $\gamma_2^0 > 0$. If the estimated $\widehat{\gamma}_2$ is significantly smaller than γ_2^0, but not significantly smaller than zero, the test wrongly concludes that there is no evidence for herding. Analogously, if the estimated $\widehat{\gamma}_2$ is significantly larger than zero, but not significantly larger than γ_2^0, the test wrongly signals that there is anti-herding. Put together, the test is biased against finding evidence for herding and towards finding evidence for anti-herding.

2.2 Cross-Sectional Absolute Deviation if There is No Herding

To implement an unbiased test of herding, we need to know the true values of γ_1 and γ_2 under the null hypothesis of no herding. To get an idea about these true values, in particular about the sign of the "herding parameter" γ_2, we now make some assumptions on the true data-generating process under H_0. Following Chang, Cheng, and Khorana (2000) we assume that the return R_{it} on stock i is described by the standard capital asset pricing model (CAPM)

$$R_{it} = R_{ft} + \beta_i(R_{mt} - R_{ft}) + u_{it}, \tag{3}$$

where R_{ft} is the risk-free rate, and R_{mt} is the return on the market. The idiosyncratic component u_{it} has mean zero and is independent of the excess market return $R_{mt} - R_{ft}$. The deviation of the return on asset i from the market return is

$$R_{it} - R_{mt} = (\beta_i - 1)(R_{mt} - R_{ft}) + u_{it}, \tag{4}$$

and the cross-sectional absolute deviation is

$$CSAD_t = \frac{1}{N}\sum_{i=1}^{N} \big|(\beta_i - 1)(R_{mt} - R_{ft}) + u_{it}\big|. \tag{5}$$

To analyze the dependence of $CSAD$ on the absolute excess market return $|R_{mt} - R_{ft}|$, we ignore the idiosyncratic components u_{it} for a moment. If $u_{it} \equiv 0$ for $i = 1, \ldots, N$, the individual absolute deviation $|R_{it} - R_{mt}|$ is a linear function of $|R_{mt} - R_{ft}|$. The cross-sectional absolute deviation becomes

$$CSAD_t|_{\{u_{it}=0, i=1,\ldots,N\}} = \left(\frac{1}{N}\sum_{i=1}^{N} |\beta_i - 1|\right) \cdot |R_{mt} - R_{ft}|\,. \tag{6}$$

In this case, $CSAD$ is indeed a linear function of the absolute excess market return. The true value of γ_1 in regression (1) is equal to the cross-sectional absolute deviation of beta from one, and the true value of γ_2 is equal to zero.

The simple linear relation in Equation (6) breaks down if the idiosyncratic components are no longer identically equal to zero. For a non-vanishing idiosyncratic component u_{it}, the absolute deviation from the market return

$$|R_{it} - R_{mt}| \quad = \quad |(\beta_i - 1)(R_{mt} - R_{ft}) + u_{it}| \tag{7}$$

is no longer a linear function of the absolute excess market return, which implies that $CSAD$ is neither. To analyze the dependence of $CSAD$ on the absolute excess market return in this general case, we look at the conditional expectation $E\left(CSAD_t | R_{mt} - R_{ft}\right)$ given the excess market return. For the conditional expectation of the individual absolute deviation $|R_{it} - R_{mt}|$, the fact that taking the absolute value is a convex function implies by Jensen's inequality that

$$E\left(|R_{it} - R_{mt}| \, |R_{mt} - R_{ft}\right) \quad \geq \quad |(\beta_i - 1)(R_{mt} - R_{ft})| \, ,$$

where the equality sign holds for $u_{it} \equiv 0$. A non-vanishing u_{it} shifts the expectation upwards, and this upward shift is the larger the smaller the absolute excess market return, i.e. the more the idiosyncratic component matters in relative terms. A large upward shift for small values of $|R_{mt} - R_{ft}|$ and a small upward shift for large values of $|R_{mt} - R_{ft}|$ in turn implies that the conditional mean $E\left(|R_{it} - R_{mt}| \, |R_{mt} - R_{ft}\right)$ is a convex function of $|R_{mt} - R_{ft}|$. The conditional expectation of $CSAD_t$ is thus a convex function of $|R_{mt} - R_{ft}|$, too. Hence, the true value of γ_2 is positive.

Chang, Cheng, and Khorana (2000) do not look at the cross-sectional absolute deviation of *realized* returns, but at the cross-sectional absolute deviation of *expected* returns. For expected returns, the relation between individual and aggregate excess returns is

$$E\left(R_{it} - R_{mt}\right) \quad = \quad (\beta_i - 1) E\left(R_{mt} - R_{ft}\right).$$

The cross-sectional absolute deviation of expected excess returns is

$$\begin{aligned} ECSAD_t &= \frac{1}{N} \sum_{i=1}^{N} \left| E\left(R_{it} - R_{mt}\right) \right| \\ &= \left(\frac{1}{N} \sum_{i=1}^{N} |\beta_i - 1| \right) \left| E\left(R_{mt} - R_{ft}\right) \right| . \end{aligned}$$

$ECSAD$ is indeed a linear function of the absolute expected excess market return.

Since expected returns are unobservable, Chang, Cheng, and Khorana (2000) replace them by realized returns, i.e. they consider $R_i - R_f$ as a proxy for $E(R_i - R_f)$ and thus $CSAD$ as a proxy for $ECSAD$. This approximation results in Equation (6) for $CSAD$

and implies a herding coefficient γ_2 which is equal to zero under the null hypothesis of no herding. However, the use of realized returns as a proxy for expected returns implies that the idiosyncratic component is ignored. This turns the true convex dependence of $CSAD$ on the absolute excess market return into a linear relation, and it eliminates the positive sign of γ_2.

The analysis so far has shown that the true value of γ_2 in Equation (1) is positive under the null hypothesis of no herding. If we make some further assumptions on the distribution of the idiosyncratic components, we can also determine the exact form of the functional dependence of $CSAD$ on the excess market return. In particular, we assume that the idiosyncratic components u_i are t distributed with $\nu > 2$ degrees of freedom, location parameter 0 and scale parameter σ_u. This assumption embeds the normal distribution as a limiting case for $\nu \to \infty$.

Equation (5) for CSAD implies that

$$E\left(CSAD_t \middle| R_{mt} - R_{ft}\right) = \frac{1}{N}\sum_{i=1}^{N} E\left(\left|(\beta_i - 1)(R_{mt} - R_{ft}) + u_{it}\right| \middle| R_{mt} - R_{ft}\right).$$

The β_i are constants, and the expectation is taken over the idiosyncratic components only. For asset i, we get

$$\begin{aligned} & E\left(\left|(\beta_i - 1)(R_{mt} - R_{ft}) + u_{it}\right| \middle| R_{mt} - R_{ft}\right) \\ = & \frac{2\sigma_u \nu}{\nu - 1}\left(1 + \frac{\mu_i^2}{\sigma_u^2 \nu}\right) f_\nu\left(\frac{\mu_i}{\sigma_u}\right) + \mu_i\left[1 - 2F_\nu\left(-\frac{\mu_i}{\sigma_u}\right)\right] \end{aligned}$$

where

$$\mu_i = (\beta_i - 1)(R_{mt} - R_{ft}).$$

f_ν and F_ν are the density and the cumulative distribution function of the t distribution with ν degrees of freedom. Hence, the expected cross-sectional absolute deviation is

$$\begin{aligned} & E\left(CSAD_t | R_{mt} - R_{ft}\right) \\ = & (R_{mt} - R_{ft})\frac{1}{N}\sum_{i=1}^{N}(\beta_i - 1)\left[1 - 2F_\nu\left(-\frac{(\beta_i - 1)(R_{mt} - R_{ft})}{\sigma_u}\right)\right] \\ & + \frac{2\sigma_u\nu}{\nu - 1}\frac{1}{N}\sum_{i=1}^{N}\left(1 + \frac{((\beta_i - 1)(R_{mt} - R_{ft}))^2}{\sigma_u^2\nu}\right) f_\nu\left(\frac{(\beta_i - 1)(R_{mt} - R_{ft})}{\sigma_u}\right). \end{aligned} \quad (8)$$

It is a nonlinear (and non-quadratic) function of the excess market return. Appendix A gives the corresponding expression for the limiting case of a normal distribution, i.e. for $\nu \to \infty$.

The expected cross sectional absolute deviation can be approximated by a linear or quadratic function when the terms $(\beta_i - 1)(R_{mt} - R_{ft})/\sigma_u$ are very large in absolute value or close to zero, respectively. If $(\beta_i - 1)(R_{mt} - R_{ft})/\sigma_u$ is close to zero, first order Taylor expansions around zero yield

$$\begin{aligned} F_\nu\left(-\frac{\mu_i}{\sigma_u}\right) &\approx 0.5 - f_\nu(0)\frac{\mu_i}{\sigma_u} \\ \left(1 + \frac{\mu_i^2}{\nu\sigma_u^2}\right) f_\nu\left(\frac{\mu_i}{\sigma_u}\right) &\approx f_\nu(0) - f_\nu(0)\,\frac{\nu-1}{2\nu}\left(\frac{\mu_i}{\sigma_u}\right)^2. \end{aligned}$$

Therefore

$$E\left(CSAD_t | R_{mt} - R_{ft}\right) \approx \frac{2\sigma_u \nu f_\nu(0)}{\nu - 1} + \frac{f_\nu(0)}{\sigma_u}\left(\frac{1}{N}\sum_{i=1}^{N}(\beta_i - 1)^2\right)(R_{mt} - R_{ft})^2, \quad (9)$$

and the expected $CSAD$ is approximately quadratic in the excess market return. The term involving the β_i can be rewritten as

$$\begin{aligned} \frac{1}{N}\sum_{i=1}^{N}(\beta_i - 1)^2 &= \left(\frac{1}{N}\sum_{i=1}^{N}(\beta_i - \hat{\mu}_\beta)^2\right) + (\hat{\mu}_\beta - 1)^2 \\ &= S_\beta^2 + (\hat{\mu}_\beta - 1)^2 \qquad (10) \end{aligned}$$

where $\hat{\mu}_\beta = N^{-1}\sum_{i=1}^{N}\beta_i$. If $(\beta_i - 1)(R_{mt} - R_{ft})/\sigma_u$ is large in absolute value, the nonlinear terms can be approximated as

$$\begin{aligned} F_\nu\left(-\frac{\mu_i}{\sigma_u}\right) &\approx \begin{cases} 0 & \text{if } \mu_i > 0 \\ 1 & \text{if } \mu_i < 0 \end{cases} \\ \left(1 + \frac{\mu_i^2}{\nu\sigma_u^2}\right) f_\nu\left(\frac{\mu_i}{\sigma_u}\right) &\approx 0. \end{aligned}$$

The approximation of the conditional expectation is then given by

$$\begin{aligned} E(CSAD_t | R_{mt} - R_{ft}) &\approx (R_{mt} - R_{ft})\frac{1}{N}\sum_{i=1}^{N}(\beta_i - 1)\left(1 - 2\cdot 1_{\{\mu_i < 0\}}\right) \\ &= |R_{mt} - R_{ft}| \cdot \frac{1}{N}\sum_{i=1}^{N}|\beta_i - 1|\,. \qquad (11) \end{aligned}$$

It is linear in the absolute excess market return and coincides with the value of CSAD for vanishing idiosyncratic components given in Equation (6). Again, Appendix A gives the formulas for the limiting case of a normal distribution, i.e. $\nu \to \infty$.

Taken together, the conditional expectation (8) of $CSAD$ can be approximated by a quadratic function of the excess return when the terms $(\beta_i - 1)(R_{mt} - R_{ft})/\sigma_u$ are small in absolute value, and by a linear function when they are large in absolute value. The size of $(\beta_i - 1)(R_{mt} - R_{ft})/\sigma_u$ depends on the time interval Δt. Since the expected return is proportional to Δt while the volatility is proportional to $\sqrt{\Delta t}$, the term $(\beta_i - 1)(R_{mt} - R_{ft})/\sigma_u$ scales with $\sqrt{\Delta t}$. The smaller the time interval, the better the quadratic approximation.

In the following, we numerically analyze the conditional expectation of $CSAD$ given the excess market return $R_{mt} - R_{ft}$. The calculations are performed using the base line scenario parameters μ_β, S_β and σ_u for the individual excess returns given in the top half of Table 1. The parameters have been calibrated to the cross section of stocks in the S&P 500 using daily prices from 07/25/2008 to 07/26/2013 provided by Thomson Reuters Datastream. Eliminating holidays, the observation period consists of $T = 1259$ trading days. Some S&P 500 stocks are not observed over the entire period. All stocks with less than 250 daily observations are deleted from the sample. The total number of remaining stocks is $N = 495$. The risk free rate R_{ft} is set to the 3-month treasury bill rate.[2] For each stock, β_i is estimated by an OLS regression of its excess return on the market excess return. Although the estimate of the cross-sectional mean beta, μ_β, is larger than unity, we set the base line parameter to the common choice of $\mu_\beta = 1$. The standard deviation S_β is estimated from the cross section of estimated β_i's, which results in $S_\beta \approx 0.4$.

We assume that the idiosyncratic components $(u_{1t}, \ldots, u_{Nt})$ are uncorrelated and jointly t distributed.[3] We fit a central t distribution with scale parameter σ_u and $\nu = 3$ degrees of freedom to the residuals pooled over time and stocks. The choice $\nu = 3$ reflects the heavy tailed nature of the residuals, yet ensures that the variance remains finite. Figure 1 depicts the distribution of the residuals and the density of the fitted t distribution (solid line). The estimate for the scale parameter σ_u is 0.01033 for daily returns, implying an annualized standard deviation for the idiosyncratic component of about 28%. For comparison, we also fitted a normal distribution (dotted line). Its goodness of fit is manifestly so much worse than the fit of the t distribution that a formal statistical comparison is not necessary.

Figure 2 depicts the conditional expectation of $CSAD_t$ as a function of the excess market return $R_{mt} - R_{ft}$. Even though the relation (8) is not exactly quadratic, it can evidently be approximated closely by the quadratic function (9), whereas the linear approximation (11) is only reasonable for extremely large excess returns. Regressing $CSAD_t$ on the absolute excess returns $|R_{mt} - R_{ft}|$ and its squares (as in (1)) is therefore likely to result in an

[2] The series has been provided by Thomson Reuters Datastream: US T-Bill Secondary Market 3 Month Middle Rate FRTBS3M.

[3] Note that zero correlation does not imply independence if the random variables are jointly t distributed. The univariate variance of a t distributed random variable with ν degrees of freedom and scale parameter σ equals $\sigma^2 \cdot \nu/(\nu - 2)$.

Individual excess returns		
	Estimate	Base line scenario
Distribution of β's		
cross sectional mean μ_β	1.1192	1
cross sectional std.dev. S_β	0.4043	0.4
Idiosyncratic components $u_{it} \sim t_3(0, \sigma_u)$		
scale σ_u	0.01033	0.01
implied annualized std.dev.	28.3%	27.4%
Number of assets		100
Excess market return $R_{mt} - R_{ft} \sim t_3(\mu_m, \sigma_m)$		
location μ_m	0.00091	0.0003
implied annualized mean	22.9%	7.5%
scale σ_m	0.00901	0.009
implied annualized std.dev.	24.7%	24.6%

Table 1: Estimated parameters and base line scenario parameters for daily excess returns on the individual stocks and the excess market return. The estimation is based on the S&P 500 and the stocks in the S&P 500 over the sample period from 07/25/2008 to 07/26/2013.

estimate $\hat{\gamma}_2$ significantly larger than zero.

Figure 3 shows the impact of the parameters σ_u, μ_β and S_β on the cross-sectional absolute deviation. The first row shows the effect of the idiosyncratic volatility σ_u, keeping constant $\mu_\beta = 1$ and $S_\beta = 0.4$. As σ_u approaches zero, the quadratic component vanishes and the curve develops a kink. In the limiting case, $CSAD$ will be given by Equation (6). An increase of σ_u leads to a higher level of the cross-sectional deviation and simultaneously to a flatter curve. In the second row we find that an increase of μ_β increases the curvature. The same effect happens if the level μ_β falls (not shown, the plot for $\mu_\beta = 0.7$ is the same as for $\mu_\beta = 1.3$). The bottom row shows the effect of the cross-sectional variation of the β_i's. The larger S_β, the more sensitive $E(CSAD_t | R_{mt} - R_{ft})$ reacts to changes in the excess market return. For a very small variation, there is hardly any effect, and the curve is virtually flat.

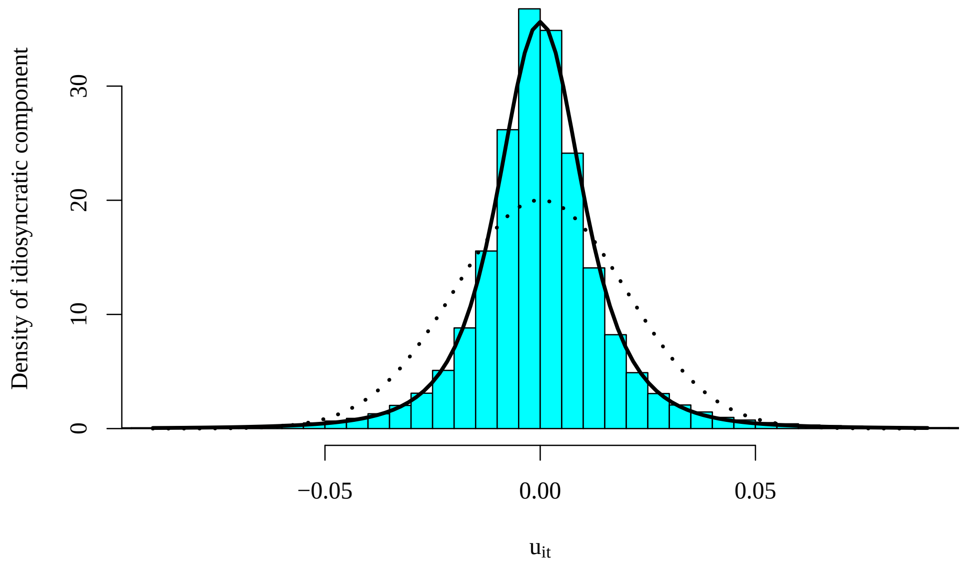

Figure 1: Distribution of the residuals, a fitted central t distribution with $\nu = 3$ degrees of freedom (solid line), and a fitted normal distribution (dotted line)

2.3 Cross-Sectional Standard Deviation if There is No Herding

The nonlinear dependence of the cross-sectional deviation on the excess market return is not specific to $CSAD$, but also holds for $CSSD$. The cross-sectional standard deviation of returns can be written as

$$CSSD_t = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[(\beta_i - 1)(R_{mt} - R_{ft}) + u_{it}\right]^2}. \tag{12}$$

If all idiosyncratic components are identically equal to zero ($\sigma_u^2 = 0$), the cross-sectional standard deviation is a linear function of the absolute excess market return:

$$CSSD_t|_{\{u_{it}=0, i=1,\ldots,N\}} \quad = \quad \sqrt{S_\beta^2 + (\mu_\beta - 1)^2}\,|R_{mt} - R_{ft}|\,. \tag{13}$$

As for $CSAD$, the presence of the idiosyncratic components u_{it} shifts this function upwards. The upward shift is largest for $R_{mt} - R_{ft} = 0$. This turns the linear relation in Equation (13) into a nonlinear, convex function.

If the idiosyncratic components $(u_{1t}, \ldots, u_{Nt})$ are uncorrelated and jointly t distributed, we can again give a closed-form solution for the conditional expectation of $CSSD$. If $\mu_i = 0$ for all i then the quadratic form

$$\frac{1}{N}\sum_{i=1}^{N}\frac{\left[(\beta_i - 1)(R_{mt} - R_{ft}) + u_{it}\right]^2}{\sigma_u^2}$$

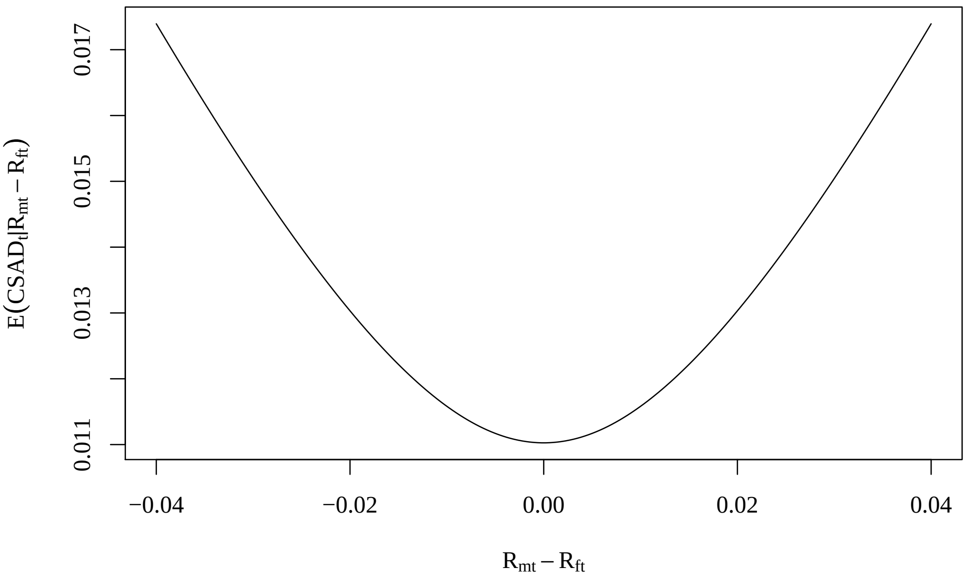

Figure 2: Conditional expectation $E(CSAD_t|R_{mt} - R_{ft})$ for the base line scenario $\sigma_u = 0.01$, $\nu = 3$, $\mu_\beta = 1$, $S_\beta = 0.4$ and $N = 100$

follows a central $F_{N,\nu}$ distribution with N and ν degrees of freedom. If $\mu_i \neq 0$ for some i, the location of the distribution is shifted, and the quadratic form does no longer follow a central, or noncentral, F distribution. Proposition 6.1 of Cacoullos and Koutras (1984) for the generalized χ^2 distribution gives

$$E(CSSD_t|R_{mt} - R_{ft}) = \frac{2\sigma_u \pi^{(N-1)/2}}{\sqrt{N}\Gamma((N-1)/2)} \int_0^\infty M(\rho)\rho^{N-1} g(\rho^2) d\rho \tag{14}$$

where

$$\begin{aligned} M(\rho) &= \int_0^\pi \sqrt{\rho^2 + 2\rho\delta\cos\theta + \delta^2}\,(\sin(\theta))^{N-2}\, d\theta \\ g(z) &= \frac{\Gamma((N+\nu)/2)}{\Gamma(\nu/2)(\nu\pi)^{N/2}} \left(1 + \frac{z}{\nu}\right)^{-\frac{N+\nu}{2}} \end{aligned}$$

with $\delta^2 = \sum_{i=1}^N \mu_i^2/\sigma_u^2$. These expressions can easily be integrated numerically. The limiting case of a normal distribution ($\nu \to \infty$) leads to a noncentral F distribution (see Appendix A).

Figure 4 displays $E(CSSD_t|R_{mt} - R_{ft})$ as a function of the excess market return $R_{mt} - R_{ft}$ for the base line scenario given in Table 1. Comparing Figure 4 to Figure 2 we find that, apart from a vertical shift, the functional forms are virtually identical. The same is true for the effect of varying the parameters σ_u, μ_β and S_β (not shown). Estimating the regression equation (2) will therefore also result in an estimate $\hat{\gamma}_2$ significantly different from zero.

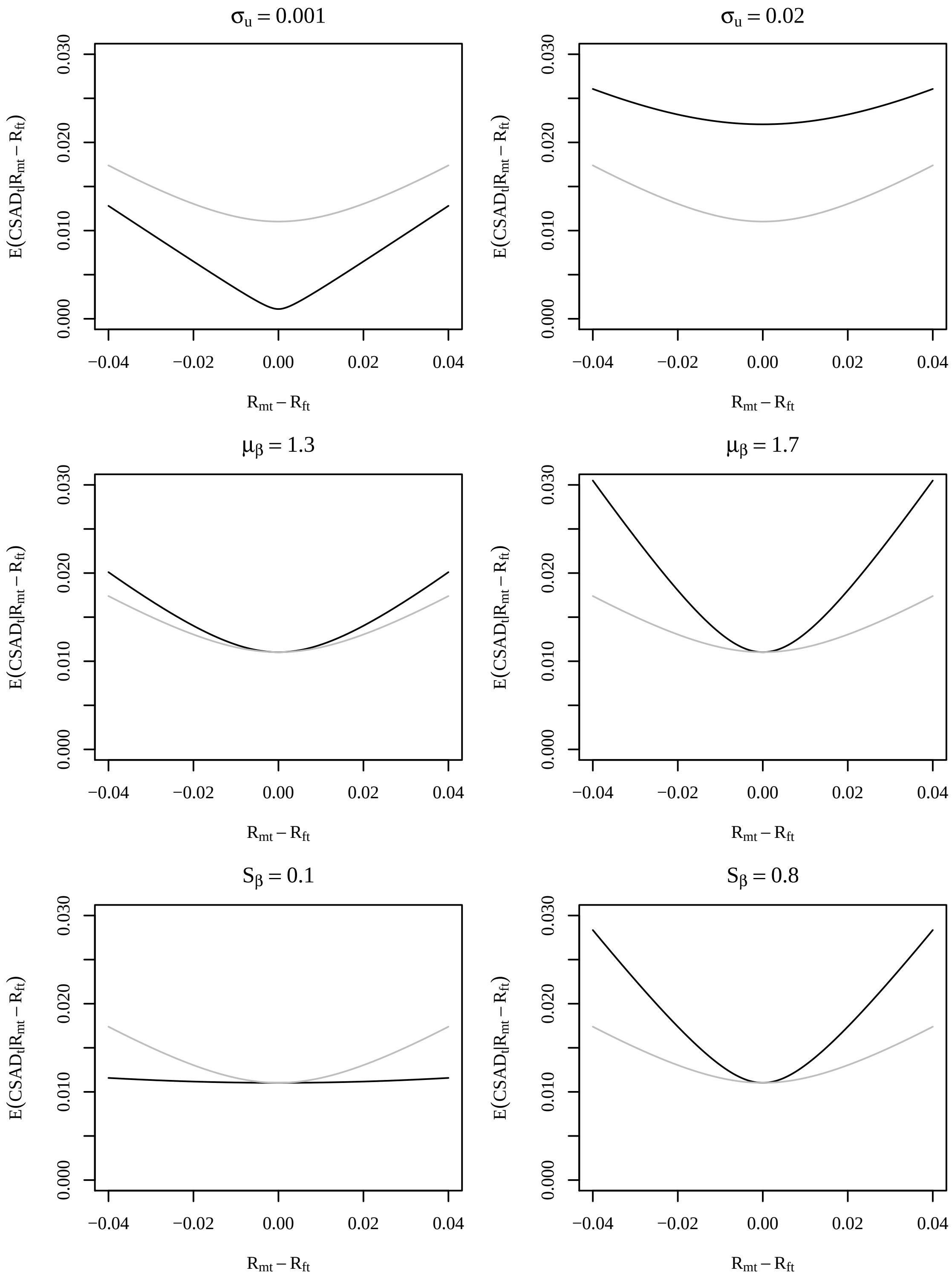

Figure 3: Conditional expectation $E(CSAD_t|R_{mt} - R_{ft})$ for the base line scenario $\sigma_u = 0.01$, $\nu = 3$, $\mu_\beta = 1$, $S_\beta = 0.4$, $N = 100$ (grey lines) and variations of each parameter

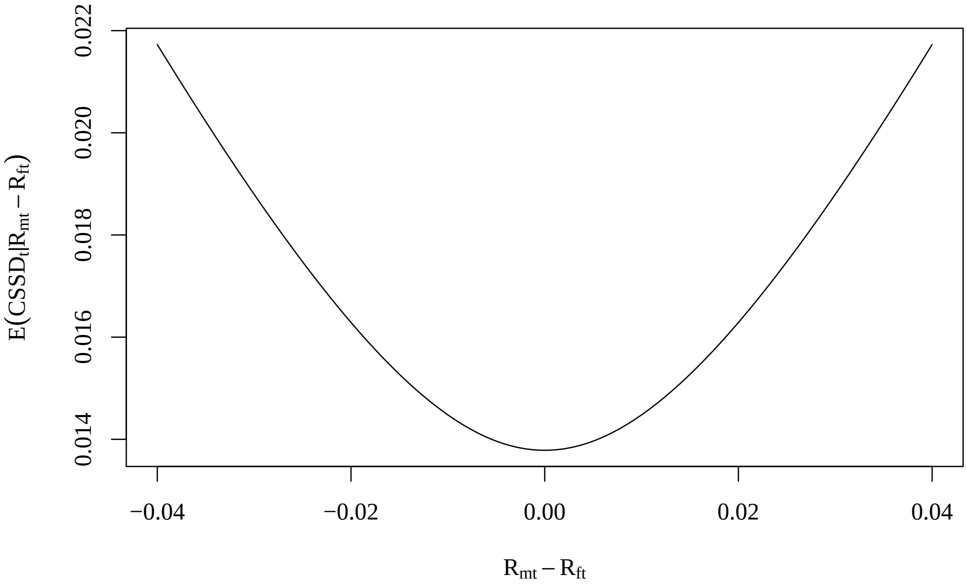

Figure 4: Conditional expectation $E(CSSD_t|R_{mt} - R_{ft})$ for the base line scenario $\sigma_u = 0.01$, $\nu = 3$, $\mu_\beta = 1$, $S_\beta = 0.4$ and $N = 100$

2.4 The impact of the parameters

Under the null hypothesis of no herding, the expected cross-sectional deviation is a non-linear and convex function of the excess market return. In fact, our examples have shown that it can be approximated rather well by a quadratic function for small excess market returns. The OLS estimate for the coefficient γ_2 in regression equation (1) or (2) is therefore likely to be significantly positive.

We now investigate the dependence of the coefficients γ_1 and γ_2 on the average μ_β, the cross-sectional deviation S_β of the betas, the scale parameter σ_u of the idiosyncratic component, and the number N of assets in more detail. The coefficients γ_1 and γ_2 (and γ_0) can be calculated by minimizing

$$\int_{-\infty}^{\infty} \left(E(CSAD_t|x) - \gamma_0 - \gamma_1|x| - \gamma_2 x^2\right)^2 f_{R_{mt}-R_{ft}}(x)dx$$

with respect to the coefficients $\gamma_0, \gamma_1, \gamma_2$. $E(CSAD_t|R_{mt} - R_{ft})$ is the conditional expectation of $CSAD$ given the excess market return. The density function of the daily excess market return $R_{mt} - R_{ft}$ is $f_{R_{mt}-R_{ft}}(x)$ which we model as a t distribution with location parameter μ_m, scale parameter σ_m, and $\nu_m = 3$ degrees of freedom. We approximate the solution to the minimization problem by running an OLS regression of

$E(CSAD_t|R_{mt} - R_{ft})$ on the excess market returns

$$(R_{mt} - R_{ft})_j = F^{-1}_{R_{mt}-R_{ft}}\left(\frac{j}{J+1}\right)$$

for $j = 1, \ldots, J$ where $F^{-1}_{R_{mt}-R_{ft}}$ is the quantile function of the excess market return. The approximation is very accurate even for moderate values of J, e.g., $J = 25$.

The parameters for the excess market return in the base line scenario are given in the bottom half of Table 1. The estimate of the location parameter μ_m is very large and implies an annualized excess return of almost 23%. We attribute this to the short sample period of only five years and thus set the location parameter equal to $\mu_m = 0.0003$ implying an expected annualized excess return of 7.5%, which is well in line with the stylized facts about the equity risk premium.[4] We assume that there are $N = 100$ stocks, the betas are normally distributed, and the idiosyncratic components follow a joint t distribution. Equation (8) then gives the conditional expectation $E(CSAD_t|R_{mt} - R_{ft})$ as a function of the excess market return.

Figure 5 depicts γ_1 and γ_2 as functions of μ_β, S_β, σ_u, and N (solid lines). In line with intuition, both γ_1 and γ_2 are always positive. In addition, the dashed lines show the functional dependence of γ_1 and γ_2 if the idiosyncratic components and the excess market return follow a normal distribution.

The upper two rows show that both γ_1 and γ_2 are increasing in the absolute deviation of the average beta μ_β from the market beta of 1 and in the standard deviation S_β. Taken together, they are increasing in the cross-sectional deviation of the beta's from the market beta. The parameters γ_1 and γ_2 are thus the larger, the less representative the stocks are. The dependence of γ_1 on the cross-sectional deviation of the betas can intuitively be explained by Equation (6) which gives $CSAD$ for vanishing idiosyncratic components. In this special case, $CSAD$ is a linear function of the absolute excess market return, and the proportionality factor is equal to the cross-sectional absolute deviation of the beta's from one. This overall picture carries over to the general case with non-vanishing idiosyncratic components. Hence, γ_1 is increasing in the absolute difference $|\mu_\beta - 1|$ and in S_β. To explain the impact on γ_2, note that the presence of the idiosyncratic components turns the linear relation (6) between $CSAD$ and the absolute excess market return into a convex function. The quadratic approximation of this function is given by Equation (9), and the sensitivity with respect to the squared excess market return is proportional to the cross-sectional squared deviation of the beta's from one. Again, this overall picture carries over to the general case in which the quadratic approximation is not perfect, and γ_2 is increasing in the cross-sectional deviation of the betas, too.

[4] Robustness checks have shown that the impact of μ_m is negligible.

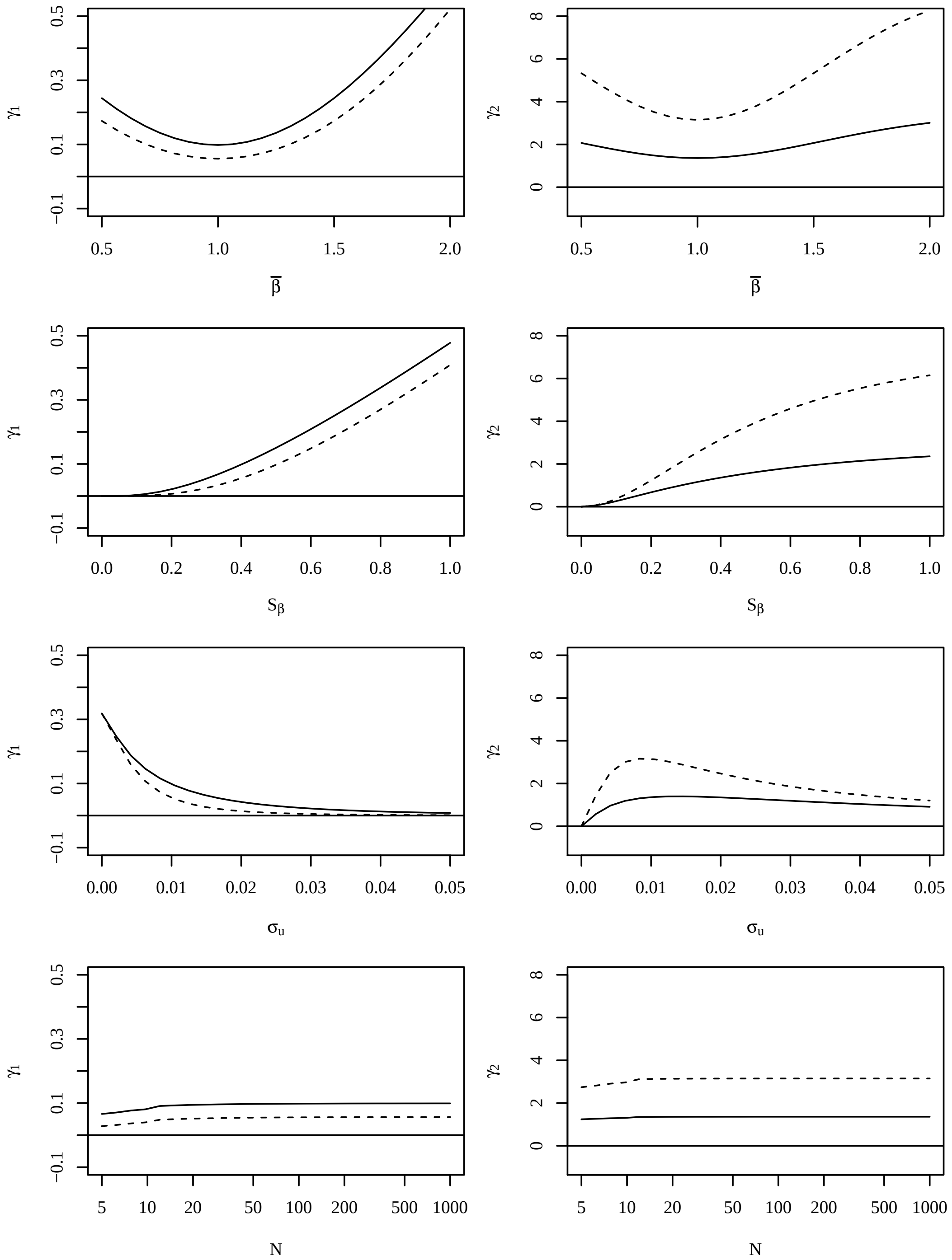

Figure 5: γ_1 and γ_2 as functions of μ_β, S_β, σ_u and the number N of stocks for $CSAD$ calculated from daily returns over five years. In the base line scenario, $\sigma_u = 0.01$, $\mu_\beta = 1$, $S_\beta = 0.4$, and $N = 100$. The solid lines show the case of t_3 distributed u_{it}, the dashed lines show the case of normally distributed u_{it}

.

The impact of the variance of the idiosyncratic components u_{it}, shown in the third row, is more involved. The linear coefficient γ_1 monotonically decreases in σ_u, whereas the herding measure γ_2 is a hump-shaped function of σ_u. To get the intuition, we rely on the linear and quadratic approximations of the conditional $CSAD$. For very small σ_u, the terms $(\beta_i - 1)(R_{mt} - R_{ft})/\sigma_u$ are large, and the conditional expectation of $CSAD$ is approximately equal to the linear function given in Equation (11). For this linear approximation, the parameter γ_1 equals the cross-sectional absolute deviation of the beta's from one, and γ_2 is equal to zero. If σ_u increases, the terms $(\beta_i - 1)(R_{mt} - R_{ft})/\sigma_u$ decrease, and the approximation shifts from the linear relation (11) towards the quadratic function (9). Hence, γ_1 decreases and γ_2 increases. Finally, note that the coefficient of the squared excess market return in the quadratic approximation (9) is decreasing in σ_u. Therefore, γ_2 will ultimately start to decrease when σ_u becomes very large.

The last row depicts the impact of the number of stocks N. Both γ_1 and γ_2 are more or less independent of the number of stocks, with the exception of a small increase if N doubles from 5 to 10.

Comparing the curves for the t distribution (solid lines) and the normal distribution (dashed lines), we find that γ_1 is smaller if the idiosyncratic component follows a normal distribution rather than a t distribution. In constrast, γ_2 is larger in the case of the normal distribution, and the impact of changing the parameters is generally more pronounced. This can be attributed to the lighter tails of the normal distribution. With a smaller probability for extreme realizations, the linear part of $CSAD$ becomes less important, which implies a smaller γ_1 and a larger γ_2.

3 Empirical Evidence and Implications for Existing Studies

The cross-sectional deviation of returns is an increasing function of the absolute excess market return. Non-vanishing idiosyncratic return components imply that this relation is convex, so that the quadratic coefficient γ_2 is positive under the hypothesis of no herding. Tests for herding that are incorrectly based on $\gamma_2 = 0$ under the null of no herding are thus biased against finding evidence for herding and towards finding evidence for anti-herding. Evidence in favour of herding when $\hat{\gamma}_2$ is significantly negative is still valid, since $\hat{\gamma}_2$ is then obviously also significantly smaller than the true positive value of γ_2. However, if $\hat{\gamma}_2$ is not significantly smaller than zero (or even positive), it is incorrect to conclude that there is no herding. Even a positive $\hat{\gamma}_2$ may still be significantly smaller than the true positive value of γ_2 and hence be evidence in favour of herding.

To take into account that $\gamma_2 > 0$ under the null hypothesis of no herding, the correct hypotheses are

$$
\begin{aligned}
H_0 &: \gamma_2 = \gamma_2^0 \\
H_1 &: \gamma_2 \neq \gamma_2^0
\end{aligned}
$$

where γ_2^0 denotes the value of γ_2 in case of no herding. In contrast to standard t test problems, not only γ_2 but also γ_2^0 needs to be estimated since it depends on the unknown parameters μ_β, S_β and σ_u. We estimate γ_2 by OLS of (1) and γ_2^0 by $\widehat{\gamma}_2^0$ as a function of the estimates $\widehat{\mu}_\beta$, $\widehat{S}_\beta$ and $\widehat{\sigma}_u$ as described in section 2.4.

The natural test statistic is

$$T = \widehat{\gamma}_2 - \widehat{\gamma}_2^0$$

and the null hypothesis is rejected if the absolute value of the realized test statistic, $|T|$, is "too large". In order to find the critical value, the distribution of T is simulated by bootstrap methods since $\widehat{\gamma}_2$ and $\widehat{\gamma}_2^0$ have a non-trivial joint distribution under the null hypothesis of no herding.

The bootstrap pseudo-samples are generated under the null hypothesis as follows. Firstly, draw $\widetilde{\beta}_1, \ldots, \widetilde{\beta}_N$ from $N(\widehat{\mu}_\beta, \widehat{S}_\beta^2)$. Secondly, for $i = 1, \ldots, N$ and $t = 1, \ldots, T$ generate new (pseudo) return observations $\widetilde{R}_{it}$ by

$$\widetilde{R}_{it} = R_{ft} + \widetilde{\beta}_i (R_{mt} - R_{ft}) + \widetilde{u}_{it}$$

where $(\widetilde{u}_{1t}, \ldots, \widetilde{u}_{Nt})$ are drawn from a central multivariate t distribution with zero correlations, common scale parameter $\widehat{\sigma}_u$ and $\nu = 3$ degrees of freedom. Obviously, the pseudo returns $\widetilde{R}_{it}$ are generated under the null hypothesis of no herding. Thirdly, the pseudo sample is used to compute $\widetilde{\gamma}_2$ and $\widetilde{\gamma}_2^0$. In the same way as for the original sample, $\widetilde{\gamma}_2$ is determined from the pseudo sample by the OLS regression (1). For $\widetilde{\gamma}_2^0$, we use the pseudo sample to estimate the betas and σ_u, and then determine $\widetilde{\gamma}_2^0$ as a function of the estimated mean beta $\widetilde{\mu}_\beta$, the estimated standard deviation of the betas $\widetilde{S}_\beta$, and the estimated scale parameter of the idiosyncratic components $\widetilde{\sigma}_u$. In this way, we take into account that these parameters have to be estimated and are thus subject to sampling errors that affect the estimate of γ_2^0 as well. Finally, $\widetilde{T} = \widetilde{\gamma}_2 - \widetilde{\gamma}_2^0$.

Let B be the (large) number of bootstrap replications. Generating B pseudo samples results in an approximation of the distribution of the test statistic. The critical value is the $(1 - \alpha)$ quantile of $|\widetilde{T}_1|, \ldots, |\widetilde{T}_B|$.

The test suggested by Chang, Cheng, and Khorana (2000) is erroneously based on testing $H_0 : \gamma_2 = 0$. In this case, the test statistic is $\widetilde{T} = \widetilde{\gamma}_2 - 0$, and the null hypothesis of no herding is rejected if $|\widetilde{\gamma}_2|$ exceeds the $(1 - \alpha)$ quantile from the bootstrap.

Using the S&P-500 sample described above, we show that the specification of the null hypothesis matters. Table 1 reports the point estimates $\widehat{\mu}_\beta = 1.1192$, $\widehat{S}_\beta = 0.4043$, and $\widehat{\sigma}_u = 0.01033$. From these estimates we calculate $\widehat{\gamma}_2^0 = 1.5491$. The linear regression (1) yields $\widehat{\gamma}_2 = 0.3474$, and the value of the test statistic is $|T| = |0.3474 - 1.5491| = 1.2017$. For $B = 25\,000$, the bootstrapped p-value of the null hypothesis of no herding is 0.016. Hence, there is clear evidence in the data against the null hypothesis of no herding. Since $\widehat{\gamma}_2 < \widehat{\gamma}_2^0$, we conclude that the data indicate herding rather than anti-herding. Under the assumption of normally distributed returns, we get $\widehat{\gamma}_2 = 1.6411$.[5] The test statistic is $|T| = |0.3474 - 1.6411| = 1.2937$, and its p-value is 0 (for $B = 25\,000$ bootstrap replications). So the test again indicates herding. In contrast, the test suggested by Chang, Cheng, and Khorana (2000) yields a p-value of 0.3237 and cannot reject the (incorrect) null hypothesis $H_0 : \gamma_2 = 0$. It leads to the conclusion that there is no herding in the market.

Put together, erroneously testing $H_0 : \gamma_2 = 0$ results in non-rejection of the null hypothesis and in the conclusion that there is no herding. In contrast, the correct test of $H_0 : \gamma_2 = \gamma_2^0$ clearly leads to a rejection of the null hypothesis and provides evidence in favour of herding. The test of Chang, Cheng, and Khorana (2000) will thus not detect herding even though it exists.

4 Conclusion

In case of herding, investors follow the collective actions of the market, and prices move more in line with each other than they normally do. The negative impact of herding on the cross-sectional dispersion of returns is used by Christie and Huang (1995) and Chang, Cheng, and Khorana (2000) to construct tests for herding. Chang, Cheng, and Khorana (2000) regress the cross-sectional absolute deviation of returns on the absolute and squared excess market return. They argue that the coefficient γ_2 of the squared excess market return is zero under the null hypothesis of no herding. A significantly negative coefficient γ_2 thus signals herding for large market movements.

We show that the coefficient γ_2 is positive under the null hypothesis of no herding. The

[5] If u_{it} are assumed to follow a normal distribution, the scale parameter σ_u equals the standard deviation, and its estimate is $\widehat{\sigma}_u = 0.0199$.

test of Chang, Cheng and Khorana thus suffers from a wrong specification of the null hypothesis. It is biased against finding evidence for herding and in favour of finding evidence for anti-herding.

The misspecification of the null hypothesis matters. For the S&P 500, the test of Chang, Cheng and Khorana finds no evidence for herding in a five-year sample from 2008 to 2013. In contrast, the test based on the correct null-hypothesis signals that there is herding. The case for herding is thus stronger than previous studies based on Chang, Cheng and Khorana suggest.

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A Normally distributed idiosyncratic terms

If the degrees of freedom parameter ν of the idiosyncratic component goes to infinity, the t distribution converges to the normal distribution, and Equations (8) and (14) can be simplified. Under normality, the expected cross-sectional absolute deviation as a function of the excess market return becomes

$$E(CSAD_t|R_{mt} - R_{ft}) = \frac{1}{N}\sum_{i=1}^{N}\mu_i\left(1 - 2\Phi\left(-\frac{\mu_i}{\sigma_u}\right)\right) + \sigma_u\sqrt{\frac{2}{\pi}}\frac{1}{N}\sum_{i=1}^{N}\exp\left(-\frac{\mu_i^2}{2\sigma_u^2}\right).$$

where $\mu_i = (\beta_i - 1)(R_{mt} - R_{ft})$. For small values of μ_i/σ_u this function can be approximated by

$$E(CSAD_t|R_{mt} - R_{ft}) = \sigma_u\sqrt{\frac{2}{\pi}} + \frac{1}{\sigma_u\sqrt{2\pi}} \cdot \left(\frac{1}{N}\sum_{i=1}^{N}(\beta_i - 1)^2\right) \cdot (R_{mt} - R_{ft})^2$$

which is quadratic in the excess market return. If μ_i/σ_u is large in absolute value, the approximation is

$$E(CSAD_t|R_{mt} - R_{ft}) = |R_{mt} - R_{ft}| \cdot \frac{1}{N}\sum_{i=1}^{N}|\beta_i - 1|$$

which is linear in the absolute excess market return.

For the cross sectional standard deviation, we get that

$$\sigma_u^{-1}\sqrt{\sum_{i=1}^{N}((\beta_i - 1)(R_{mt} - R_{ft}) + u_{it})^2}$$

follows a noncentral χ distribution with N degrees of freedom and noncentrality parameter

$$\delta^2 = \frac{1}{\sigma_u^2}\sum_{i=1}^{N}(\beta_i - 1)^2(R_{mt} - R_{ft})^2.$$

Hence, the conditional expectation of $CSSD_t$ is

$$E(CSSD_t|R_{mt} - R_{ft}) = \sqrt{\frac{\pi\sigma_u^2}{2N}}\, L_{1/2}^{(N/2-1)}\left(\frac{-\delta^2}{2}\right)$$

where $L_{1/2}^{(N/2-1)}$ is the generalized Laguerre polynomial (Olver, Lozier, Boisvert, and Clark, 2010, 18.11.2, p. 448).