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A Direct Test of Rational Bubbles

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Abstract

The recent introduction of new derivatives with future dividend payments as underlyings allows to construct a direct test of rational bubbles. We suggest a simple, new method to calculate the fundamental value of stock indices. Using this approach, bubbles become observable. We calculate the time series of the bubble component of the Euro-Stoxx 50 index and investigate its properties. Using a formal hypothesis test we find that the behavior of the bubble is compatible with rationality.

1 Introduction

On 30 June 2008, Eurex introduced its first futures contracts on the dividends of a major European stock index, the Dow Jones Euro Stoxx 50. Ever since, market participants have been trading expectations about index dividends. As the number of contracts got further expanded in May 2009, today's investors are able to price possible earnings of the upcoming ten years. This paper analyzes these new contracts and examines the question whether trading dividend expectations separately can contribute to the long-lasting discussion about rational bubbles.

There is a large and still growing number of papers trying to test for rational bubbles; a recent overview is provided by Gürkaynak (2008). In the early eighties, Shiller (1981) and Grossman and Shiller (1981) criticized simple present-value models for stock prices based on a test for variance bounds. They argued that if the discounted stream of expected dividends was indeed the optimal forecast of a stock price, this forecast should be more volatile than empirical prices themselves. In fact, the bounds imposed by the variance of ex-post rational prices were exceeded by their empirical counterparts. These papers, as well as later works by Campbell and Shiller (1988a, 1988b), did not directly link their arguments to bubbles. However, Tirole (1985) and Blanchard and Watson (1982) argued that these kinds of variance bounds did not hold if bubbles existed. West (1987) proposed a further test which made use of the fact that one can estimate the parameters needed for the calculation of discounted dividends in two different ways. By testing whether these ways led to the same results he tested for speculative bubbles. He found that the U.S. stock market data usually rejected the null hypothesis of no bubbles. Concerns about this test approach were raised by West himself as well as by Flood, Hodrick and Kaplan (1994). They showed that a rejection might happen because of other factors than a bubble and even if the model did not have any problems detectable by specification tests. In addition, Diba and Grossman (1988) reported empirical tests for the existence of explosive rational bubbles in stock prices. They concluded that the existence of explosive bubbles can be rejected. However, Evans (1991) disagreed and showed that standard tests might fail to detect explosive patterns of periodically collapsing bubbles. A further specific type of rational bubbles depends exclusively on aggregate dividends. These bubbles have been termed 'intrinsic' by Froot and Obstfeld (1991). They found evidence for a strong nonlinear relationship between prices and dividends and interpreted these findings as a rejection of the hypothesis that there was no bubble. In contrast, Driffill and Sola (1998) argued that the explanatory contribution of this kind of bubbles was low when they included both regime switching fundamentals and intrinsic bubbles into a model. All of these different arguments and counter-arguments are summed up by Gürkaynak (2008) stating that "for every test of bubbles, there is another paper that disputes the particular 'bubble' interpretation." (p. 182). Until today, there has been no truly satisfying result on this topic.

Our paper suggests a new, direct way to calculate and observe bubbles and to test if they are rational. The main idea is to elicit the market expectations about future dividend streams from futures with dividend payments as underlyings. Doing so, we find evidence of what is commonly known as a rational bubble. It seems likely that extreme and erratic movements of index prices can occur independent of the evolution of expected future dividend payments.

Our paper is structured as follows: Section 2 presents the dividend futures and describes the data. Since this kind of derivatives has only been introduced relatively recently and is not yet standard, we delve rather deep into the details of these contracts. In section 3 we motivate the test procedures and report the results. Section 4 concludes.

2 Dividend futures

The Dow Jones Euro Stoxx 50 Price Index (called Price Index in the following), introduced on 26 February 1998 by Stoxx Ltd., has become one of the leading European stock indices (see STOXX Ltd., 2009c, for more detailed information about this and other indices). The Euro Stoxx 50 Price Index consists of fifty companies in the Eurozone and aims to provide a "Blue-chip" representation of them. The index is anchored at 31 December 1991 with a base value of 1000 points; historical data is available back to 31 December 1986. Every year in September, the composition of the fifty stocks is examined. Their weighting is based on the free float market capitalization. All weightings are capped at a maximum of 10 percent. The weighted free float market capitalization is divided by an index divisor in order to keep the index steady through changes caused by corporate actions (STOXX Ltd. 2009b, 31).

The Dow Jones Euro Stoxx 50 DVP Index ("dividend points index", simply called Dividend Index in the following) was launched by Stoxx Ltd. on 16 June 2008. It consists of ordinary un-adjusted gross dividends paid by the companies listed in the Price Index (Thomson Reuters 2008, 1). It cumulates all dividend payments – measured in index points – from the beginning of the year until the current date. It is reset annually on the third Friday in December (Eurex Frankfurt AG 2008a, 1). For instance, a value of 100.10 as on 1 October 2009 states that 100.10 index points of the Price Index have been paid as dividends so far. All calculations are in close accordance to the Price Index: Dividend points are the amount of ordinary cash dividends in euro weighted by the market capitalization, and normalized by the same denominator as the Price Index.

More technically, both indices use the same formula, one with prices and the other with gross dividends. The exact calculation formula can be found in the Dow Jones Stoxx Dividend Points Calculation Guide (STOXX Ltd. 2009a, 1-4).

Shortly after introducing the Dividend Index, Eurex launched futures contracts on it. Starting on 30 June 2008, the Dow Jones Euro Stoxx 50 Index Dividend Futures (called Dividend Futures in the following) offered the opportunity to trade expectations about annual dividend payments with underlyings ranging from December 2008 until December 2014 (Eurex Frankfurt AG 2008b). Later the range of underlyings was expanded further into the future: When the 2008 futures contract ended on 22 December 2008, a contract on the annual dividends of 2015 was introduced. Three further derivatives based on the dividends of 2016 to 2018 were introduced on 4 May 2009 (Eurex Frankfurt AG 2009, 1-2). Since then, all dividend expectations[1] of the upcoming 10 years can be traded as futures contracts.

The main specifications of the futures contracts are: The Dividend Index serves as the underlying, whereas the Price Index is the reference equity index. Its price is determined in index points representing the gross dividends payed by the constituent companies of the index, computed to one decimal place. The contract size comprehends EUR 100 per index dividend point. All futures are cash settled on the first trading day after the third Friday in December of their respective year. The contract has no position limits, hence its price can move as much as market participants want it to. As usual, closing prices are either paid or received via margin calls (Eurex Frankfurt AG 2008a, 1-2). According to Eurex, the final settlement price is "the cumulative total of the relevant gross dividends declared and paid by the individual corporations of the underlying equity index as calculated in the form of index points by Stoxx Ltd for the contract period. The final settlement price is calculated at 12:00 CET on the last trading day" (Eurex Frankfurt AG 2008a, 2).

In a press release of 5 June 2008 a member of the Eurex board states that "... the exchange listing of the dividend element of the index separates the dividend risk and increases investors ability to focus on the fundamentals that determine equity values" (Eurex Frankfurt AG 2008b). This statement is closely related to the objective of this

[1]Notice that the futures prices equal the expectations only under the risk neutral probability measure.

paper: We analyze whether this statement is true and in how far this market's explication of investors' beliefs can contribute to the detection and measurement of asset price bubbles.

There are two main reasons why we base our analysis on Eurex futures rather than on corresponding certificates issued by financial companies such as Société Générale: First, futures are more frequently traded and more liquid than the certificates and thus a more accurate measure of market expectations. Second, certificates issued by a private corporation face a higher issuer risk than futures issued by the trading exchange. Despite the fact that the certificates have initially been introduced to allow trading of dividend expectations among private investors, we regard them as less useful for our purpose as all findings would be contingent on the assumption that the issuer risk can be neglected. This problem is less severe for derivatives issued by the exchange itself.

In our analysis we used the time series of the daily Price Index from 1 January 1987 to 8 September 2009 as provided by Datastream. Daily values of the Dividend Index and the corresponding Dividend Futures originate from Bloomberg. The time series of the Dividend Index ranges from 2 January 2005 to 8 September 2009. The Dividend Index time series will later help to estimate a dividend share function, since the payment stream of index-dividends significantly differs from the one of a single stock. While the latter is commonly paid on a yearly basis, the former are distributed over many different days. Nonetheless, the relative distribution over the year is quite constant over time.

As to the Dividend Futures, the data situation is more complex: The first future series on dividends due in the years 2008 until 2014 start on 30 April 2008. When the contract on the 2008 dividends expired in December 2008, a follower based on the dividends of 2015 was emitted. This time series starts on 22 December 2008. On 4 May 2009, three new contracts were introduced based on dividends in 2016, 2017, and 2018. All time series are observed until 8 September 2009. Some descriptive statistics about the time series used in this paper are given in table 2 in the appendix.

Figures 1 and 2 graph the time series of the eleven dividend futures prices (2008 until 2018). The futures prices for 2009 and the years thereafter dropped very sharply during the turmoil following the collapse of Lehman Brothers on 15 September 2008. The decline continued until spring 2009. Since then there is a slow but steady upturn.

While figures 1 and 2 depict the evolution of the prices for the individual dividend futures over the observation period, their prices can also be graphed for individual days along the futures' exercise dates on the abscissa. Figure 3 shows the prices for dividend futures with expiry dates 2008 through 2018 for selected days. We term this graph the dividend expectation curve. The top curve shows that on 30 June 2008 the market expected dividends of roughly 150 for 2008, slightly less for 2009, and about 140 for the years 2010 to 2014. Three month later, on 1 October 2008, after the collapse of Lehman Brothers, the market expected the same dividends as before for 2008 and 2009 but less than before for the years 2010 to 2014. During October 2008 it became evident that the financial crash had severe effects on the real economy. The market's expectations about future dividends declined sharply: on 3 November 2008 the entire dividend expectation curve is far lower than on 1 October 2008. This trend continued until 1 April 2009. Since then expectations about future dividends recovered and the curve shifted upward.

3 Calculating the fundamental value

The fundamental value of the stock index is calculated under the assumptions of the simple efficient market model (see e.g. Cuthbertson and Nitzsche, 2005, or Campbell, Lo and MacKinlay, 1997). Let P_t denote a stock (or index) price at the beginning of time period t and D_t the dividend paid between time t and time $t+1$. The efficient market model states that the expected return,

$$R_{t+1} = \frac{P_{t+1} + D_t}{P_t} - 1, \tag{1}$$

given all information available at time t, is constant,

$$E_t\left(R_{t+1}\right) = r. \tag{2}$$

Hence "one can never know that stocks are a better or worse investment today than at any other time" (Shiller, 1987). Inserting (1) in (2) and solving for P_t gives

$$P_t = \frac{E_t\left(P_{t+1} + D_t\right)}{1+r}.$$

Iterating over time and applying the law of iterated expectations yields

$$P_t = E_t\left(\sum_{i=0}^{T} \delta^i D_{t+i}\right) + E_t\left(\delta^T P_{t+T}\right) \tag{3}$$

with $\delta = 1/(1+r)$. If the transversality condition is imposed, the second expectation converges to zero as $T \to \infty$. However, a violation of the transversality condition cannot be ruled out. If it does not hold, rational bubbles can occur. The fundamental value of the stock (or index) is defined as the first expectation of (3) as $T \to \infty$, i.e., the expected net present value of all future dividend payments,

$$\begin{aligned} P_t^f &= E_t\left(\sum_{i=0}^{\infty} \delta^i D_{t+i}\right) \\ &= \sum_{i=0}^{\infty} \delta^i E_t\left(D_{t+i}\right). \end{aligned} \tag{4}$$

The fundamental value of the stock index can be calculated using the information contained in the dividend expectation curves. Let A_j denote the cumulated dividend payments of year j at the end of that year, and let $E_t^{\mathbb{Q}}(A_j)$ denote its expectation under the risk-free probability measure given all information available at time t. As we frequently have to handle daily and annual time scales simultaneously, the following two mappings help simplify the notation: First, define $y(t)$ as a mapping from time t (measured on a daily time scale) to the year in which t lies. Second, define $doy(t)$ as the day-of-year of time t, e.g. if t is 1^{st} February 2009, then $y(t)$ is 2009 and $doy(t)$ is 32.

Using this notation, the dividend expectation curve of day t (e.g. one of the curves shown in figure 5) is given by the points

$$\Big(y(t), E_t^{\mathbb{Q}}\left(A_{y(t)}\right)\Big), \Big(y(t)+1, E_t^{\mathbb{Q}}\left(A_{y(t)+1}\right)\Big), \ldots, \Big(y(t)+J, E_t^{\mathbb{Q}}\left(A_{y(t)+J}\right)\Big) \tag{5}$$

where $y(t), \ldots, y(t)+J$ are the years for which Dividend Futures are available. Although the number of available years J is changing a couple of times in our data set, we keep the notation simple and suppress the dependence of J on time t. Of course, the form and position of the dividend expectation curve will in general change from day to day, as new information arrives.

We assume that the observable prices of the Dividend Index are true indicators of the expected future dividend payments. In particular, we presuppose that the Dividend Index itself is not influenced by bubbles. This assumption seems reasonable in light of experiments showing that bubbles are rare in markets where (a) the payment date is known and (b) the participants' experience is mixed (Dufwenberg, Lindqvist and

Moore 2005). In addition, the time series shown in figures 1 and 2 are apparently not bubbly.

As the fundamental value is to be calculated under the physical measure rather than under the risk-free measure, the expectations $E_t^{\mathbb{Q}}(A_j)$ have to be transformed using

$$E_t(A_j) = e^{(r-r_f)\Delta} \cdot E_t^{\mathbb{Q}}(A_j)$$

where r is the required rate of return, r_f is the (constant) risk-free rate and Δ is the time difference between time t and the end of year j. A positive risk premium (i.e. $r > r_f$) implies $E_t(A_j) > E_t^{\mathbb{Q}}(A_j)$.

In contrast to the expected cumulated dividend payments of year j, $E_t(A_j)$, the expectation $E_t(D_{t+i})$ in (4) refers to dividends payable on a single day $t+i$. In order to compute this value for day $t+i$ we develop a method to convert the expectations about cumulated annual dividends, $E_t(A_j)$, to expectations about daily dividend payments, $E_t(D_{t+i})$ where day $t+i$ is in year j, i.e. where $y(t+i) = j$. Figure 4 shows the daily prices of the Dividend Index from 3 January 2005 to 8 September 2009. Obviously, dividend payments are unevenly distributed over the year. This fact has to be taken into account for discounting. While the absolute amount of payments differs, the relative pattern of payments does not change much from year to year. Typically the largest share of dividends is paid in spring. We construct an estimate of the normalized dividend distribution of an average year in the following way:

Let $DVP^{(j)}(d)$ denote the Dividend Index at day $d = 1, \ldots, 365$ of year j. The Dividend Index is only observed on trading days $d_1^{(j)} \leq \ldots \leq d_{n^{(j)}}^{(j)}$, where $n^{(j)}$ is the number of observed trading days in year j. We normalize both the time dimension and the amount of dividends to unity for each year. At the normalized trading days

$$\tau_i^{(j)} = \frac{d_i^{(j)}}{365}$$

the share of dividends already paid is given by

$$F^{(j)}(\tau_i^{(j)}) = \frac{DVP^{(j)}(365 \cdot \tau_i^{(j)})}{DVP^{(j)}(365)}.$$

For τ between observed days we use linear interpolation. The typical share of dividends already paid in the first proportion τ of a year is then estimated as[2]

$$F(\tau) = \frac{1}{4} \sum_{j=2005}^{2008} F^{(j)}(\tau). \tag{6}$$

The estimate $F(\tau)$ is shown in figure 5. The typical share of total annual dividends paid on the d^{th} day of a typical year is calculated as

$$F\left(\frac{d+1}{n}\right) - F\left(\frac{d}{n}\right)$$

where n is the number of days per year. Note that this approach is flexible with respect to the number of days per year: setting $n = 250$ (trading) days is often easier to handle than $n = 365$ (calendar) days.

For calculating the fundamental value, we distinguish three future time periods:

(A) the rest of the current year $y(t)$,

(B) the years $y(t) + 1, \ldots, y(t) + J$ over which the dividend expectation curve (5) of day t extends,

(C) the years $y(t)+J+1, y(t)+J+2, \ldots$ far in the future for which no explicit dividend expectations are available at time t.

We describe the contributions to the fundamental value of these three periods in turn:

Period A: Let t_A^1 be the last day of the current year. Then the contribution of the current year to the fundamental value is

$$P_t^{f,A} = \sum_{i=0}^{t_A^1 - t} \delta^i E_t\left(D_{t+i}\right). \tag{7}$$

Of course, at time t it is already known that the absolute amount of cumulated dividends in the current year is $DVP^{(y(t))}(t)$. The expected cumulated amount for the entire year is $E_t\left(A_{y(t)}\right)$. Hence, the expected amount of dividends to be paid during the remaining

[2] To simplify the calculation and the notation we ignored dividend information from 2009.

days of year $y(t)$ is $E_t\left(A_{y(t)}\right) - DVP^{(y(t))}(t)$. We use a re-normalized version of (6) to distribute this amount over the remaining days. For $doy(t)/n \leq \tau \leq 1$ define

$$\tilde{F}(\tau) = \frac{F(\tau) - F(doy(t)/n)}{1 - F(doy(t)/n)}$$

as the typical dividend distribution between time t and the end of the year. Using

$$E_t(D_{t+i}) = \left[E_t\left(A_{y(t)}\right) - DVP^{(y(t))}(t)\right] \cdot \left[\tilde{F}\left(\frac{doy(t+i)+1}{n}\right) - \tilde{F}\left(\frac{doy(t+i)}{n}\right)\right]$$

as the expected dividend payment for day $t+i$ we can easily calculate (7).

Period B: This period covers all years of the dividend expectation curve apart from the current year. The expected cumulated dividend of year $y(t)+j$ is $E_t(A_{y(t)+j})$ for $j \in \{1, \ldots, J\}$. Since a large share of the dividend payments occurs early in the year, we distribute the expected cumulated amount over the year using the function $F(\tau)$ again. Let $t_B^0 = t_A^1 + 1$ and t_B^1 be the first day and last day of period B, respectively. Then the contribution of period B to the fundamental value is

$$P_t^{f,B} = \sum_{i=t_B^0-t}^{t_B^1-t} \delta^i E_t\left(A_{y(t+i)}\right) \cdot \left[F\left(\frac{doy(t+i)+1}{n}\right) - F\left(\frac{doy(t+i)}{n}\right)\right]. \tag{8}$$

Period C: Unfortunately, the dividend expectation curves (5) do not extend into the very far future. Rather, they are cut off after 8 to 10 years; the latest expiry date in our data set is 2018. Despite the fact that dividends expected to be paid in the far future are heavily discounted, they may nevertheless contribute substantially to the fundamental value and cannot be neglected. Of course, forming expectations about dividend payments ten years in the future is difficult and the information set is extremely crude. Looking at the dividend expectation curves in figure 5 (in particular at the log scaled curves in the lower panel) suggests that market participants follow a simple strategie to form their expectations about the far future: They simply assume that dividends grow at a constant rate toward the end of the dividend expectation curve. Extrapolating this constant growth rate we can approximate the contribution of period C to the fundamental value. Using the last three observations of the dividend expectation curve of day t, one can fit an ordinary linear regression line to the logarithm of the expected cumulated dividends,

$$\log E_t^{\mathbb{Q}}\left(A_{y(t)+j}\right) = \alpha_t + \beta_t \cdot (y(t)+j) + \varepsilon_j \tag{9}$$

for $j = J-2, J-1, J$. For instance, we use the observations for 2016, 2017, and 2018 of the dividend expectation curve of 8 Sep 2009 to approximate the growth rate. Note that (9) refers to expectations about cumulated annual dividends under the risk-free measure. However, the slope β_t of the regression line can also be used as an estimate of the expected constant growth rate in the far future under the physical measure, since

$$\log E_t\left(A_{y(t)+j}\right) = (r - r_f) \cdot n \cdot j + \log E_t^{\mathbb{Q}}\left(A_{y(t)+j}\right)$$

and the term $(r - r_f)\, nj$ is merged with the intercept α_t in (9). Figure 6 indicates that the slope coefficient β_t (thin line) moves rather erratically from day to day which is not surprising given that only the last three observations of each dividend expectation curve are taken into account. As such a volatile behavior of the expectations about the far future is unreasonable we apply cubic smoothing splines[3] to the time series. Let $\tilde{\beta}_t$ be the smoothed estimate of the expected growth rate in the far future; the thick line in figure 6 shows its evolution. Apparently, this expectation is not constant but does not change radically, either. Let t_C^0 be the first day of period C. Then,

$$\begin{aligned} P_t^{f,C} &= \sum_{i=t_C^0-t}^{\infty} \delta^i E_t\left(A_{y(t)+J}\right) \tilde{\beta}_t^{y(t+i)-(y(t)+J)} \\ &\quad \times \left[F\left(\frac{doy(t+i)+1}{n}\right) - F\left(\frac{doy(t+i)}{n}\right)\right]. \end{aligned} \tag{10}$$

The contribution of period C is the expected present value of dividends payed beyond year $y(t)+J$. The expectation of those payments is approximated by the latest available value of the dividend expectation curve $E_t(A_{y(t)+J})$ growing each year by a factor $\tilde{\beta}_t$. The expected annual payments are distributed within the years according to the cumulative dividend share function. If the expected growth factor $1+\tilde{\beta}_t$ is less than $1/\delta$ the infinite sum (10) converges.

Obviously, for all three periods A,B, and C, it is necessary to have (a) the required rate of return r or, equivalently, the discount factor δ, and (b) the risk free rate of return r_f. We follow the standard approach in the literature and approximate r by the average long term rate of return of the stock index. Fitting a linear time trend to the daily time

[3] We utilized the command smooth.spline of the statistical programming language R, version 2.10.1, with the default settings.

series of the log of the stock index from 1 January 1987 to 8 September 2009, we find a slope (i.e. average growth rate of the stock price) of $r = 0.03143\%$ per trading day if the number of trading days is $n = 250$ per year; in annualized terms the required rate of return is about 7.858%. As the observation period is rather short, we regard the risk -free rate r_f as a constant.[4] We set the risk free rate to the mean Euribor-12-month-rate over the observation period, $r_f = 0.01244$ (or, annualized, 3.110%).

The total fundamental value of the stock index at day t is the sum of the three components,

$$P_t^f = P_t^{f,A} + P_t^{f,B} + P_t^{f,C},$$

and the bubble component of the stock index price is,

$$B_t = P_t - P_t^f.$$

Figure 7 depicts the daily time series of the Price Index (P_t) and its fundamental value (P_t^f). Obviously, the Price Index is always above the fundamental value, positively indicating the existence of a bubble.

4 Testing rationality

The empirical evidence presented in the previous section shows that a bubble exists. We now investigate if the bubble can be characterized as rational. A bubble is called rational if

$$E_t\left(B_{t+1}\right) = (1 + r)\, B_t. \tag{11}$$

Rational bubbles are expected to grow at rate $(1 + r)$. Condition (11) defines a martingale if the factor $(1 + r)$ is neglected. The process

$$C_t = \delta B_{t+1} - B_t$$

[4] See Kraft (2004) for portfolio optimization with stochastic interest rates and the influence of the time horizon on the results compared to deterministic interest rates.

with $\delta = 1/(1+r)$ is a martingale difference sequence because

$$\begin{aligned} E_t(C_t) &= E_t(\delta B_{t+1} - B_t) \\ &= \delta E_t(B_{t+1}) - B_t \\ &= 0. \end{aligned}$$

Figure 8 shows the time series $C_1, \ldots, C_T$ derived from the bubble time series computed in the previous section and shown in figure 7.

The hypothesis that the bubble is a rational bubble can now be tested using a formal statistical test of the martingale difference property. If the null hypothesis is rejected, one can conclude that the bubble is likely to be irrational and needs to be explained by arguments from behavioral finance and behavioral economics. Recently, a number of statistical hypothesis tests for the martingale difference property or, equivalently, of the martingale property have been developed, e.g. Deo (2000), Domínguez and Lobato (2003), Kuan and Lee (2004). A comprehensive overview is given the Escanciano and Lobato (2009). The test of Deo (2000) is based on all autocorrelation coefficients of the time series; simulation studies suggest that it is more powerful than tests based on the usual Ljung-Box statistic. Both Dominguez and Lobato (2003) and Kuan and Lee (2004) propose consistent tests based on the equivalence of the martingale difference property and the orthogonality $E(C_t \cdot f(C_{t-1}, C_{t-2}, \ldots)) = 0$ for all (measurable and square integrable) functions f. For details about the test procedures we refer the reader to the survey of Escanciano and Lobato (2009).

As there are no a-priori reasons why one of the martingale difference tests is superior for our testing problem, we applied all three tests to the time series $C_1, \ldots, C_T$. The null hypothesis is always that the time series $C_1, \ldots, C_T$ has been generated by a martingale difference sequence.

Table 1 presents the test results. Of course, the individual p-values have to be interpreted carefully when multiple hypothesis tests are performed, as the overall p-value is in general larger than the individual p-values. However, the martingale difference property cannot even be rejected by almost all of the tests individually, at the usual significance levels. We can thus conclude that bubbles may be regarded as rational. The observed bubble term does not significantly violate (11). Of course, this result does not

Test	Statistic	lags	p-value
Deo (2000)	CVM		0.1914
Dominguez and Lobato (2003)	C	1	0.7490
	K	1	0.4550
	C	2	0.0440
	K	2	0.1580
Kuan and Lee (2004)	$\mathcal{I}_g$	1	0.8851
	$\mathcal{I}_g$	2	0.1027

Table 1: Test results of the martingale difference tests (p-values)

imply that models built on behavioral arguments are wrong or useless. However, there is no obvious need to explain bubbles by irrational behavior as their time series properties are consistent with what is commonly referred to as rationality.

A possible reason why our test procedure fails to detect irrationality may be the relatively small sample size. Since the dividend futures were only introduced in 2008 there are just about 300 observations available for the test. The test decision might, of course, change when the sample size grows as time goes by. Another shortcoming of our investigation is the fact that there were no obvious large bubbles (neither inflating nor bursting) during the observation period. Maybe some historical bubbles (say, the dotcom bubble) would have been detected as irrational. Since the Dividend Index and other derivatives based on dividend expectations are now available it is just a matter of time until the next apparently irrational bubble can be investigated by the test proposed in this paper.

5 Conclusion

Using prices of recently introduced derivatives on expected future dividends of index stocks, one can calculate the fundamental value of an index. The difference between an index and its fundamental value is the bubble. The approach suggested in this paper has the advantage that the bubble term is observable, at least if some mild assumptions about expected dividends in the far future hold. In contrast to the existing literature the question if a bubble exists can be answered easily: Yes, there is a bubble component in the price of the Euro Stoxx 50 Price Index. Having established its existence, we suggest a way to test if the bubble is rational. Recently developed tests of the martingale difference property do not indicate irrationality. It is reasonable to assume that the bubble is rational. Hence, common behavioral arguments are not required to explain deviations of the index price from its fundamental value. Notwithstanding our findings, we do of course accept the fact that behavioral patterns influence and shape many human decisions and, hence, may also influence financial markets. However, if such behavioral patterns lead to martingale properties in the bubble component, we might need to rethink the way we distinguish between what we call rational bubbles and behavioral (irrational) bubbles.

As to the limitations of our study, the main shortcoming is the rather short observation period. The Dividend Index was only launched in June 2008, and the number of daily observations in the study is just above 300. While the impact of the Lehman Brothers crash is part of our time series, we lack any obvious candidates for rapidly inflating bubbles. As more data become available, future research will be able to tell if bubbles in (even) more exuberant times can be regarded as rational, too.

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A Appendix

Dow Jones Euro Stoxx 50	Start	End	# of obs.	min	max
Index	01 Jan 87	08 Sep 09	5919	615.9	5464.43
DVP Index	03 Jan 05	08 Sep 09	1173	0.87	158.59
Div. Future 08	30 Jun 08	19 Dec 08	125	152.30	159.20
Div. Future 09	30 Jun 08	08 Sep 09	304	88.10	147.00
Div. Future 10	30 Jun 08	08 Sep 09	304	54.00	137.00
Div. Future 11	30 Jun 08	08 Sep 09	304	51.70	138.20
Div. Future 12	30 Jun 08	08 Sep 09	304	53.70	139.20
Div. Future 13	30 Jun 08	08 Sep 09	304	54.50	140.20
Div. Future 14	30 Jun 08	08 Sep 09	304	55.50	141.30
Div. Future 15	22 Dec 08	08 Sep 09	179	57.20	98.80
Div. Future 16	04 May 09	08 Sep 09	92	75.50	100.50
Div. Future 17	04 May 09	08 Sep 09	92	76.00	101.70
Div. Future 18	04 May 09	08 Sep 09	92	76.70	102.30

Table 2: Descriptive statistics for the index, the dividend point index, and the dividend futures with expiry dates 2008 to 2018

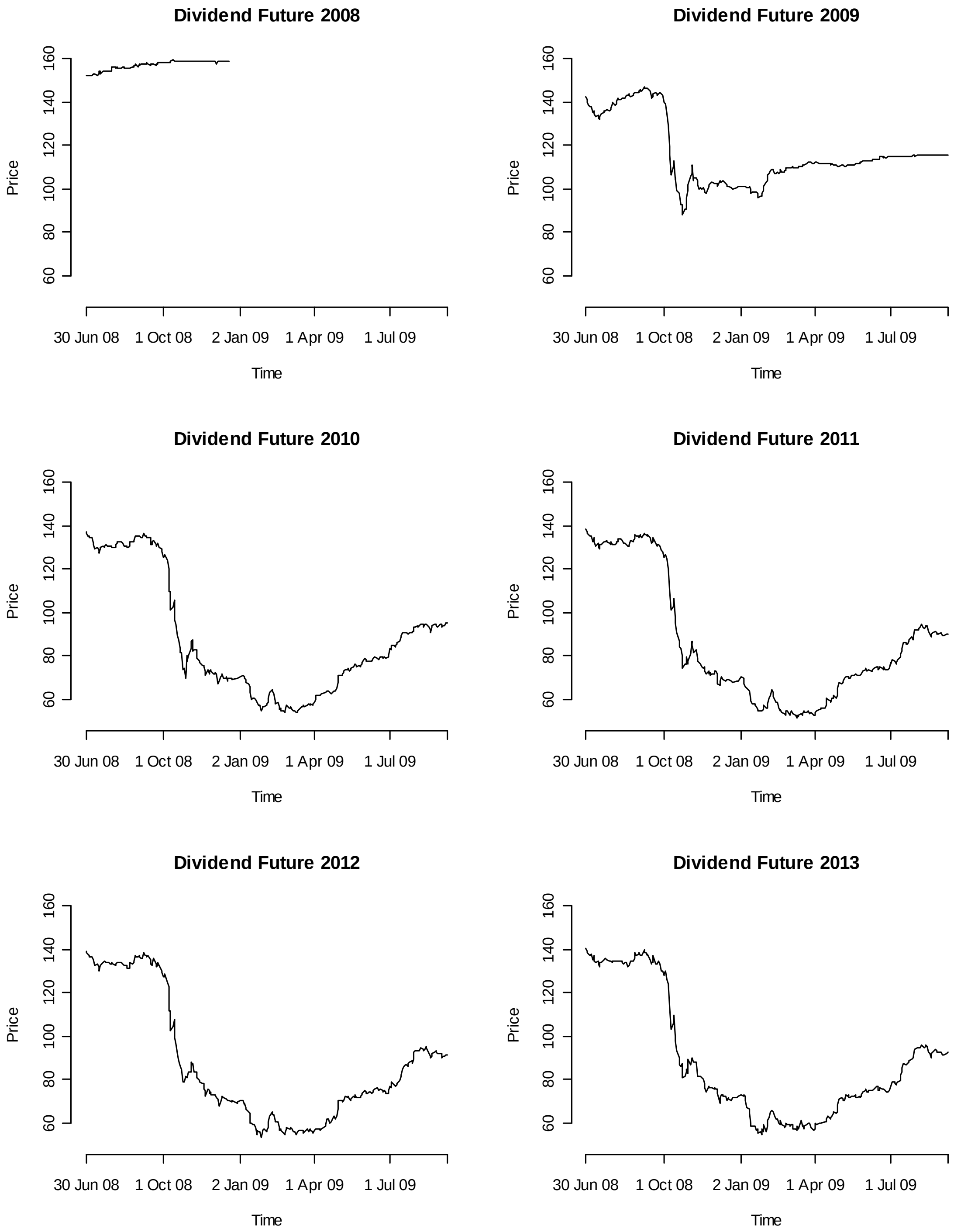

Figure 1: Time series of the Dow Jones Euro Stoxx 50 Index Dividend Futures for the years 2008 to 2013

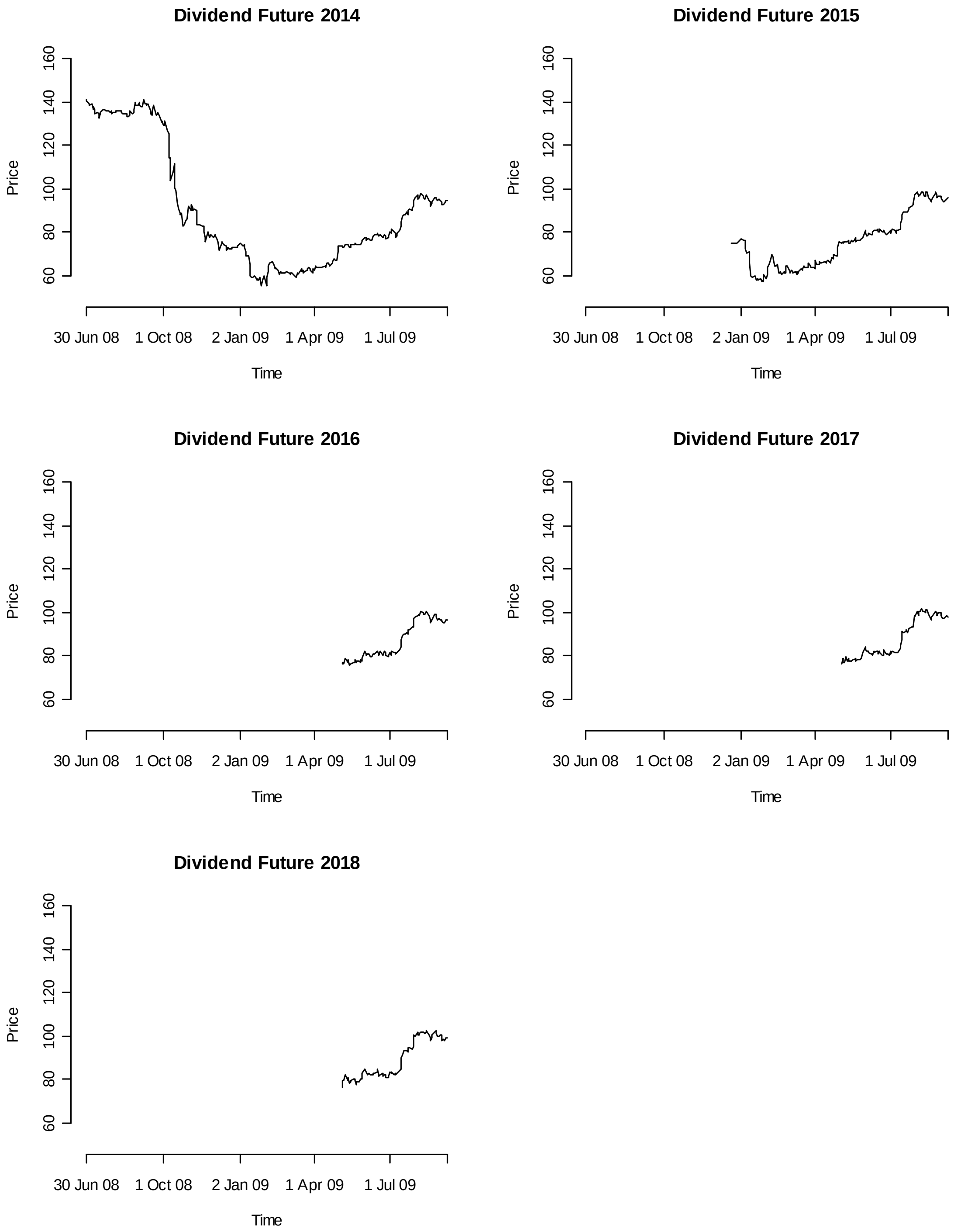

Figure 2: Time series of the Dow Jones Euro Stoxx 50 Index Dividend Futures for the years 2014 to 2018

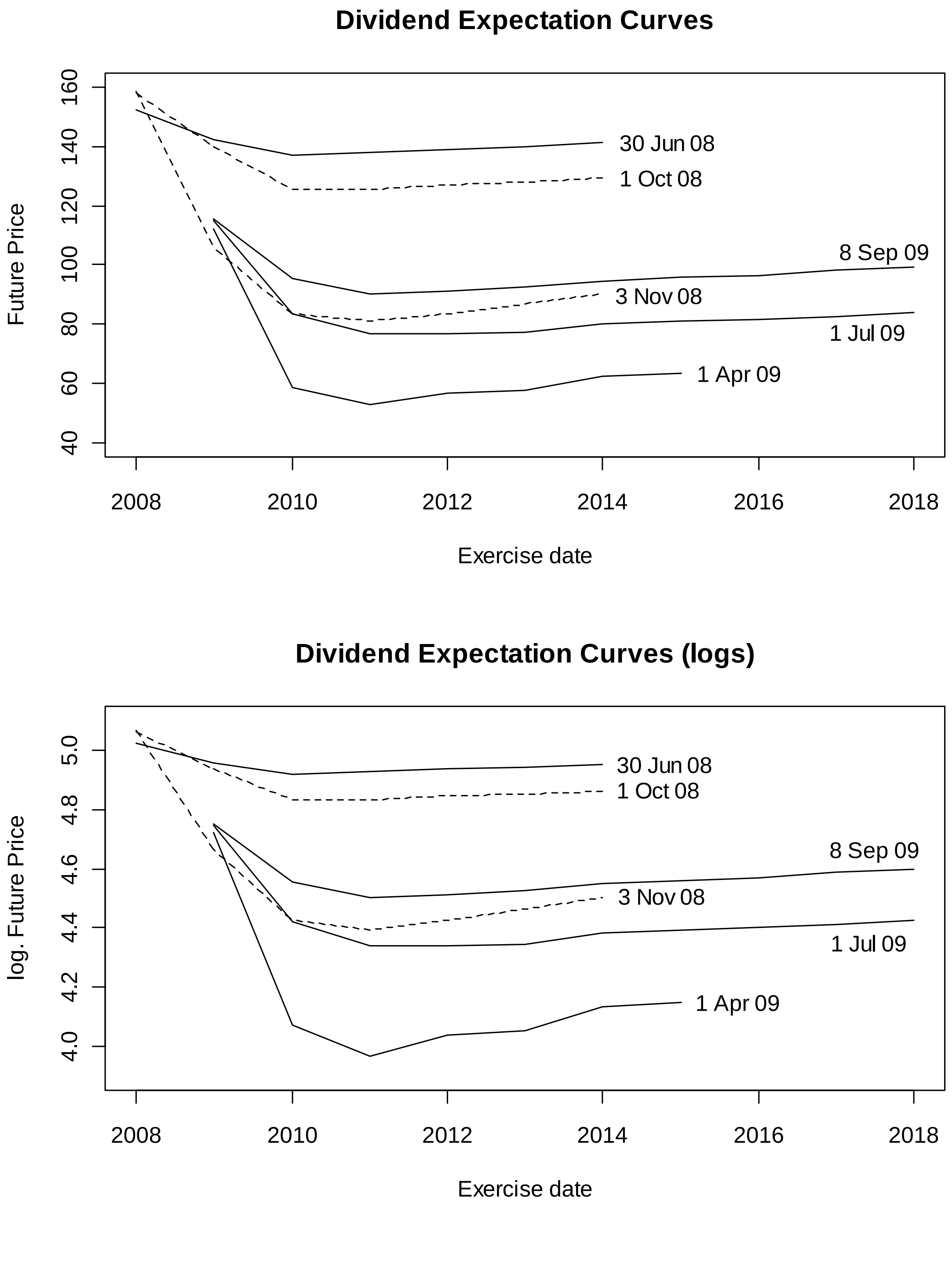

Figure 3: Dividend future prices (top) and their logarithms (bottom) on selected days for exercise dates 2008 to 2018

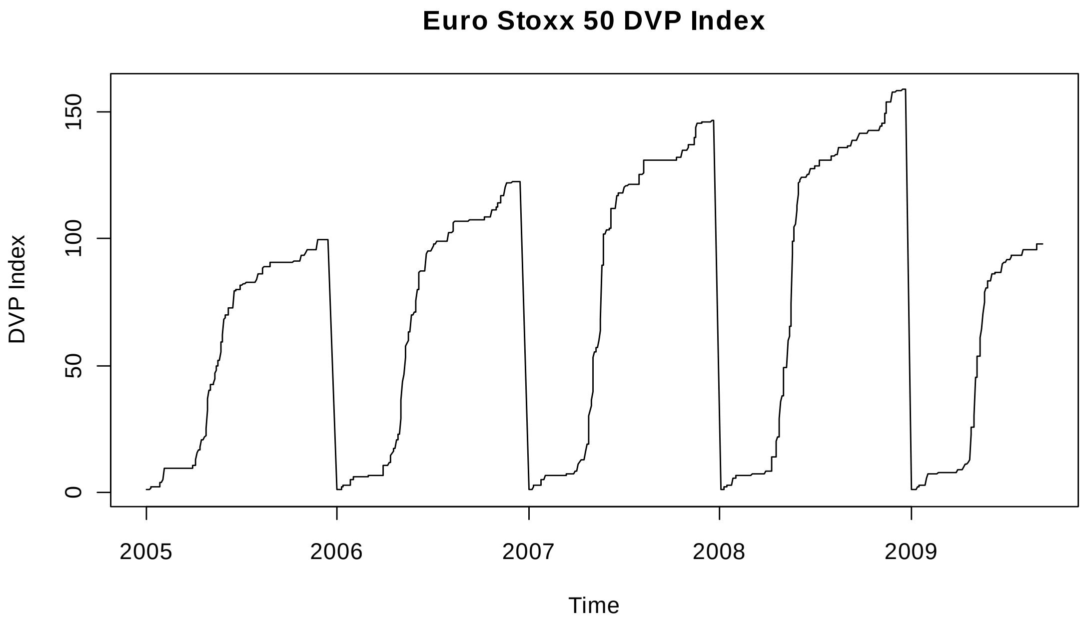

Figure 4: Euro Stoxx 50 DVP Index from 3 January 2005 to 8 September 2009

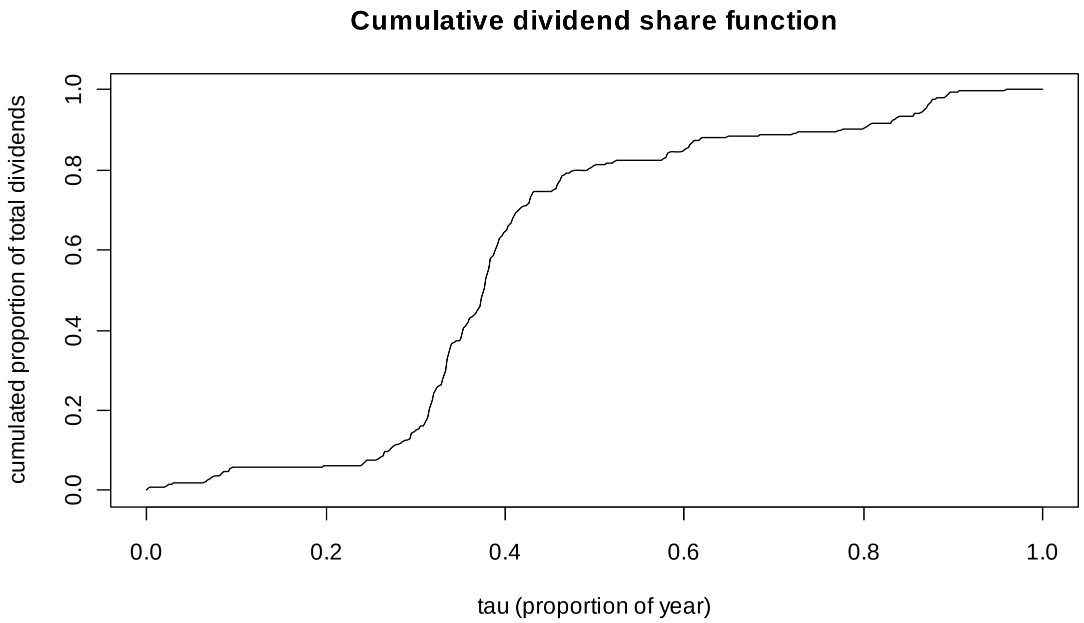

Figure 5: Typical share of dividends paid in the first proportion τ of a year

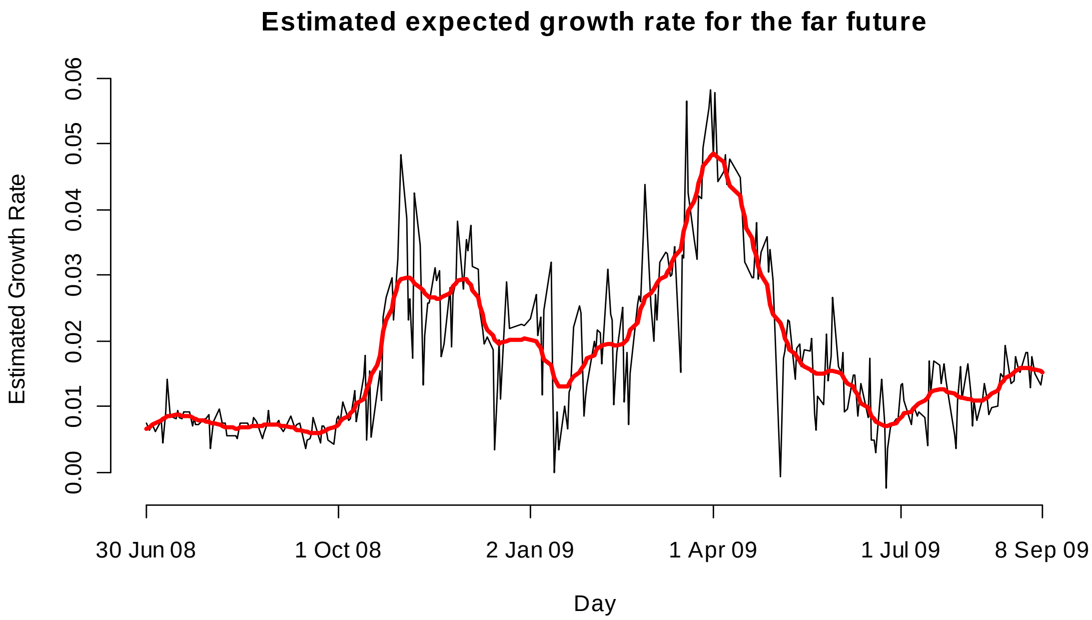

Figure 6: Estimated expected dividend growth rate for the far future and its smoothed estimate

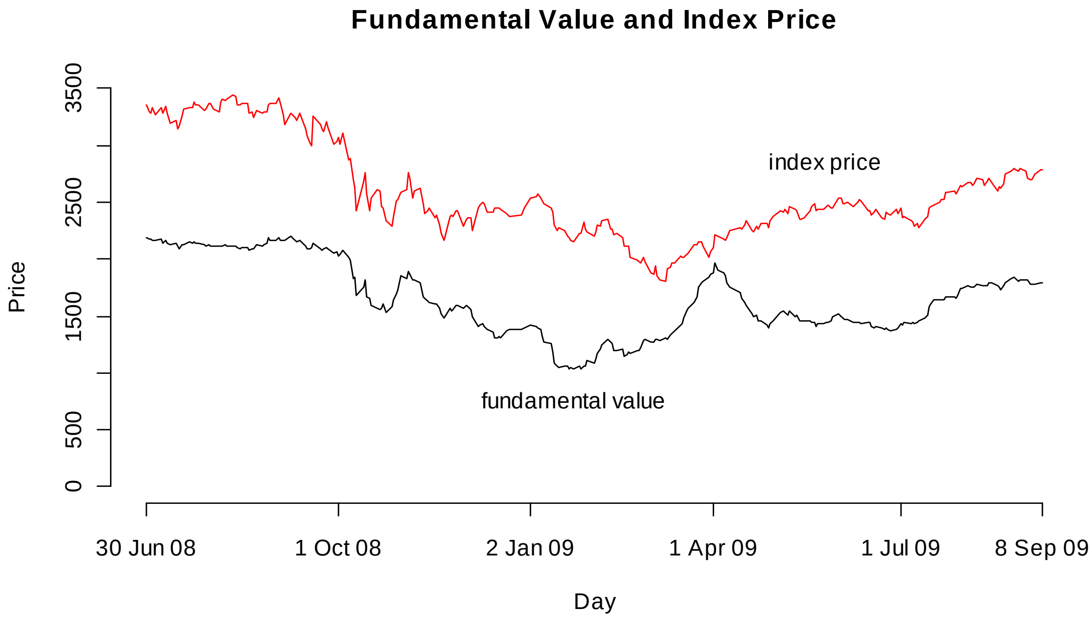

Figure 7: Fundamental value and Euro Stoxx 50 Price Index

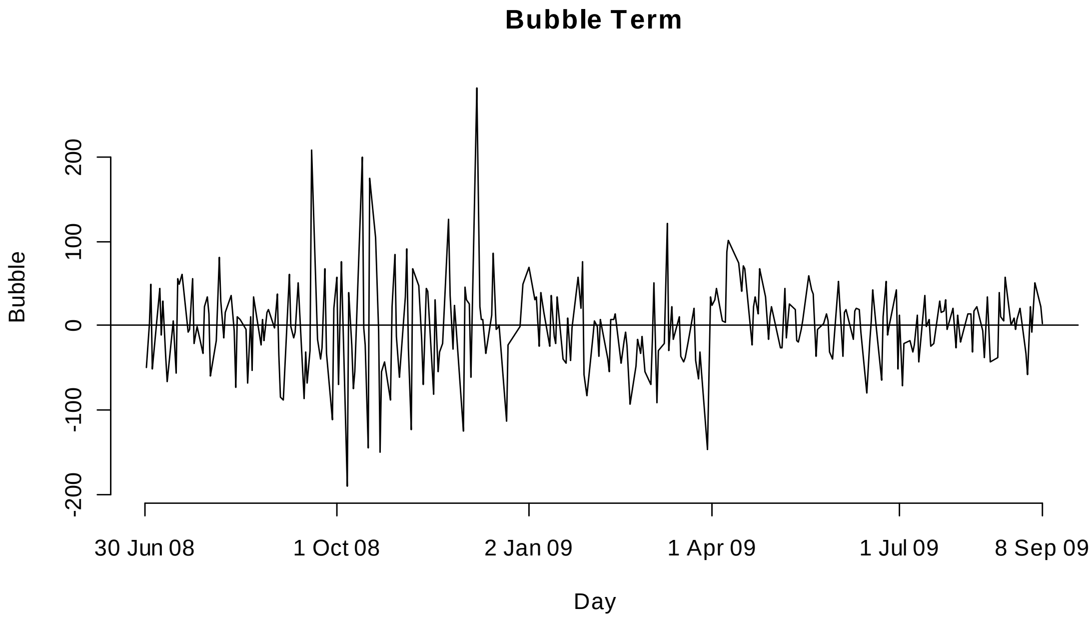

Figure 8: Evolution of the bubble term