Verifying Time Inconsistency of the ECB Monetary Policy
by a Regime-Switching Approach

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Abstract

This work aims to verify whether there is an inflation bias in the Euro-area monetary policy. One verify the presence of a strategic repeated game between the European Central Bank and the market in setting actual and expected inflation and what the features of this game are. In particular, the aim is to verify whether the market fixes the expected inflation strategically and how the probability about the kind of monetary policy is formed. One concludes that the market behaves strategically but the inflation bias does not emerge from data. The ECB is not deemed credible with a probability of one third. However, it is clear that the source of credibility cannot be the lack of commitment of the bank, but it can be the partial inability to control or to communicate economic shocks. These conclusions are robust to two different estimation strategies which are both based on regime-switching regressions.

JEL classification: C51, C73, E61

Key words: Time Inconsistency of monetary policy, Repeated Game, Markov-switching model.
Introduction

Since its first appearance, the models of Kydland and Prescott (1977) and of Barro and Gordon model (1983a) of time-inconsistency of low-inflation monetary policy has often been considered a reference point for setting monetary policy strategies and statutes of many central banks. For example, the European Central Bank (ECB) monetary policy strategy indicates price stability as its primary objective. However, despite many theoretical developments, relatively few attempts have been made to empirically test this. This verification is also useful for the ECB staff, since some authors have argued that its credibility has deteriorated in the past years (see, among the most recent, Geraats et all, (2008), Geraats (2008)) and that the medium term objective of maintaining price stability (below but close to 2%) is time-inconsistent “by construction”, see for example Geraarts (2008). On the other side, some policy insiders have cast some doubts about the practical relevance of these models in explaining inflation, see for example, Blinder (1997) and McCallum (1995). This work aims to verify the presence of the inflation bias in the Euro-area monetary policy also by directly testing whether there is indeed a strategic game between the central bank and the public and what the features of this game are. This is a crucial assumption for reputational time-inconsistency models. If it is not verified, the central bank’s incentive of expanding output by deviating from the inflation target is not compromised by some kind of “punishment” of market participants. In particular, one can verify, at each point of time, whether and which equilibrium the two players (central bank and market) achieve. Moreover, one can test the kind of strategies the two players may play. For example, players can run some trigger strategies (i.e. a “tit-for-tat” strategy) – if this is the case, then the central bank may not find it profitable to systematically expand output by setting

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1 See among others: Berlemann (2005)
inflation higher than its predetermined target. If players act independently, then the central bank could set inflation in a freer way.

Consider for a while the basic model of Barro and Gordon (1983b), both players have two pure strategies at their disposal at each point of time of the repeated game – the target inflation and the higher inflation level – implying four outcomes of the game, at each period\(^2\). In order to verify these outcomes, firstly a bivariate model with regime-switching (see Hamilton 1989, 1990, 1996) is performed where the variables are the actual inflation and the market expectation on inflation. This serves to verify whether there are the typical elements of a repeated game, for example, whether the central bank “leads” the game and whether the market promptly “follows” it. This is the reverse of the usual approach in the game theory. In fact, the usual approach is to specify the pay-offs and then determine the optimal strategy. Here, pay-offs are not observed, but the strategies that players adopt are. This allows one to make inferences about the nature of the game. In this particular context the crucial assumption is that players play only pure strategies each period but, in order to allow switching among equilibriums of the game one has to assume external noises imperfectly monitored by the market. Nevertheless, this approach may be too simplistic; in fact, the market may not know for certain what type of central bank it is dealing with. If this is so, the market may find it optimal to differentiate its strategy each period, that is, it may play a mixed strategy each period since it considers the probability of facing either kind of central bank. In this context, an appropriate econometric model is needed, allowing one to estimate the market probability of facing a credible central bank. For these reasons an alternative specification is necessary. It will be shown that this probability can be found by regressing the market’s expectation on inflation on a constant and

\(^2\) It will be seen later that not only is this basic framework considered but also important variants of it.
on the actual inflation; note that this probability may be time-varying and dependent upon other variables (for example, the past actual inflation), hence regime-switching regressions with time-varying transitions probabilities (see Diebold et al., 1994) are performed for the above specification. Comparing the performance of both specifications for the expectation on inflation, it should be possible to conclude whether or not the market reacts to the monetary policy performance and, if so, how it reacts. If, for the Euro-area, the market fixes the expectation on inflation strategically, that is, considering what type of central bank it is dealing with and what is its performance in steering inflation, then a strong determination in fighting inflation is justified and it is the only way to conduct monetary policy which increases social welfare. In sum, this paper proposes two major contributions; the first consists of providing further evidence about the Euro-area monetary policy; the second is methodological as it shows how one can estimate the relevant parameters of a generic repeated game.

The paper is organised as follows: in the first section, useful references of the literature are outlined along with the estimation strategy. The second section deals with the issue of the mixed strategies and the way to estimate the market probability of facing a credible central bank. In the third section, estimates are provided; conclusions then follow. The Euro-area data are taken into consideration; for the expected inflation, the expectations of the professional forecaster are used; the data are provided by the ECB.

**The theoretical background and the estimation strategy**

According to Kydland and Prescott (1977) or to Barro and Gordon (1983a), if the central bank considers the market expectation of inflation as given, it has an incentive to set the inflation level (slightly) higher than its target in order to expand output; the inflation level the central
bank will set is not the target level but the value which maximizes the welfare function subject to a supply curve. A problem arises when market participants can solve this maximization problem as the central bank does: the market anticipates the central bank’s choice and the monetary policy is no longer able to expand output but can only steer inflation.

A solution for this situation refers to as reputation\(^3\). In this case, the central bank can build its reputation in a multiperiod setup, in order to convince the market that its commitment is credible. Indeed, several variants of the basic model are built which differ from alternative assumptions about the information set of the market; each model then, leads to different equilibriums. In this paper, one verifies three kinds of setups where the central question consists of defining the information set of the market. More precisely, consider the following definition provided in Persson and Tabellini, 1990, (note 28, section 3): incomplete information implies that some players do not know some characteristic of some of their opponents; imperfect information implies that some players do not know the actions of some of their opponent, or of nature.

At this junction, two particular aspects should be clarified. First, the particular reference model(s) to be chosen should be enough sophisticated to ensure a good description of the Euro-area data. For example, it is hard to think that a perfect and complete game may satisfactorily describe the main features of the (Euro-area) actual and inflation expectations because the equilibriums outlined in the theory\(^4\) predict constant actual and expected inflation, unless one introduces some source of variability. Second, in order to verify robustness of estimations, it should be appropriate not to rely only on a particular (class of) model.

\(^3\) The reader may soon refer to the models of Backus and Drifill (1985), Ball (1995), Barro (1986), Barro and Gordon (1983b), Canzoneri (1985). They are extensively described later on in the paper.

\(^4\) See the model of Barro and Gordon (1983b) and Al-Nowaihi and Levine (1994) described later.
Turning to the theoretical background, a seminal paper of this kind is that of Barro and Gordon (1983b). One postulates a “single-period punishment” trigger strategy⁵ which enforces an equilibrium which “turns out to be weighted averages of those from discretion and those from the ideal rule”; under the assumption the both the (inflation) preference parameter and the discount factor are unobservable both to the central bank and to the market, this average is constant over time. If, instead the central bank has private information about those parameters, the enforceable equilibrium implies a time-varying inflation and a constant expected inflation⁶. Hence, the paper of Barro and Gordon (1983b) provides a model with complete and perfect or imperfect information. The latter information set is also considered by Canzoneri (1985)⁷ and by Herrendorf (1999)⁸ among others.

Another strand of literature assumes incomplete (perfect and imperfect) information. In particular, the market forms its expectations by considering the possibility of facing a central bank which may or may not be credible; by the expression “central bank’s credibility” is meant (unless differently specified), that the central bank is either not interested in expanding output or is able to commit⁹. This inference is based on Bayesian’s principles according to which it is possible to form the probability of facing a credible central bank on the basis of the available information, including past inflation. In this context, both the central bank and the

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⁵ This “tit-for-tat” strategy implies: the “follower” plays, at period $t+1$ what the “leader” played at period $t$; if this is the case, the central bank is “punished” whenever it plays an inflation level above its target.

⁶ Note however that, Al-Nowaihi and Levine (1994) argue that this kind of strategy may lack from credibility because the public should punish itself. They instead extend the model of Barro and Gordon (1983b) introducing the “chisel-proof” credibility and they show that a trigger strategy with time-varying period punishment can credibly support a positive inflation rate but lower than the discretionary level (the punishment length depends on usual parameters).

⁷ This author has shown that whenever the money growth rate is larger than a predetermined value, the market best strategy is to “punish” the central bank. This implies an equilibrium where there are occasional periods of (high) inflation (whenever the instrument variable is higher than a particular threshold).

⁸ This author proposes a more transparent monetary policy by establishing a fixed exchange rate policy in order to avoid these occasional shifts of inflation.

⁹ Walsh, (2010) in chapter 7 reports a vast survey of authors who assume either of the two meanings of credibility.
market may play mixed strategies each period (with probabilities strictly less than one). Among
the class of games of (incomplete and) perfect information, one can include Backus and Driffl
(1995), Barro (1986), Cukierman and Laviatan (1991), and Ball (1985) (but see also
Walsh (2010) pag 385 and references therein). Among the class of games of incomplete and
imperfect information, one can include the following papers highlighting the relationship
between credibility from the one side and announcements, transparency and targeting from
the other side: Cukierman and Meltzer (1986), Faust and Svensson (1998, 1999), Walsh

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10 In this paper, the central bank can be optimizer (wet) or inflation fighter (dry), the market does not know it but
fully observes the policy and infers the type by a Bayesian rule. The equilibrium in a finite-period game depends
on the values of the relevant parameters, it may consist in an actual inflation that is initially zero in order for the
central bank to build its reputation. If the wet central bank inflates then it reveals itself and from then on it always
inflates and expectations adjust accordingly.
11 He shows how reputation gives the uncommitted policymaker an incentive to masquerade as the committed type.
In this way, it is able to create high inflation which is surprising initially, but subsequently becomes anticipated.
12 In this paper, the wet and dry central bank are allowed to make announcement and they differ in their ability to
commit. Equilibriums may be pooling where at the beginning both types do not inflate and at the end they reveal
theirseleves. It may be separating where the dry central bank, at the beginning inflates at a lower rate than the
discretionary level and at the end does not inflate; the wet central bank always inflate at a discretionay rate. A
mixed strategy equilibrium is also possible.
13 Ball (1995) assumes that the central bank type follows a Markov process. The economy is subject to occasional
discrete shocks observed by the market. The outlined equilibriums involve the wet type not to inflate when the
shock is null but to inflate at the discretionary rate otherwise. In this case, the central bank reveals itself, the market
expectations are formed accordingly and the wet type inflate until it takes over.
14 Winkler (2000) provides a conceptual framework in which different aspects of transparency have been outlined.
15 The central bank preference evolves according to an AR(1) process. The market observes the policy instrument
but neither the central bank preferences nor the economy disturbances. They show that a certain degree of
“ambiguity” is optimal for the central bank because it enables the central to create surprise inflation whenever it
cares more about output. On the same time, it allows the central bank to create negative surprise inflation
whenever it cares more about inflation. The inflation bias is proportional to the inflation control error.
16 By modifying the control error and the objective function of the model of Kukierman and Meltzer (1986) Faust
and Svensson (1998) show that “ambiguity” is not welfare improving although the central bank may have no
interest in transparency since it may reveal its true identity. In Faust and Svensson (1999) they consider the model
of Faust and Svensson (1998) and analyze the optimal choice of control and transparency under commitment and
discretion.
17 The author shows that the targeting rule reduces the inflationary bias but distorts the central bank reaction to the
economic shocks. This distortion is eliminated through announcements.
18 The author argues that transparency reduces inflation bias and makes the monetary policy more flexible in order
to respond to the economic shocks. In this respect, it is useful to provide the market the central banks forecasts as
they represent a sufficient statistic of the central bank private information.
19 The author shows that transparency is not welcome when the central bank enjoys low-inflation credibility and
there is need for output stabilization policy.
One strand of empirical research has focused on the relationship between independence of the central bank and the inflation performance, see Alesina (1988), Grilli et all (1991), Cukierman et all (1992) and Krause and Méndez (2008) all concluding that such a relationship on a panel of central banks is negative, at least for the developed countries\(^\text{22}\). Another strand of empirical applications tests the implications of the Barro and Gordon model (1983a) about inflation and unemployment; see for example Ireland (1999) Christiano and Fitzgerald (2003) who concluded that the inflation bias is present in the US economy; however, Doyle and Falk (2008), after testing for structural breaks and inflation spill-over concluded that this evidence can only be accepted for the US among the OECD countries; in particular, the authors noted that in many western European countries, the recent decline in inflation was not associated with a fall of the unemployment rate as postulated by the Barro and Gordon model of 1983a. Bae (2011) relaxes the assumption of non-stationarity assumed in Ireland (1999) and finds that the inflation bias in the US economy is only present in 1960s and 1970s. Berlemann (2005), using polling data concluded that the evidence was in favour to the Barro and Gordon model\(^\text{23}\). Sachsida et al (2011) redo the same analysis by dividing the US inflation according to the chairmanship and by using more efficient unit root tests, they conclude that the inflation bias only emerged during the Greenspan chairmanship. Pierdzioch and Stadtmann (2011) applied a

\(^{20}\) The author allows the central bank to choose either the monetary targeting regime or the inflation targeting regime. He shows that, for most parameter values the advantage of communication are outweighed by the advantage of inflation control when choosing inflation targeting for both dependable and not dependable central banks.

\(^{21}\) There is a trade-off between accountability and stabilization that depends on the weight to place on achieving an inflation target. The author shows that this weight is related to the ability to monitor the central bank; this ability can also be related to the “transparency” of policy.

\(^{22}\) For an extensive review on the topic of central bank independence and its economic performance see Cukierman (2008).

\(^{23}\) In particular, he detects inflation bias in Denmark, United Kingdom and United States. In Austria, Australia and Germany he finds evidence that the monetary policy was successful in solving the time-inconsistency problem.
rolling Cointegration tests to Euro-area data, they found no evidence about the inflation bias.\footnote{The authors assumed a priori a five years rolling window for the Cointegration test.}

The major shortcoming of the latter empirical approach is that it is able to identify the interactive behaviour between the central bank and the market only when it turns out to be unsolved that is when the inflation bias emerges; nothing can be said when instead, even in the presence of a strategic game, the discretionary bias is solved.\footnote{For example, only based on the past evidence that the inflation bias is absent in the Euro-area, in the future the ECB could in principle be tempted to provide surprise inflation to boost the economy.}

Put in other words, it neglects the theoretical development of almost thirty years. Another body of research postulates the asymmetry in the preferences in the Barro and Gordon model: see Ruge-Murcia (2003, 2004), Doyle and Falk (2010); the authors however provide contrasting evidence.

Another strand of empirical research attempts to verify the effect of announcements on the central bank’s credibility, see, for example, Johnson (2002) and Demir and Yigit (2008) who find that announcements have a positive impact on the decline of inflation expectations.

Regarding the empirical analysis of the ECB performance, Fendel and Frenkel (2005) stated that the ECB applied a Taylor-type rule to its monetary policy and that the implied inflation target are close to the range of target that the ECB announced. Goldberg and Klein (2005) tested the effect of the ECB’s announcements on the yield curve and exchange rates and showed that the market’s perception of the ECB inflation aversion is increased during its first six years. Geraats (2008) argued that credibility of the ECB achieving price stability is gradually eroded to critically low levels, this is due to a lack of transparency about objectives, forecasts and decision-making. Ullrich (2008) shows that ECB communication does significantly influence the expectation formation.
The estimation strategy of this paper is based on three broad setups. The first two setups consider incomplete and perfect games and complete and imperfect games; complete and perfect games are also considered here as special cases. To this purposes one will use bivariate Markov-switching model (BMS). The third setup considers games with incomplete and imperfect information, the next section shows how to estimate the probability of facing either kind of central bank consistently with this kind of games. Put in other words, the first two approaches examine the joint behaviour of the actual and the (market) expected inflation; the third approach attempts to estimate the market probability of facing a credible central bank under the presence of economic disturbance.

Note however, that in order to verify the presence of a repeated game one should indeed also test (and not assume) the assumption that the central bank does not believe the market playing strategically. This may occur either because the central bank may not assume rationality of the market or because it may assume some market’s coordination failure. But, if the market indeed behaves strategically (and the central bank does not believe that), then one should experience in the data the realization of that trigger strategy “threatened” by the market.26 Put in other words, the central bank may have to learn the market behaviour.27 If it is so, occasional shifts between high and low inflations are possible not only in Canzoneri (1985) but also in Barro and Gordon (1983b) and Al-Nowaihi and Levine (1994) because of learning; in all of these cases trigger strategies may be pulled by the market and hence tested.

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26 It should be used in order to convince the central bank that indeed it behaves strategically.
27 In investigating the causes of the US “Great Inflation”, Primiceri (2006) among others argued that the Federal Reserve was learning about the key parameters of the economy.
More particularly, in order to test these trigger strategies one assumes that the behavior of the central bank and of the market is considered in a repeated game\textsuperscript{28}, both the market’s and the central bank’s sets of strategy are initially formed by two values of inflation: the target level ($\pi^T$) which the central bank is committed to and the value of the inflation which maximizes the welfare function ($\pi^*$), in other words, the “expanding output” or discretionary inflation\textsuperscript{29}. Recall that in the paper of Barro and Gordon (1983b) one postulates a “single-period punishment” trigger strategy and in Al-Nowaihi and Levine one shows that “chisel-proof” credible trigger strategy implies a time-varying period punishment. Finally, players may run a “infinite-period punishment” trigger strategy, i.e. players cooperate until someone stops cooperating. If players act independently, then the central bank can set inflation in a freer way\textsuperscript{30}. Note that, both the central bank and market play pure strategies each period but they may randomize them along the periods.

Furthermore, in this paper, the set of pure strategies is defined as the set known strategies which includes the inflation target and the “expanding output” inflation, bearing in mind that these strategies can be randomized along the periods (as in the first setup); by mixed strategy one means an average of the above strategy that each player may choose in each period\textsuperscript{31}.

Going ahead in reporting the reference literature, one may consider Hamilton (1995) describing regime switching in the money supply\textsuperscript{32}.

\textsuperscript{28} see Gibbons (1992).
\textsuperscript{29} Estimates will assume also three possible values of the actual and expected inflations.
\textsuperscript{30} Note that independence in the context of a repeated game implies that the ECB has an expanding output inflation as a dominant strategy.
\textsuperscript{31} In other words, each strategy is played by a probability strictly between 0 and 1.
\textsuperscript{32} He showed, in the context of a money demand model, that rational agents evaluate the probability that the government raises money growth $g$, for a future period. The process for the money supply may be:

$m_{t+1} = m_t + g + e_{t+1}$
In Smith et al (2000), a repeated game between Greece and Turkey regarding the arms race was modelled as a bivariate regime-switching process\textsuperscript{33}. Since the strategies are two for each country (high or low military expenses), a 4-regime Markov-switching (with mean switching) model can capture all the possible combinations of strategies of the two players\textsuperscript{34}.

Furthermore, a two by two probability transition matrix (between high and low expenses) is associated to each country; this transition probability matrix for each player \( j: j=1,2 \) to move between states’ expenses, is:

<table>
<thead>
<tr>
<th>Period ( t )</th>
<th>Period ( t+1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>((H</td>
</tr>
<tr>
<td>Low</td>
<td>((L</td>
</tr>
</tbody>
</table>

\begin{equation*}
\begin{pmatrix}
(H | H) \\
(L | H)
\end{pmatrix}
\end{equation*}

Probability transition matrix for both players to move between \( H \) and \( L \).

If \( g \) can assume only two unknown values (to the market): \( g=0, g>0 \), then, conditional on the available information, the market may expect:

\begin{equation*}
\begin{cases}
m_{t+1} = m_t + g + e_{t+1} & \text{with probability } d; \quad f. \quad \text{And } g > 0. \\
m_{t+1} = m_t + e_{t+1} & \text{with probability } 1-d; \quad 1-f.
\end{cases}
\end{equation*}

where the probability \( d \) is conditional on the information that the economy is in the zero inflation regime and the probability \( f \) is conditional on the information that the economy is in the high inflation regime. Then, Hamilton showed that the actual inflation rate would be:

\begin{equation*}
pr_{t+1} - pr_t = g + e_{t+1}
\end{equation*}

where \( pr \) is the logarithm of the price level. If \( e_{t+1} \) is zero on average, then \( E_t[p_{t+1}] - pr_t = g \); since \( g \) can take only two values, the process for the inflation rate can be captured by a 2-regimes Markov-switching model.

The Hamilton model can be enhanced by assuming that also the government forms a probability belief about the market behaviour in setting the expected inflation. In fact, if the market’s expectations on inflation are high, then it might be not optimal for the government to set a low inflation level. But this is the case when the government’s decision process for setting \( g \) (i.e. its objective function) depends on output gap and inflation and a Lucas supply curve applies. Hence, the theoretical framework of the Barro and Gordon model suggests that switches be captured both in the market expectation (similarly to the Hamilton model) and in the government’s inflation policy.

\textsuperscript{33} The authors aimed to describe the features of the potential outcomes of the game by switching the means of the military expenses of the two countries, in order to verify whether there were independent or strategic actions between them.

\textsuperscript{34} High for Greece and high for Turkey; high for Greece and low for Turkey; low for Greece and high for Turkey; low for Greece and low for Turkey.
where each element of the table indicates the probability of moving from either state ($H$ or $L$) to the other. Furthermore, the 4 by 4 transition probability matrix of the bivariate Markov-switching (BMS) model gives the transitions of each combination of strategies for the 2 players. By looking at this 4 by 4 matrix, one can verify whether there has been a strategic game between the two players. When players behave independently, all the values of table 2 are obtained by multiplying each element of the transition probabilities of each player.

In this latter case, this 4 by 4 probability transition matrix can be represented as (with $j=1,2$):

$$
\begin{array}{c}
(H \mid H)_1 (H \mid H)_2 (H \mid H)_1 (H \mid L)_2 (H \mid L)_1 (H \mid H)_2 (H \mid L)_1 (H \mid L)_2 \\
(H \mid H)_1 (L \mid H)_2 (H \mid H)_1 (L \mid L)_2 (H \mid L)_1 (L \mid H)_2 (H \mid L)_1 (L \mid L)_2 \\
(L \mid H)_1 (H \mid H)_2 (L \mid H)_1 (H \mid L)_2 (L \mid L)_1 (H \mid H)_2 (L \mid L)_1 (H \mid L)_2 \\
(L \mid H)_1 (L \mid H)_2 (L \mid H)_1 (L \mid L)_2 (L \mid L)_1 (L \mid H)_2 (L \mid L)_1 (L \mid L)_2 \\
\end{array}
$$

*The set of all probabilities possibilities obtained by the combination of each outcome of table 1 when $j=1$ with each outcome of table 1 when $j=2$.

Alternatively, one can consider the case where players behave according to some strategies. If, for example, player 1 “leads” and player 2 plays a “tit-for-tat” strategy and if in period $t$, player 1 has played $H$, the probability that, in period $t+1$, player 2 plays $H$ is 1, whatever strategy he or she has played in period $t$ (and the probability to play $L$ is 0):

$$(H \mid H)_2 = (H \mid L)_2 = 1$$ and $$(L \mid H)_2 = (L \mid L)_2 = 0.$$ So, if for example, player 2 leads the game, the above matrix becomes:

$$
\begin{array}{c}
(H \mid H)_2 0 (H \mid H)_2 0 \\
(L \mid H)_2 0 (L \mid H)_2 0 \\
0 (H \mid L)_2 0 (H \mid L)_2 \\
0 (L \mid L)_2 0 (L \mid L)_2 \\
\end{array}
$$

*This table is obtained by assuming in table 2 that player 2 “leads” the game and player 1 plays a “tit-for-tat” strategy.*

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35 Player 2 plays, in $t+1$ the strategy of player 1 played in time $t$. 

13
In principle, the approach of Smith et al. (2000) can be followed here in order to model the repeated game between the central bank and the market. Hence, as a first estimation strategy, in this paper one uses a BMS model where each variable (actual inflation and expected inflation) is subject to two (or three) regimes each. The latent variables which govern the processes of the actual inflation and the market expectation are called $S_t^C$ and $S_t^M$ respectively; both can take two values each. Furthermore, since the BMS model can take four regimes, it is possible to define a new latent variable $S$ such that the following correspondences hold:

$$
S_t = 1; \quad S_t^C = H; \quad S_t^M = H; \\
S_t = 2; \quad S_t^C = H; \quad S_t^M = L; \\
S_t = 3; \quad S_t^C = L; \quad S_t^M = H; \\
S_t = 4; \quad S_t^C = L; \quad S_t^M = L;
$$

However, the following important shortcoming must be considered. This approach omits modelling the autocorrelation of the actual and expected inflations. Furthermore, it implies that at each point of period, the two players play a pure strategy (high or low military expenses); meanwhile, in the game between the central bank and the market, one should not exclude the possibility of playing a mixed strategy, in the sense of an average strategy, at each point of period. This issue is the subject of the next section.

**A method for estimating the probability of the central bank’s type.**

This section formally shows how one can estimate the probability of facing either type of central bank in a context of incomplete and imperfect information. In this case, the market should play a mixed strategy where the probabilities associated to the pure strategies are strictly

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36 C stands for central bank and M for the market.
37 For the sake of simplicity, the regimes for the central bank and the market are directly called high inflation (H) and low inflation (L).
38 And over time they may randomise between the two strategies.
less than one. The derived model also allows autocorrelation of the two variables. Also using a TVTP econometric model one can show how the probability of facing either type of central bank depends on some exogenous regressors. A mixed strategy is appropriate whenever the market is unsure about the type of the central bank; hence, for each period, the market’s best choice is to form a probability \( p \) on the possibility of facing a credible central bank. In this situation, the market should:

- decide the best strategy to play when it is clear what type of central bank it is facing;
- form a probability of the type of central bank; also on the basis of the probability distribution of the shocks of the economy.

The aim of this section is to show how one can estimate this probability consistently with the third setup of the model outlined above. In particular, it will be shown that in order to find this probability, the market expectation on inflation has to be appropriately regressed on a constant and on the actual inflation only.

Now, it is useful to resume the main features of a general model from which one can derive either setup outlined before as a particular case:

1) A Lucas-type supply function:
   \[
   y_t = \bar{y}_t + c(\pi_t - E_{t-q}^M[\pi_t]) + e_t, \quad c>0, \quad q>0;
   \]
   where \( y_t \) is the actual output, \( \bar{y}_t \) is the potential output, \( \pi_t \) is the actual inflation, \( E_{t-q}^M[\cdot] \) stands for market expectation set in period \( t-q \).

2) The link between actual inflation and the policy instrument:
   \[\pi_t = f(\Delta_i_t) + \nu_t;\] where \( \Delta_i_t \) is the variation of the policy instrument and \( \nu_t \) is an error term.

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39 The time varying transition probability (TVTP) model allows the regressions parameters to switch between regimes and the transition probability (between regimes) to be a logistic function of some exogenous variables. This model is perfectly capable of capturing smooth switches between regimes.
3) The central bank’s utility function is $L_{t+1}^{CB} = E_{t}^{CB} \left[ \sum_{j=0}^{J} \theta^{j} U_{t+j} \right]$ where

$$U_{t} = \frac{1}{2} w(y_{t} - \bar{y}_{t} - k)^{2} - \frac{1}{2} (\pi_{t} - \pi^{T})^{2}$$

and $E_{t}^{CB}[\cdot]$ denotes the central bank expectation set in period $t$, and $w=0,1$: is the type of monetary policy.\(^{40}\)

4) The market is rational and has a loss function of the following form, conditional on the available information (specified above): $L_{t-q}^{M} = \sum_{k=0}^{\infty} \omega^{k} \left( \pi_{t+k} - E_{t+k-q}^{M} [\pi_{t+k}] \right)^{2}$.

5) The private sector’s expectations are assumed to be determined prior (in $t-q$) to the central bank’s choice of its instrument (in $t$).

6) In general terms, it is assumed that the central bank, in period $t$, observes $e_{t}$ but not $v_{t}$.\(^{41}\)

Based on (some of) these assumptions, Barro and Gordon (1983a) derived the (discretionary) equilibrium (rate of) inflation: $\pi^{*}_{t} = f(\Delta_{t}) + v_{t} = cwk + \pi^{T} - \left( \frac{cw}{1+c^{2}w} \right) e_{t} + v_{t}$. Furthermore, in this paper, further assumptions are made for $e$ and $v$:\(^{43}\)

$$v_{t+1} = \alpha + \beta v_{t} + e_{t+1}, \quad |\beta|<1, \quad e_{t+1} \approx i.i.d(0, \sigma_{e}^{2}); \tag{1}$$

$$e_{t+1} = \gamma + \varphi e_{t} + \zeta_{t+1}, \quad |\varphi|<1, \quad \zeta_{t+1} \approx i.i.d(0, \sigma_{\zeta}^{2}) \tag{2}$$

The rest of this section is devoted to describing how one can estimate the probability of facing either type of central bank having assumed the third setup. The aim now is to provide a

\(^{40}\) Only for the sake of simplicity, it is assumed here that the central bank is always able to commit. So, if $w=1$, then the monetary policy cares about output; if $w=0$ the reverse is true.

\(^{41}\) One could have assumed a different timing for the shock and the decision of the central bank in order for $v$ not to be observed, that is, in time $t$, the central bank observes $v_{t-1}$. Neither representation hinges on the outcome of the model.

\(^{42}\) Although here the inflation target different from zero has been inserted.

\(^{43}\) Where the notation “i.i.d.” stands for identically and independently distributed.
specification of the market inflation’s expectation both consistent with the main findings of quoted literature and with a regime-switching framework.

As reported in point 3), the actual inflation should equal the discretionary equilibrium inflation if w=1 or the target level if w=0. In this case the actual inflation can be modelled as:

\[ \pi_t = \pi^T + w(\pi^*_t - \pi^T) + \epsilon_t \]  

(3)

That is,

\[
\begin{cases}
  \pi_t = \pi^T + \epsilon_t, & \text{if } w = 0 \\
  \pi_t = \pi^*_t + \epsilon_t, & \text{if } w = 1
\end{cases}
\]

where \( \epsilon_t \) is defined in eq. (1) and \( \pi^*_t \) in point 3). The inflation set by the policy maker is \( \pi_{w,t}^\circ \) (with \( w=0,1 \)) such that: \( \pi^T = \pi_{w=0,t}^\circ + E^CB_t[v_t] \) and \( \pi^*_t = \pi_{w=1,t}^\circ + E^CB_t[v_t] \) that is, the (adjusted) expanding output inflation. It can be easily shown that eq. (3) is equivalent to the following equation:

\[
\pi_t = \{\pi^T - E_t[v_t] + w(\pi^*_t - \pi^T)\} + \{E_t[v_t] + \epsilon_t\}
\]

(4)

where the first bracket represents the component of actual inflation determined by the policy maker and the second bracket represents the component determined by the economy.

To understand the mechanism of eq (4), first suppose that \( E[v] = \epsilon = 0 \); eq. (4) simply states that if the central bank cares about output (\( w=1 \)), then \( \pi = \pi^* \). If the central bank does not care about output (\( w=0 \)) then \( \pi = \pi^T \). Now, consider the presence of the shock; the predictable part of it, \( E[v] \), must be considered in setting the target inflation or the expanding output inflation as described above; in the former case, the central bank sets it equal to \( \pi^\circ = \pi^T - E[v] \) and in the

---

44 Note that the specification of eq.(3) is also consistent with the Hamilton model outlined in the previous section.

45 Remember that only the shock of the policy instrument function is not known to the central bank.
latter case: \( \pi^\circ = \pi^* - E[v] \). However, the central bank cannot sterilize the unpredictable part of \( v, \varepsilon \), thus it affects actual inflation \( \pi \) directly.

Recall that the market is not supposed to know the realisation of the shocks of the economy and hence the (adjusted) expanding output value \( \pi^* \) but only its process (since it depends on unobserved variables such as \( v \) and \( e \)); Annex A shows that the following process can be derived for the discretionary equilibrium inflation value from the basic model along with other assumptions made so far:

\[
\pi^*_{t+1} = \theta + \varphi \pi^*_t + \xi_{t+1}; \quad \xi_{t+1} \approx i.i.d(0, \sigma^2) \tag{5}
\]

where \( \theta \) and \( \varphi \) are parameters defined in Appendix A and in eq. (2) respectively. Now, the focus is shifted to the market. In order for the market to form its expectations, the (adjusted) expanding output inflation \( \pi^* \) must be forecast. Note that, if the central bank is supposed to care about output \( (w=1) \), then, by virtue of eq. (3), the best inference\(^{46} \) of \( \pi^*_t \) is \( \pi_t \).

Furthermore, going by eq. (5), the best market forecast of \( \pi^*_{t+1} \) is the actual inflation \( \theta + \varphi \pi^*_t = \theta + \varphi \pi_t \). Thus, the best forecast of \( \pi^*_{t+1} \) is \( \theta + \varphi \pi_t \), conditional on the fact that the market believes it is facing a central bank caring about output and it is involved in the discretionary equilibrium.

When the market believes that the central bank does not care about output, then it sets inflation expectation on the target level \( \pi^T \) because, as before, the central bank sets the actual inflation on that target level, also considering the available information on \( v \). When the market is not sure about the kind of central bank, by virtue of the law of the iterated expectation, it forms the following expectation:

\(^{46} \) Set \( w=1 \), solve for \( \pi^* \) and apply the expected value operator.
\[ E_i^M[\pi_{i+1}] = (1 - p_i)\pi^T + p_i(\vartheta + \varphi \pi_i) \]

(6)

where \( p_i \) is the probability of facing a central bank caring about output. Hence, the market sets its expectation on inflation as an average of the target inflation and the actual inflation weighted by the probability of the central bank playing either a low (target) inflation or a high inflation, conditional on the available information.

The fact that the above reasoning leads to eq. (6), can be also verified by applying the expected operator on both sides of eq. (3); therefore:

\[ E_i^M[\pi_{i+1}] = \pi^T + E_i^M[w](E_i^M[\pi_{i+1}] - \pi^T) + E_i^M[\varepsilon_{i+1}] \]

(7)

\[ = \pi^T + p_i(\vartheta + \varphi \pi_i - \pi^T) + 0 \]

\[ = (1 - p_i)\pi^T + p_i(\vartheta + \varphi \pi_i) \]

\[ (1 - p_i)\pi^T + p_i\vartheta + p_i\varphi \pi_i \]

(8)

having noted that \(^47\) \( p_i = E_i^M[w], \) having already established that \( E_i[\pi_{i+1}^*] = \vartheta + \varphi \pi_i, \)

\[ \text{cov}[w, \pi^*] = 0 \] and having observed that \( E_i^M[\varepsilon_{i+1}] = 0. \)

Suppose that the analyst may observe both actual and expected inflation \(^48\); then he or she may attempt to estimate the probability \( p \) of eq. (6). Having regressed the market’s expectation on inflation on a constant \( \hat{\alpha} \) and on the actual present inflation \( \pi \) (call its parameter \( \hat{\beta} \)), from eq. (6) it is easy to derive the following relationships: \( \hat{\alpha} = (1 - \hat{\rho})\pi^T + \hat{\rho}\vartheta \) and \( \hat{\beta} = \hat{\rho}\varphi. \)

\(^{47}\) \( w \) is a Bernoullian variable.

\(^{48}\) The target inflation level is assumed to be known.
Furthermore, in principle, \( \vartheta \) and \( \varphi \) can be estimated by using further information; see Annex B for an estimate of \( \varphi \).

Against this background, the estimation strategy regarding eq. (6) is based on the following observations:

1. The regression based on eq. (6) should provide, together with the estimates of \( \varphi \), an estimate of \( p \) consistent with the probability property: \( 0 \leq \hat{p} \leq 1 \). If this property holds, one cannot exclude that the market is inferring the type of central bank it is facing. However, this is not enough to conclude that the market behaves strategically.

2. The regression based on eq. (6), in which parameters are allowed to vary across regime, allows the researcher to verify whether the estimated probability is time-varying and whether it depends on variables suggested by the theoretical model, i.e. some measure of inflation exceeding the inflation target. If this probability changes according to these values, one can conclude that the market is indeed behaving strategically.

**Empirical results**

The “ECB Survey of Professional Forecasters” is considered. This is a quarterly survey of expectations of the rate of inflation (Harmonised Index of Consumer Prices, \( HICP \)) for several horizons. All participants are experts affiliated with financial or non-financial institutions based within the European Union. See Garcia (2003) for a description. One assumes that the relevant horizon is one year ahead\(^{49} \). The actual inflation is labelled as \( HICP \). Data range from 1999:Q1

\(^{49}\) The relevant horizon of the inflation expectation is implicitly defined in the short-term Phillips curve. If the central bank is tempted to expand output above its potential level, this should be made in the short run.
to 2011:Q1 (quarterly percentage changes), that is, from when the ECB came into office. The source of the data is the ECB. Data are shown in graph 1.

In graph 1, the expected and actual inflations are depicted. The expectation regards the quarterly percentage changes of inflation of one year ahead. The shaded line shows a band of possible values for the ECB inflation target that is between 1.7% and 1.9%. So far, the actual inflation level has often been above this band. The expectations on inflation are set below this band before 2000:Q2, between 2003:Q1 and 2003:Q4 and from 2009 on. After 2006:Q1 they are set above the band but after 2008:Q3 they are set below again (apart from the last two quarters). Note that only for the first part of the sample and in 2009 the actual inflation has been below the expected one. In practice, the data roughly show the following features:

- actual and expected inflations below the target level but the actual inflation below the expected inflation;
- actual inflation above the target level and expected inflation on the target level;
- actual and expected inflation above the target level and the actual inflation above the expected inflation;
- actual and expected inflation on the target level.

These features may be differently interpreted according to whether the assumptions of either setup outlined in the previous section is accepted. However, although these are rough observations, the econometric estimates should be consistent with them.

Now, in order to verify the first and the second setups, estimates consistent with the approach of Smith et al. (2000) are presented and some comments follow. These estimates are based on a BMS model where each variable (actual and expected inflations) are subject to changes in regimes. The data seem to swing between three equilibrium levels rather than two (say low, target and high inflation) hence the choice of estimating a BMS model with both two and three regimes seems appropriate. This serves to take into account that the ECB has not a fully control of inflation.

One begins by testing how many regimes are statistically present in the data. The next table shows the maximum likelihood values as well as maximum likelihood based criteria, AIC and BIC\(^{50}\), which are used to establish the best regime specification.

<table>
<thead>
<tr>
<th></th>
<th>LL</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>BMS-2</td>
<td>-24.3</td>
<td>88.6</td>
<td>126.4</td>
</tr>
<tr>
<td>BMS-3</td>
<td>9.9</td>
<td>148.2</td>
<td>307.1</td>
</tr>
</tbody>
</table>

\(^{50}\) These criteria penalize the models which have a larger number of parameters. The model with the lowest level of parameters in these tests should be chosen.
According to these results, both AIC and BIC lead to choice of the BMS-2 model. However, for reasons explained below, it will also be interesting to examine the results of the MBS-3 model. The next table reports the relevant parameters of the MBS-2 model.

<table>
<thead>
<tr>
<th></th>
<th>Regime L</th>
<th>Regime H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Actual Infl.</td>
<td>1.68*</td>
<td>2.16*</td>
</tr>
<tr>
<td>St. Dev. Actual Infl</td>
<td>1.16*</td>
<td>0.24*</td>
</tr>
<tr>
<td>Mean Expected Infl.</td>
<td>1.71*</td>
<td>1.74*</td>
</tr>
<tr>
<td>St. Dev. Expected Infl.</td>
<td>0.10</td>
<td>0.36*</td>
</tr>
</tbody>
</table>

The table shows the estimated means and standard deviations of the bivariate regime switching model of the actual and expected inflations. The sign (*) shows the significance at the 95% level of significance.

Recall that the BMS-2 model estimated four regimes, each of which is obtained by the combination of the regimes of the actual and expected inflation (as shown in the preceding section).

Graph 2 shows the smoothed probabilities of the MBS-2 model against the actual and the expected inflations.
At this junction, it should be clear how to interpret the four regimes. For example, regime 1 (both high actual and expected inflations) has roughly occurred between 2006 and 2007; regime 2 (high actual inflation and low expected inflation) has occurred between 2000 and 2005, from 2010:Q2 on and so on. Although this model is to be preferred to the BMS-3 model according to the AIC and BIC, it is not thoroughly convincing. In fact it is also important to note that, for the actual inflation, the regime labelled L (lower mean) also has a very high standard deviation (see table 5). This explains the fact that in regimes 3 and 4 both very low and very high values of the actual inflation may occur; however, there is no clear theoretical explanation for this. Furthermore, for the expected inflation the means of the two regimes both lay below 2% (although values larger than this threshold have occurred); again, there is no clear interpretation of it, since all models classified in the first setup (with incomplete or perfect information) predict periods of both high and low inflations, at least after that the market has had the possibility (the time) to adjust its expectation. Turning to the second setup (with complete or imperfect information), estimates and the smoothed probabilities of the BMS-3 model are shown in table 6 and graph 3 respectively.

Table 6: Estimated means and standard deviations of the BMS-3 model

<table>
<thead>
<tr>
<th>Regime L Mean Actual Infl.</th>
<th>Regime M Mean Actual Infl.</th>
<th>Regime H Mean Actual Infl.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90*</td>
<td>2.16*</td>
<td>3.18*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>0.60*</td>
<td>0.24</td>
<td>0.59*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Regime L Mean Expected Infl.</th>
<th>Regime M Mean Expected Infl.</th>
<th>Regime H Mean Expected Infl.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.37*</td>
<td>1.73*</td>
<td>2.05*</td>
</tr>
</tbody>
</table>

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<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12*</td>
<td>0.10</td>
<td>0.13*</td>
</tr>
</tbody>
</table>

The table shows the estimated means and standard deviations of the bivariate regime switching model of the actual and expected inflations. The sign (*) shows the significance at the 95% level of significance.

In the context of the MBS-3 model, the nine regimes should be interpreted as follows:

Table 7: combination of the single regimes of the actual and expected inflations

<table>
<thead>
<tr>
<th>Regimes</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual inflation</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>L</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>Expected inflation</td>
<td>H</td>
<td>M</td>
<td>L</td>
<td>H</td>
<td>M</td>
<td>L</td>
<td>H</td>
<td>M</td>
<td>L</td>
</tr>
</tbody>
</table>

51 If it is not clear, the reader may turn back to the latter section, to see this.
52 In fact, according to the model of Ball (1995) the market should promptly “follow” the central bank when it steers down inflation.
The following graph shows the probabilities for these regimes to occur.

Note that by imposing three regimes on the model, the first shortcoming outlined for the BMS-2 is no longer found – the BMS-3 model, for the actual inflation, does not estimate any regime with large swings within the L-regime (as the BMS-2 model did). Indeed, this L-regime (for the actual inflation) has been split in two parts while the H-regime of the BMS-2 model has remained as the same as in the BMS-3 model. At this junction, one can also consider the BMS-3 model has a natural statistical extension of the BMS-2 model for which the shock of the games of the first setup allows inflations to be set also below the target. Comparing the mean values of the expected and actual inflations, one may note that the “distances” between both inflations is diminished, however, in each regime, the means of the expected and actual inflations remain statistically different. This opens the question about what the source of non-credibility emerges. If this source was the lack of commitment then the market should have

\[ P(r=1) \]

\[ P(r=2) \]

\[ P(r=3) \]

\[ P(r=4) \]

\[ P(r=5) \]

\[ P(r=6) \]

\[ P(r=7) \]

\[ P(r=8) \]

\[ P(r=9) \]

---

53 Likelihood ratio tests are performed leading to this conclusion at the 95% level of significance.
fixed its expectation not (statistically) different from the ECB and the inflation bias should be present. If however, there was a lack of ability then the market should fix its expectation consistently with the persistence of the economic shock and with the ECB ability to neutralize it. It is clear from the estimates that the second source of inability emerges from data (see also Geerart (2008)) since the inflation bias has not emerged.

It is also useful to analyse the actual sequence of regimes. According to the BMS-2 model, the sequence of the regimes is as follows: L,H; H,L; H,H; L,H; H,L. Note that in these regimes, the market “has played” what the ECB played in the previous regime\(^5\), hence, there is evidence that players behave strategically and consistently for example with switches of Ball (1995) but also with the (constant parameters) model of Barro and Gordon (1983b) and with that of Al-Nowaihi and Levine (1994) once one assumes “learning” for the central bank. However, since the duration of the regimes is longer than a quarter and it is rather time-varying, there is no evidence of a prompt and regular response by the market to the movements of the actual inflation as postulated in Barro and Gordon (1983b)\(^5\), it is instead consistent with the model of Canzoneri (1985) once the shock is assumed to be persistent and with the model of Ball (1995) where the process of the central bank type is a Markov process\(^5\). Suppose the economic shock (as in Canzoneri (1985)) or the present preference about output (as in Ball (1995)) are transitory, then it is optimal for the market to wait for (as in Al-Nowaihi an Levine (1994)) the expiration of the transitory change which generally may last for more than one period; this may also support the idea that the market wants to gather enough information before “punishing” the

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\(^5\) In fact, up to the beginning of 2000, the central bank played L; the market subsequently played L and so on.

\(^5\) it is instead consistent with the “several periods” punishment of Al-Nowaihi and Levine (1994) where parameters are varying over time.

\(^5\) For a further interpretation, see Henckel et al (2011) who assumed that the market uses the principle of inferential expectation to test the null hypothesis that the central bank inflates at the discretionery rate.
central bank, as in Henckel et al (2011). Now, it is worth testing a variant of the “tit-for-tat” strategy – it is assumed that, given a strategy played by the leader, the follower does not necessarily have to play the same strategy in the next period; in each of the following periods the follower can either not change his/her strategy or play what the leader played. This implies different restrictions on the transition probability with respect to those reported in table 3.

Suppose that the central bank leads, the transition probability matrix for the BMS-2 model becomes:

| (H | H) | (H | H) | (H | L) | (H | H) | 0 |
|------|------|------|------|----|
| 0    | (H | H) | (L | L) | (H | L) | (H | H) | 0 |
| (L | H) | (L | H) | (H | L) | (L | H) | 0 |
| 0    | (L | H) | (L | L) | (L | L) | (L | L) | (L | L) |

This table shows all the probabilities possibilities of table 1, imposing a variant of the “tit-for-tat” restriction outlined in the text.

In order to test this null hypothesis, a likelihood ratio test is performed and this hypothesis is not rejected by the data. So, if one assumes that the BMS-2 model provides a good description of the data, one should also conclude that the central bank and the market behave strategically, although the market reacts to the central bank policy with some lags.

According to the BMS-3 model, the sequence of the regimes is the following: L,L; M,M; M,H; H,H; L,L; M,M. In this case, there is no immediate evidence that players behave strategically at least in the sense specified above; rather, they seem to act simultaneously. A likelihood-ratio test to verify independency is performed, which leads one to not reject the hypothesis that the

---

57 Since the log-likelihood value with this restriction is -25.8, the likelihood ratio is $2\times(-24.3+25.8)=3.0$, which is well below the chi-sq with 8 degrees of freedom at the 95% level of significance: 15.5.
central bank and the market do not behave strategically. However, as noted in Smith et al. (2000) and references therein, independence does not imply that series cannot change the regime simultaneously as seems to be the case here. It simply means that the unobserved states can be modelled as an independent Markov chain, which does not exclude the possibility that their behaviour is affected by a common shock as postulated by model with imperfect information and imperfect control of the actual inflation. This is also consistent with the fact that the support of the shock can be either positive or negative since one observes periods of both low, medium and high actual and expected inflations.

To summarize, according to the MBS-2 model, the market responds strategically to the central bank’s behaviour although with some irregular delay. According to the BMS-3 model, players respond strategically to the same common shock. Both models lead to conclude that the market behaves strategically; however, looking at the mean values of the actual and expected inflations within regimes there is no evidence of the inflation bias since these means are statistically different. Furthermore, since according to the BMS-2 estimation results there is no prompt response of the market to the actual inflation’s fluctuations, one can also conclude that both estimations suggest the presence of an unobservable (to the market) shock which is important in determining the strategy of the market.

The second strand of estimations focuses on verifying the hypothesis that the market is not sure about the type of central bank it is dealing with in a context where external shocks are (at least partially) unverifiable from the market. Hence, it is important to establish, how the market forms the probability of facing either type of central bank (consistently with the third setup of

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58 The likelihood value, under the null, is -0.2; the likelihood ratio is 2*[9.9+0.2]=20.2. This value is far below the chi-sq(51) value at the usual critical levels. In fact: 51=63-12 where 63 (=81-9) is the number of the free parameters in the unconstrained transition probability and 12 (=2*(9-3)) is the number of the free parameters in the constrained transition probability.
the paper where the game is assumed to be incomplete and imperfect). To this purpose, the model of eq.(6) is estimated. Recall that the aim of this model is to directly estimate this probability. The reference regression, based on eq. (6), is as follows:

\[ E_t[\pi_{t+4}] = \alpha_r + \sum_{j=0}^{I} \beta_{ij} \pi_{t-j} + \sum_{i=1}^{J} \phi_{ni} E_{t-1}[\pi_{t+4-i}] + \rho_t; \]

\[ \rho_t = iid(0, \psi_r^2); \]

with \( r=1,2, \ldots \) and the time-varying transition probability:

\[ \Pr[w_t = r \mid w_{t-1} = r] = \exp(\lambda_{tr} + \lambda_{2r} z) / (1 + \exp(\lambda_{tr} + \lambda_{2r} z)) \]

Eq. (9) and (10) allow the parameters to vary across \( r \) regimes. Recall that \( E_t[\pi_{t+4}] \) is the market expectation on inflation formed on the basis on several other indicators besides the actual inflations\(^{59}\). After an appropriate selection of the number of regimes \( r \) and the number of lags \( I \) and \( J \), the best econometric model is a TVTP model with two regimes and one lag of the inflation expectation. The variable in the transition probability function \( (z) \) is the first lag of the 3-quarters average of the actual inflation. The estimated parameters are shown in table 9.

\(^{59}\) One assumes that the Lucas supply function is the following: \( y_{t+4} = \bar{y}_{t+4} + c(\pi_{t+4} - E_t[\pi_{t+4}]) + \epsilon_{t+4} \), that is, the output \( y \) of next year (at time \( t+4 \)) depends on the difference between the next year’s actual inflation and the market’s expectation of it based on the current year’s information (at time \( t \)) and the shock. See Annex B for its parameters estimates.
Table 9: estimates of TVTP model based on eq. (6).

<table>
<thead>
<tr>
<th></th>
<th>Regime 1</th>
<th>Regime 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\alpha}_j$</td>
<td>0.63*</td>
<td>0.85**</td>
</tr>
<tr>
<td>$\hat{\beta}_{0j}$</td>
<td>0.02</td>
<td>0.23**</td>
</tr>
<tr>
<td>$\hat{\phi}_j$</td>
<td>0.66**</td>
<td>0.23*</td>
</tr>
<tr>
<td>$\hat{\lambda}_{1j}$</td>
<td>3.50*</td>
<td>6.23**</td>
</tr>
<tr>
<td>$\hat{\lambda}_{2j}$</td>
<td>-1.14*</td>
<td>-1.70**</td>
</tr>
<tr>
<td>$\hat{\sigma}_j$</td>
<td>0.03*</td>
<td>0.09*</td>
</tr>
</tbody>
</table>

The signs (*) and (**) show the significance at the 95% and at the 99% level of significance, respectively.

Graph 4 shows the inferred probabilities of the two regimes occurring.

Annex B shows that an estimate of $\varphi$ is 0.7; hence, since it holds that $\hat{\beta} = \hat{p}\hat{\phi}$, the probability of facing a not credible central bank is, for the second regime, $0.23/0.7 = 0.33$ and, since for the first regime the estimated parameter is not significant, this probability can be considered equal
to 0. The inferred (smoothed) probability of the second regime occurring shows that for a large part of the sample, in particular before the beginning of 2006 and from 2008:Q2, the market formed a higher probability that the central bank is not credible: 0.33. Between 2006:Q2 and 2008:Q2, the ECB built the best reputation. However, from the beginning of 2008 on, the market again went back to believing that the ECB has a roughly 1/3 chance of not being credible. To make the point better, consider the transition probability function, where the marginal impact of $z$ is not constant but dependent upon $z$. In order to understand the mechanism behind this, it is worth examining the behaviour of the two transition probabilities as a (logistic) function of $z$. Graph 5 shows the transition probabilities for values of $z$ ranging between -0.9 and 3.6.

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60 It is easy to show that the long run value of the inflation expectation is 1.86% (consistent with the ECB target) meanwhile in the second regimes, this long run value depends on the long-run value of inflation conditional on being in regime 2.
61 Whose parameters are reported in table 9.
62 These are roughly the maximum and minimum values of $z$ in the sample.
Note first, that the behaviour of these probabilities changes considerably between large and small values of $z$, where large and small are defined with respect to the inflation target value (say 1.8%); that is, the transition probability (as a function of $z(-1)$) of remaining in regime 2 start decreasing after this threshold value and the slope of the transition probability of remaining in regime 1 becomes steeper. In particular, for larger values, the probability of remaining in regime 2 (with higher $\hat{\beta}_{0j}$) is stronger than the probability of remaining in regime 1. For values of $z(-1)$ smaller than 1.8%, the difference between the two probabilities is smaller. Hence, in particular on the latter case, the probability (conditional on past information) of each regime occurring heavily depends on the same probability of the previous period because it weights the transition probabilities. In fact, for example, for regime 2 it holds that:

$$
\Pr[r_t = 2 | I_{t-1}] = 
$$

$$
pr[r_t = 2 | r_{t-1} = 2, z_{t-1}]pr[r_{t-1} = 2 | I_{t-1}] + (1 - pr[r_t = 1 | r_{t-1} = 1, z_{t-1}])pr[r_{t-1} = 1 | I_{t-1}];
$$

All this implies that values of $z$ larger than 1.8% may make the market form a higher probability of facing a not credible central bank, but the reverse is not necessarily true. At each period, this depends on the probability which the market formed in the previous period. This supports the idea (see for example Cukierman and Meltzer (1986) or Henckel et al (2011)) that credibility can be built or compromised only along a relative long period of time. However, the speediness of the adjustment to the new regime is larger toward the “credibility” regime than toward the “non-credibility” regime. This fact may stem from the good reputation of the bank in fighting inflation.

63 More precisely, one means the smoothed probability (and not the transition probability defined in eq(11)).

64 Put it differently, due to the behaviour of the transition probabilities as function of the $z$ (past average inflation) the market react differently to low and high values of $z$. Should the market observe a low value of $z$, the inflation
The TVTP model also estimates as starting values the ergodic probabilities of each regime occurring. These can be interpreted as a-priori beliefs whose formation does not depend on the actual inflation behaviour (of the sample) but on other information, such as the degree of reputation which the ECB inherited from the national central banks, and the features of the mandate embedded in its statute. The ergodic probability of being in regime 1 (with $\hat{\beta}_{01} = 0$) is 0.22. This means that, at the beginning of the sample, the market was relatively sceptical about the ECB’s credibility to meet the target inflation. At the same time, if a probability of 0.22 for regime 1 is inserted in eq.(11) as a probability of period $t-1$, even for values of $z$ well below 1.8%, the probability of remaining in regime 1, in period $t$, is still only roughly 0.22. This means that either the ECB had to build its own reputation or that the statutory mandate is not completely appropriate. About the latter question Cukierman (2000) argues that since the ECB mandate does not uniquely regards price stability a maximal level of transparency is not optimal which, however implies a cost in terms of partially loosing credibility.

Note, however, that generally when the central bank is believed not to be credible (in the sense explained above), the question of what the source of non-credibility is remains open. For example, consider the period after 2008Q1. At that period, the market again changed its belief about the ECB credibility. At this junction, one may consider the following hypotheses about the potential sources of non-credibility (see also Geraarts (2008)):

- the policy instrument and the policy implementation were ineffective in steering such a high inflation;

\[\text{65 The Maastricht Treaty states that without prejudice of price stability, the ECB should help promote the economic policy of the European Community.}\]
- the ECB cares about the output (and it was deemed to fall well below its potential level due to the oil shock).

More precisely, the loss of credibility can be ascribed to a lack of inflation control, which, together with the uncertainty of the type of the central bank, should cause a certain amount of the inflation bias (as in Cukierman and Meltzer (1986)); this is pretty plausible for the ECB as it can be still considered as a novel institution (see Cukierman (2000)); in other words, there is lack of ability in fighting inflation. Secondly, one might argue that the central bank is believed to commit to pursuing low inflation, its inflation control is sufficiently precise but there is a lack of transparency\textsuperscript{66} in the phase of explaining the shock, as argued in Geraarts (2008). This is consistent with the theoretical point of view of Walsh (1999, 2003) who argues the announcements and transparency help solving the conflict between (inflation) targeting and (output) stabilization.

Finally, one might also argue that these oil shocks (and the financial crisis) were perfectly observable by the market and hence the context of incomplete and imperfect game collapses to an incomplete but perfect game; hence for example, according to the model of Ball (1995), these are times where supply shocks occur and the central bank caring about output reveals itself and the market accordingly revise its belief about the type of the central bank it is facing. In this case, the lack of credibility depends on the revealed lack of commitment. However, in the light of the results of the first strand of estimations it should be now clear which kind of source of non-credibility presented above may be emerged from the data.

\textsuperscript{66} However, this lack of transparency may be deliberated, see Cukierman and Meltzer (1986) or Jensen (2001).
Conclusions

All estimations of this paper lead to conclude that the ECB and the public behave strategically in setting the actual and expected inflation respectively meaning that the ECB conduct of monetary policy does influence the market expectations and vice versa. However, one can conclude that the ECB is deemed as a central bank determined in maintain low inflation since all the evidence also leads to conclude that the discretionary equilibrium never occurred. However, either the market perceives the ECB as not fully able to face the economic shock or there is a communication problem. This conclusion can be considered as robust in that it relies on several specifications (setups) encompassing the large majority of the models emerged in this literature. It is also worth to stress that this paper significantly contributes to the empirical research by providing a new methodological tool to statistically measure the features (strategies) of a repeated game.

Going into the details of the single estimations, one can resume the results as follows. The first strand assumes games with incomplete and perfect or with complete and imperfect information. Imposing perfect information on the data means that actual and expected inflation are allowed to vary between two regimes each only. One has verified that the market follows the ECB behaviour although with irregular delay. This is evidence that the market behaves strategically but, two facts remain unclear in this context.

The first one regards the appropriateness of the imposition of only two regimes in the Euro-area data. Indeed, they switch between at least three equilibriums including values (of actual and expected inflations) well below the claimed target of (below but near to) 2%. To overcome this problem a three regimes model has been also estimated whose interpretation leads to a complete

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67 This is consistent with the model outlined in Ball (1995) but also with the model of Barro and Gordon (1983b) and in Al-Nowaihi and Levine (1994) with “learning”.

35
but imperfect information setup. Results in this context show that actual and expected inflations move simultaneously in the same direction. This is consistent with the model outlined in Canzoneri (1985), in there the market and the central bank behave strategically and that strategies are affected by a common shock.

The second shortcoming of the two-regimes switching model\(^{68}\) regards the reason why the market needs (irregular) time in order to change their strategy. In order to overcome this problem, the more general assumption of an incomplete and imperfect information setup is considered. In this context, one allows the market to infer the type of central bank it is dealing with in an environment of imperfect monitoring of the economic shock. In fact, the reason why the market adjusts its expectation to the actual inflation with (irregular) delay may also depend on its assessment of the kind of central bank. To this purpose, a different regime-switching model is used, allowing the estimate a time-vary market’s probability to face either kind of central bank. One finds that for most of the sample, the public perceives the ECB as non-credible with a probability of 33\% and the switch between regimes depends on the past actual inflation. However, a decrease of the past inflation has a larger effect on the increase of the credibility than the increase of past inflation on the decrease of credibility. This source of non credibility may not necessary stem from the ECB performances. In fact, the degree of credibility embedded in that statute and/or inherited from the Euro-area national central banks may also play their own role. This “a priori” credibility has been found very restrained.

Throughout this paper one has shown that the potential source of non-credibility of the ECB is not its will to expand output by the surprise inflation; rather it should lay either in its limited ability of controlling inflation or in the transparency about the way it conducts monetary policy.

\(^{68}\) It should be now clear that this shortcoming is also valid for the three-regimes model.
Establishing which one of these aspects may play the most important role is, however, beyond the objectives of this paper.

References


Annex A

Define $h = c_{w,k}$ and $g = - \left( \frac{cw}{1 + c^2w} \right)$ so that the discretionary inflation becomes

$$\pi^*_t = h + ge_t + v_t$$

(a.1)

Now, having assumed that:

$$e_t = \gamma + \varphi e_{t-1} + \zeta_t \quad \zeta_t \approx iid(0, \sigma^2)$$

(a.2)

It is easy to see from (a.1) that $e_{t-1} = (\pi^*_{t-1} - h - v_{t-1})/g$ and hence by substituting out in (a.2) and then in (a.1) one obtains:

$$\pi^*_t = h - \varphi h + g\gamma + \varphi\pi^*_{t-1} + g\zeta_t + v_t - \varphi v_{t-1}$$

(a.3)

Note that $v_t = \alpha + \beta v_{t-2} + \varepsilon_t = \alpha \sum_{i=0}^{j} \beta^i + \beta^{j+1} v_{t-j-1} + \sum_{i=0}^{j} \beta^i \varepsilon_{t-i}$ and that

$$\varphi v_{t-1} = \varphi(\alpha + \beta v_{t-2} + \varepsilon_{t-1}) = \varphi \left[ \alpha \sum_{i=0}^{n} \beta^i + \beta^{n+1} v_{t-n-2} + \sum_{i=0}^{n} \beta^i \varepsilon_{t-i-1} \right]$$

and considering sufficiently large $j$ and $n$ such that $\beta^{j+1} \approx 0$ (and $n=j$) one obtains:

$$v_t - \varphi v_{t-1} \approx \frac{(1 - \varphi)\alpha}{1 - \beta} + \varepsilon_t + (1 - \varphi) \sum_{m=0}^{n} \beta^m \varepsilon_{t-m}$$

(a.4)

having assumed stationarity for $v$. One can substitute the above expression in (a.3) and hence one obtains

$$\pi^*_t = h - \varphi h + g\gamma + \varphi\pi^*_{t-1} + g\zeta_t + \frac{(1 - \varphi)\alpha}{1 - \beta} + \varepsilon_t + (1 - \varphi) \sum_{m=0}^{n} \beta^m \varepsilon_{t-m}$$

(a.5)

Setting $(1 - \varphi)h + g\gamma + \frac{(1 - \varphi)\alpha}{1 - \beta} = \varnothing, g\zeta_t + \varepsilon_t + (1 - \varphi) \sum_{m=0}^{n} \beta^m \varepsilon_{t-m} = \zeta_t$ one obtains

$$\pi^*_{t+1} = \varnothing + \varphi\pi^*_{t} + \zeta_{t+1}; \quad \zeta_{t+1} \approx i.i.d(0, \delta^2);$$

(a.6)
The iid property of $\zeta_t$ comes also from the fact that it is the sum of iid processes and that $E[e_t v_t] = 0$ and hence $E[\zeta_t e_{t-q}] = 0 \ \forall q \in T$.

**Annex B**

In order to find an estimate of $\varphi$ of eq.(2), the following Lucas supply function of point 1) section 2: $y_{t+4} = \bar{y}_{t+4} + c(\pi_{t+4} - E_t[\pi_{t+4}]) + e_{t+4}$ with a MA(1) term is estimated (see also note 60). The potential output can be estimated by a Hodrick-Prescott filter (with 1600 as the smoothing parameter). The OLS estimates of the Lucas supply equation are reported in table 10:

**Table 10: OLS estimates of the Supply curve.**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potential Output</td>
<td>1.68**</td>
</tr>
<tr>
<td>HICP actual</td>
<td>0.25</td>
</tr>
<tr>
<td>HICP expected(-4)</td>
<td>-0.35*</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.70**</td>
</tr>
</tbody>
</table>

*The signs (*) and (**) show the significance at the 95% and at the 99% level of significance, respectively.*