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Böhm-Bawerk meets Keynes – what does determine the interest rate, and can the latter become negative?

**Zusammenfassung**


**Summary**

100 years after Böhm-Bawerks death and nearly 70 years after Keynes has died there is still fundamental controversy about the factors which determine the interest rate in the long run. While Economists in the Austrian tradition see it as solely driven by real phenomena, Keynesian authors mainly stress the monetary factors. Likewise, the current phase of low interest rates is explained in most different ways by prominent economists. While many blame the expansive monetary policy, others point to excess capital supply in ageing industrial states. The present paper seeks to combine these explanations by the use of a stock-flow-consistent macro-model. It is argued that theories in the tradition of Böhm-Bawerk and Keynes respectively do not at all preclude each other but, on the contrary, can nicely be combined.

JEL-Klassifikation: E10, E40, E50
JEL-Schlüsselwörter: public debt, stock flow consistent model, monetary policy
I. Introduction

100 years after Böhm-Bawerks death and nearly 70 years after Keynes has died there is still fundamental controversy about the factors which determine the interest rate in the long run. While Economists in the Austrian tradition see it as solely driven by real phenomena (see e.g. Block 1999; Huerta de Soto 2006), Keynesian authors mainly stress the monetary factors. Likewise, the current phase of low interest rates is explained in most different ways by prominent economists. While many blame the expansive monetary policy, others point to excess capital supply in ageing industrial states.

This paper uses a macroeconomic model similar to that in van Suntum (2013). Like the latter, it is both stock-flow consistent and fully micro-founded, but simplified in some respects in order to concentrate on the interest issue. We have private households who maximize an inter-temporal utility function, where not only consumption and savings but also wealth in the form of both real capital and liquidity is included. Private firms produce a single commodity (corn), which can be used for consumption and investment alike. By this simplification we circumvent the problem of an unequivocal definition of capital in a world with heterogeneous goods, which had so much puzzled Böhm Bawerk and many other authors working in this field. Moreover, capital goods live only for one period, and then vanish. Thus, problems with properly defining the roundaboutness of production are also absent in our model.

While – unlike in my former contribution - the existence of both a private bank sector and multiple interest rates is neglected in this paper, we still have both a central bank and a government. Hence, monetary impacts on the interest rate can be examined and the impact of public debt as well. On the other hand, again unlike in our former paper, we do not explicitly include a foreign sector. Moreover, no attempt is made to construct any dynamics, but the analysis is solely restricted to long-term steady state results.

Stock-flow-consistency means that there is no flow (like e.g. investment) without the corresponding stock (real capital) in the model and vice versa. Pioneering work in this respect has been done by Tobin (1969), Taylor (2004) and Godley and Lavoie (2007 I; II). Most important with respect to our issue, a change in the quantity of money can disturb the optimal size and composition of private wealth, thereby influencing the interest rate. So, unlike in common DSGE-models, stock-flow-consistent models allow for more general results in the long run. Moreover, they are both consistent with and presentable in terms of the System of National Account.¹

The remainder of the paper is organized as follows: In Section II, the model is introduced and Böhm-Bawerks three reasons for the existence of an interest rate are linked to it as well as Keynes’s monetary concept of liquidity preference. Lastly, a most simple micro-founded macro-model is created where both real and monetary determinants of the interest rate are combined. In Section III, we derive the steady state equilibrium equations and discuss the most interesting results concerning the determinants of interest. In particular, the steady state impact of monetary policy on interest, total output, and the price level is investigated. Moreover, we compare our model with Samuelson’s approach from (1958) and ask, under which circumstances the interest rate could even become negative. Section IV summarizes and points to some limitations of the model.

¹ For an overview on this class of models see e.g. Papadimitriou, D. / G. Zezza (2011) and Taylor (2008).
II. The model

1. Flow optimization by private households: Böhm-Bawerk’s first two causes

In order to derive Böhm-Bawerk’s first two causes for the existence of an interest rate\(^2\) we make use of a simple OLG-model with only two generations, the young and the elderly (Diamond 1965). With respect to flows, the representative individual maximizes the following logarithmic utility function, which is standard in modern textbooks (see e.g. Romer 2006, 51):

\[
(1) \ln U_f = \frac{1}{2 - \theta} \ln C_1 + \frac{1 - \theta}{2 - \theta} \ln C_2 \quad \text{with } 0 \leq \theta \leq 1
\]

In equation (1), \(C_i\) denotes his consumption quantities in Period \(i \in \{1, 2\}\) of his life, while \(\theta\) is the rate of time preference. Note that, even with \(\theta = 0\), there could be an incentive to antedate consumption because of its diminishing marginal utility in each Period. In particular, as Böhm-Bawerk rightly stated, with the expectation of a rising income, consumption tends to be less valuable in future than today. Hence, starting with \(C_1 = C_2\), individuals would try to shift some part of their consumption from Period 2 to Period 1, which in turn would immediately generate a positive rate of interest. This is nothing else than Böhm-Bawerks “first reason”.

His “second reason” points to a psychological law, according to which future needs are systematically underestimated. Here it is where the modern notion of time preference comes into play. Even if marginal utility were constant, a positive \(\theta\) would also create a tendency to antedate consumption. In the extreme case where \(\theta = 1\) consumption would only occur in Period 1 while in Period 2 it were zero.

Hence, in our textbook-utility function two of Böhm-Bawerks main arguments are already incorporated. By delogarithmizing, (1) can be transformed into the more convenient form:

\[
(2) U_f = C_1^{(2 - \theta)} C_2^{(1 - \theta)/(2 - \theta)}
\]

We assume that individuals earn original income only when they are young, while just consuming their wealth including interest in the second period of their life. The respective budget constraints are then given by

\[
(3) Y_{H1} = C_1 + S_1 \\
(4) S_1 (1 + i_v) = C_2
\]

where \(S_1\) denotes savings in the first period of an individual’s life and \(i_v\) is the average interest paid on savings\(^3\). Applying the conventional Lagrange-method yields the following set of optimal flows:

\[
(5) C^*_1 = \frac{Y_{H1}}{2 - \theta} \\
(6) C^*_2 = \frac{(1 - \theta)(1 + i_v)Y_{H1}}{2 - \theta} \\
(7) S^*_1 = \frac{(1 - \theta)Y_{H1}}{2 - \theta}
\]

\(^2\) For a short overview see e.g. Ekelund/Hebert (2007), pp 313.

\(^3\) Because part of wealth is held as liquidity, \(i_v\) is generally not the same as the capital market interest rate, see below.
Leaving optimization of private wealth to a later section, for now we leave the household sector by only noting that, according to (7), savings are independent from the interest rate but a certain fraction of income which only depend on the rate of time preference.

2. The productive sector: Böhm-Bawerk’s third cause

Unlike other Austrians like e.g. Fetter (1914), Böhm-Bawerk did also adjudge the production side of the economy an impact on the interest rate. He famously derived his famous “third cause” from the roundaboutness of production, thereby creating a temporal capital theory which has many similarities to that of Thünen.

In our model, we acknowledge this issue by incorporating a most simple production function where capital is the only factor of production:

\[ Y = K_F^\beta \]

where \( Y \) is gross domestic production, \( K_F \) is real capital (seed), and \( \beta \) is the relevant elasticity which is assumed to be less than unity. Hence we have positive, but diminishing marginal returns on capital as it is generally assumed in neoclassical production theory as well. Moreover, we assume that the seed completely vanishes in one period and, hence, gross capital income must cover both capital regeneration and pure interest. Thus for the distribution of income we have

\[ (1 + i)K_F = \beta Y \]
\[ \Pi_F = (1 - \beta)Y = Y_{H1} \]

where \( i \) is the capital market interest rate and \( \Pi_F \) is the firm’s profits which accrue to individuals when they are young. As an illustration, one can conceive that the latter both operate the farms and supply them with capital in the first Period of their life, while just reaping the fruits of their savings after retirement.

It is worth noting that with \( 0 < \beta < 1 \) we have \( Y > K_F \) if \( K_F < 1 \) and vice versa. In other words, it can well be that gross output \( Y \) is lower than capital input, in which case the net rate of return turns negative. We will come back to this issue when discussing the possibility of negative interest rates.

3. Private wealth optimization: introducing money and bonds

Now we return to our private households and see how they optimize their stocks, which consist of both bonds and liquidity in our model. Bonds are issued either by firms, by the government, or by the central bank. By purchasing them, private households provide capital \( K_H \) to the capital market and receive the respective interest rate \( i \). It is assumed that bonds have a maturity of just one Period and must then be renewed as the case may be.

Liquidity in the form of paper money is held by private households for the two main reasons that have already been given by Keynes: On the one hand, money is needed to carry out daily market transactions. We assume that it is proportional to nominal income, i.e. we have

\[ L_T = \bar{\alpha}Y_p \]

On the other hand, money can also be held as a luxury good which is used for speculative purposes or just as a liquid form of wealth. Only in the latter case, however, it is dealt with as part of the individual’s utility function in our model. On contrast, the transaction motive is viewed as a pure need which does not autonomously add to the individual’s wellbeing. Otherwise, there would be double-counting with the consumer goods which have already been recognized in the flow-part of our utility function (1).

Formally, we assume the following stock-part of the utility function:
\[ (12) U_S = \left( K_H \right) \left( L_s / p \right) \]

where \( K_H \) is the individual capital supply and \( L_s / p \) is that part of liquidity which is held in excess of pure transaction needs (i.e. the Keynesian idle balance). Both are defined in real terms and weighted by their respective rate of return, which is the capital market interest rate \( i \) in case of \( K_H \) and the non-pecuniary advantage of liquidity \( l \) in case of \( L_s \).

Total utility is assumed to be simply the sum of (1) and (12), so we have no problems with maximizing the two parts separately. The volume of \( V = K_H + (L_s / p) = S \) is known from (7). So, by employing (10) in addition, we can easily maximize (11) in order to get the optimal structure of individual wealth:

\[ (13) K_H^* = \frac{i}{i + l} \left( \frac{1 - \theta}{2 - \theta} \right) (1 - \beta) Y \]

\[ (14) (L_s / p)^* = \frac{l}{i + l} \left( \frac{1 - \theta}{2 - \theta} \right) (1 - \beta) Y \]

Obviously, according to (13) and (14), individual save the more in the form of bonds, the higher the interest rate is, and tend to hold the more liquidity, the higher the respective non-pecuniary advantages are. This is in accordance with both intuition and Keynesian liquidity preference theory.

The average interest on savings which we have referred to in (4) above can be calculated as follows:

\[ (15) i_s = \frac{i K_H}{K_H + (L_s / p)} = \frac{i K_H}{V} \]

With this supplement of individual utility maximizing in terms of stock, two purposes have been achieved: First, like in Böhm-Bawerk’s theory but in contrast to many others, stocks come into play in addition to flows in the explanation of interest, as it was rightly demanded by Tobin. Second, unlike Böhm-Bawerk, we have thereby created a link between real and monetary determinants of interest, and at the same time between Austrian and Keynesian theory.

4. The public sector

Liquidity does not fall from heaven in our model, but is brought into circulation by a monetary authority. We restrict our analysis on open market policy, i.e. the central bank purchases bonds \( K_M \) (defined in terms of commodity units) at the capital market in order to create money. In addition, we assume that there is some initial amount of paper money which is \( M \) (defined in nominal terms). It greatly facilities calculations when the amount of \( K_M \) issued is defined as a fraction \( m \) of total income. We then have for the real money supply:

\[ (16) \frac{M}{p} = \frac{\tilde{M}}{p} + mY \]

Because the monetary authority earns interest \( i \) like any other provider of capital, there accrue central bank profits which are assumed to fall to the government. For simplicity, they are assumed to be the only source of public receipts. However, the government can also take credit at the capital market by issuing public bonds \( K_G \), which are defined in commodity units as before. Again, we define the volume of these bonds as a fraction \( g \) of total income. Moreover, we assume that government receipts are either consumed or required for paying interest on public debt. Thus the steady-state government’s budget equation is given by
(17) \( i(K_M - K_G) = iY(m - g) = C_G \)

Note that public debt reduces public consumption in the steady state unless we have a negative interest rate. Only in transition periods, which are not in the scope of the present paper, can the government extent its primary expenses by taking additional debt, as is well known from conventional growth theory.

III. Equilibrium conditions and steady state results

1. Equilibrium interest rate and price level

After all these preparations, it is time to make our model work. First, we derive the steady state equilibrium conditions. With respect to the capital market, we have

(18) \( K_H + K_M = K_f + K_G \)

with capital supply at the left hand side and capital demand at the right hand side of the equation. By using (9), (13) and (17) this can be transformed into

(19) \[ \frac{i}{i+l} \left( \frac{1-\theta}{2-\theta} \right)(1-\beta)Y + mY = \left( \frac{\beta}{1+i} \right)Y + gY \]

After cancelling \( Y \) in (19) and some manipulation of terms we get the following quadratic equation for the interest rate:

(20) \[ i = \frac{-A_1}{2} \pm \sqrt{\left( \frac{A_1}{2} \right)^2 - A_2} \]

with

\[ A_1 = \frac{(1-\theta)(1-\beta) - \beta + (m-g)(1+l)}{(2-\theta)(1-\beta) + m-g} \]
\[ A_2 = \frac{(m-g-\beta)l}{(2-\theta)(1-\beta) + m-g} \]

Obviously, all variables in (20) are exogenous so both interest and all other endogenous variables in real terms can be easily calculated.

Concerning nominal terms, we need to know the commodity price level \( p \), which can be calculated from the monetary equilibrium condition. The latter is given by

(21) \[ \frac{M}{p} = \frac{\bar{M}}{p} + mY = (L_s/p) + (L_f/p) \]

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4 More precisely, this is the case unless the economy’s growth rate (which is zero in our model) exceeds the interest rate.
where money supply stands on the left hand side and real liquidity demand on the right hand side, the latter being the sum of idle liquidity \((L_a / p)\) and liquidity held for transaction purposes \((L_T / p)\). From (21) the price level can then easily be derived:

\[
(22)\ p = \frac{\bar{M}}{(L_a / p) + (L_T / p) - mY}
\]

In the appendix, some numerical examples are given with the whole set of equilibrium values for all variables in the model. In the following, we do not provide any formal proofs but restrict ourselves to highlight some very general results which have been derived by both numerous simulations and economic intuition. Most of them are also easily verified by a quick inspection of the relevant equations.

2. **What does actually drive the interest rate and why?**

While we have verified all the three of Böhm-Bawerk’s causes for the existence of an interest rate in the Sections above it is also true that both fiscal and monetary have an impact on it as well. Formally, this can be derived from (20), where neither \(g\) nor \(m\) cancel out. On the other hand, \(\bar{M}\) does not appear in (20), while it is proportionate to the price level according to (21). This seems to support the widespread belief that a pure increase in the quantity of money does only lead to inflation but does not affect the interest rate, at least not in the steady state.

Indeed, one has to be quite careful here. Yes, a simple multiplication of money “by helicopter” would actually only increase the price level. The same is true for the - perhaps more realistic case - that the central bank purchases any commodities for freshly printed Dollar notes. So far, the Austrian view is supported by our model.

However, things are different with the normal case of debt money, as it has been already detected by Metzler (1951). The reason is that, in this case, the central bank does not only print notes, but also acts as a supplier of capital at the market. While this can thoroughly be viewed as a cheat, because no “real” savings stand behind this capital offer, it nevertheless decreases the interest rate, even in the long run according to (20). This would not be the case only if private households were aware of the fact that the central bank is lastly their own, because with a given amount of public goods, its profits tend to reduce their tax burden by the same amount. In this case of Ricardian equivalence they would namely reduce their own savings accordingly, so that we would ideally have \(\Delta K_M = -\Delta K_H\) and no impact of monetary policy on the interest rate would result. However, it is not very likely that Ricardian equivalence in this extreme form really exists. So the central bank can principally reduce the interest rate by whatever extent, even in the steady state. Moreover, because of (9), this would c.p. also increase both the capital stock and total output in the economy.

However, these benefits would generally go along with a change in the commodity-price level, according to our simulations. Interestingly, either temporary inflation or temporary deflation could occur, depending on the value of exogenous variables. Normally, one would expect a rising price level, because of the increase in money supply. However, also real output increases and, even more important, so does the demand of idle money because of the decreasing interest rate. If the latter effects exceed the former, deflation rather than inflation will occur.\(^6\)

Although, in either case, this would be only a one-off effect, without permanent to occur, it cannot be neglected from a welfare point of view. Indeed, future generations would all benefit from a permanently higher income and wealth. However, in case of inflation the generation in the transition Period would suffer from a devaluation of their savings, while in the deflationary scenario debtors who live in the transition Period would correspondingly suffer. Hence, no clear Paretian improvement can be achieved an expansionary monetary policy like this.

Concerning public debt taking, there is definitely an increasing impact on the long term interest rate. Formally, this can again be derived from (20). The economic the reason is that we have assumed that all government receipts are consumed. So any increase in public debt causes a crowding out at the capital market without any

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\(^5\) Formally, we have \(\partial i / \partial m < 0\) in that equation.

\(^6\) See the examples in the appendix, where we have an inflationary effect with a relatively low parameter \(\bar{a}\), and a deflationary effect with higher values of \(\bar{a}\). According to (11) \(\bar{a}\) is the relation of transaction liquidity and total income.
compensation in the form of public investment. Correspondingly, the higher public debt is in relation to total income, the lower is both the latter and the private capital stock.

It is also quite obvious what happens in the model when additional public debt is financed by additional debt money, i.e. in the case of monetizing public debt. Formally, we then have an equal increase in $\pi$ and $m$, which lastly cancels out in (20). In this case, nothing changes in the steady state except the price level, which definitely increases according to (22). The interest rate, total income and also its distribution would all be the same as before in the long run, including the relative size of the public sector! Indeed, there are now higher interest expenses of the government, but these are exactly outweighed by correspondingly higher central bank profits. So the government can spent neither less nor more for public consumption than in the initial steady state.

Again, however, this is not true for the transition periods. In these Periods, the government can indeed spend more money, because of the temporary net-receipts from additional public debt. Again, the costs are lastly born by those who had saved before and are now partly expropriated by inflation. Although the latter will end after the steady state has been reached again, the onetime increase in price level leaves the transition-people with a corresponding loss in wealth.

To summarize, both the Austrians and the Keynesians are right: Yes, printing money in order to decrease the interest rate does work even in the long run. But also yes, this is a fraud, the costs of which are imposed by the way of temporary inflation or deflation on people who live in the transition Period. This appears therefore a highly questionable way of spurring growth, although finally a new steady state with both a higher total income and a higher price level can be reached.

On the other hand, the view of some extreme Austrians asserting that the interest rate was solely determined by time preference in the long run is clearly false. A positive rate of time preference rate is neither necessary nor sufficient for a positive rate of interest, according to our model. It is just only one of several factors – both real and monetary – which have an impact. On the other hand, our results also prove wrong those who believe that interest would instantly vanish if only the evil impatience of people could be overcome or be compensated by some public measures.

3. **Can the interest rate be negative in the long run?**

Samuelson (1958) famously showed in his path-breaking OLG-model that the interest rate can well get negative in a pure barter economy without the existence of durable goods. In that model, three generations exist, the young, the middle aged and the elderly. The youngest generation has no wealth and only a small income, so they seek to increase consumption by taking credit. When they want to invest in a new firm, for instance, they will have to take even more credit. On contrast, in the middle of their life they do not only pay back the credit but also save as a provision for their retirement. Finally, in the last phase of their life, they consume the whole of their wealth including interest. Thus, in summary, in this three-Period life cycle model the middle generation gives credit as a net saver for the respective young generation in order to receive interest from the latter when they are old.

Now assume that population decreases at a certain rate, i.e. there are less young people and more elder people per head of the middle-aged. Obviously, interest should then go down, because there are more savers and less borrowers in each time Period. As Samuelson has shown, it could even happen that interest becomes negative, at least in a pure barter economy where neither durable goods nor money is available for the storage of wealth of the middle generation. Suppose, for instance, Robinson Crusoe ages and wants to provide for his elder days when he will no longer be able to work as intensive as before. If there is no Friday to whom he can give credit, he will have to save some apples or cattle although he knows that part of them will already have spoiled or died until he can consume them. In other words, Crusoe will be well prepared to accept a negative interest rate on his savings in order to survive at all. This is basically the same with several generations, when savings tend to exceed capital demand even at a zero interest rate, so the latter will again turn negative in a barter economy without durable goods.

Only the existence of stable money could prevent that, because in this case savers would always have a preferable alternative to lending their assets for less than nothing. Thus stable money creates the so-called lower zero lower bound to expansionary central bank policy. However, as Samuelson argued, this would even worsen the welfare situation, because the interest rate should always equal the growth rate of an economy according to the so-called golden rule. So with a declining population, a negative interest rate would indeed be desirable to
prevent dynamic inefficiency. Moreover, it is often argued that a negative interest rate would be necessary to overcome situations like the Keynesian liquidity trap. Silvio Gesell famously sought to generate it by levying a tax on holding cash. Even modern economists like Mankiw (2009) favor similar measures in order to spur economic growth in times of crisis.

So it is interesting to ask whether our model allows for a negative interest rate or not. First, some differences must be pointed to in comparison with Samuelson’s approach. In particular, we have explicitly regarded only two generations instead of three. One can well assume, however, that there is indeed another, young generation (children) which is fed by the adult and, hence, included in the latter’s consumption, so this is a minor point.

The more important difference to the common life cycle model is that we have implicitly taken account for inheritance. In our model, the young generation immediately receives all the firm’s profits as their income and, hence, effectively owns them. So they do not have to take any credits for consumption but, on the contrary, are net savers themselves in order to provide for their retirement. For only equity is handed down to the younger generation, while borrowed capital sensibly remains in the possession of the elderly.

Thus, in our model, a decreasing population would generate less savers, but relatively more elder people who seek to dis-save by selling bonds to the market or turning their liquidity into consumption. Moreover, with an unaltered number and size of transmitted firms, total capital demand for the sake of real investment would be the same as before. Thus we should c.p. expect an increasing rather than a decreasing interest rate in the ageing society! On the other hand, many other effects – e.g. at the labor market - would have to be taken into account, which are not covered by our simple model. At least, however, it throws seriously doubt on the common belief of the capital melt down hypothesis.

Nevertheless, the capital market interest rate can well become negative in our model, if only the central bank increases debt money by a sufficient amount. Formally, even with non-pecuniary return on holding liquidity \( l > 0 \), a sufficiently high \( m \) will ultimately yield a capital market interest rate \( i < 0 \). In this case, individuals would not only hold liquidity as their only asset, but even turn to borrowers at the capital market, i.e. we would have \( K_H < 0 \). This in turn implies that the central bank would then be the only supplier at the capital market.

Moreover, the net rate of return on real capital \( K_F \) is also negative in such a scenario, i.e. for marginal gross return we have \( dY/dK_F < 1 \). This is also a questionable outcome from a welfare economic point of view, although total firm profits remain positive in our model.

To some extent, this scenario is similar to the current situation of financial repression in the European Monetary Union. In particular, the interest rate is below the inflation rate and, hence, implies a negative real interest, at least for cash-holders and holders of relatively liquid nominal assets. It is sometimes argued this cannot be blamed to the ECB, because the negative interest was caused by “real” reasons like in the Samuelson model. However, in the light of our model, this line of argument is not very convincing. First, the model puts considerable doubt on the so-called capital melt down hypothesis when inheritance is recognized, as it should be. Second, we could show that the central bank can indeed turn the capital market interest rate negative, even in the steady state, if only sufficiently strong - although temporary - inflation is allowed for. On contrast, according to our model, a negative interest rate is impossible in the steady state without an extremely expansive monetary policy like this.

IV. Summary

The interest theories by Böhm-Bawerk and Keynes are thoroughly consistent with each other. As has been shown above, both real and monetary factors determinate the interest rate. In particular, unless an extreme form of Ricardian equivalence is assumed, the central bank can indeed permanently lower the interest rate to a voluntarily chosen or even negative level. In the model, this leads to an increase in both total output and the capital stock. However, this necessarily comes at the cost of inflation. Although the price will only temporarily

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7 However, as numerical simulations have shown, it can never happen that \( l + l < 0 \), i.e. the respective denominator in (13) and (14) remains always positive. At least, this is true when we do not allow for a negative price level, which would apparently do not make any sense.
change, this is sufficient to make such an expansive monetary policy quite questionable from a welfare economic point of view. In particular, some of those who live in the transition Period are partly deprived of their wealth.

We have also argued that Böhm-Bawerk’s three “real” causes for the existence of an interest rate are both still valid and fully consistent with contemporary economic theory. On contrast, it is not possible to reduce them to just one reason like e.g. the concept of intertemporal discounting. Even in a pure barter economy we would have to take at least one additional factor into account, namely the decreasing marginal utility of consumption in any period. Hence, adverse opinions like e.g. in Herbener (2011) appear to be clearly invalid in the light of our analysis.

Of course, we have to admit that our analysis has some limitations. In particular, the two-part utility function and the simple production function without labor which were employed might be a matter of debate. Moreover, there is neither a foreign sector nor a private financial sector in our model. On the other hand, as was shown in van Suntum (2013), the incorporation of these refinements does not change anything in principle. For example, when foreign bonds are only imperfect substitutes to domestic bonds, the central bank has still the power to manipulate the capital market interest rate at home.

Another limitation of our model is the restriction to a one period maturity of capital and only two Periods of life for individuals. These simplifications have been mainly chosen in order to simplify calculations because, with allowing for more Periods, compound interest would creep into the equations and the model would no longer be solvable other than by numerical methods. Indeed, it would be interesting to have a corresponding more sophisticated model, in particular for the purpose of analyzing transition Periods in more detail. As long as only steady states are considered, however, this limitation appears of less, if any relevance.

So hopefully this paper could shed some light on the long-term determinants of the interest rate at least and will yield fruitful subsequent research on this fascinating issue.

References


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Appendix: Some numerical examples

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<tr>
<th>Exogenous Variables</th>
<th>Symbol</th>
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<td>0,20</td>
</tr>
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| Endogenous Variables                  |        |                                      |                                      |
| capital market interest rate          | i      | 0,09  | 0,01  | -0,05  | 0,29  | 0,09  | 0,01  | -0,05  | 0,29  | 0,09  |
| real transaction liquidity            | \(L_1/p + \alpha Y\) | 0,08  | 0,08  | 0,08  | 0,08  | 0,08  | 0,08  | 0,08  | 0,08  |
| productive capital (+ investment)     | \(K_{F}=1\) | 0,07  | 0,08  | 0,08  | 0,06  | 0,07  | 0,08  | 0,08  | 0,07  |
| total output (GDP) \(V\)              |        | 0,7670  | 0,7785  | 0,7527  | 0,7670  | 0,7670  | 0,7737  | 0,7785  | 0,7670  |
| real capital offer individuals \(K_{1}\) | 0,07  | 0,07  | 0,07  | 0,13  | 0,07  | 0,07  | 0,07  | 0,13  |
| real capital demand government \(K_{0}\) | 0,00  | 0,00  | 0,00  | 0,08  | 0,08  | 0,00  | 0,00  | 0,08  |
| real capital offer central bank \(K_{ra}\) | 0,00  | 0,07  | 0,16  | 0,00  | 0,08  | 0,00  | 0,07  | 0,16  |
| check: capital market in equilibrium \(Y = \gamma s\) | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  |
| real firm profits = income young individuals \(\prod r_{1} = \prod s_{1}\) | 0,69  | 0,70  | 0,70  | 0,68  | 0,69  | 0,69  | 0,70  | 0,68  |
| real idle liquidity \(L_{0}/p\)       | 0,16  | 0,22  | 0,31  | 0,09  | 0,16  | 0,16  | 0,22  | 0,31  |
| price level                           | \(p\) | 423,14  | 426,19  | 435,10  | 507,19  | 626,48  | 129,13  | 129,16  | 129,10  | 144,03  | 143,57  |
| individual's total wealth \(V = KH + US/p\) | 0,23  | 0,23  | 0,23  | 0,23  | 0,23  | 0,23  | 0,23  | 0,23  |
| individual's consumption in 1. Period \(C_{1}\) | 0,46  | 0,46  | 0,47  | 0,45  | 0,46  | 0,46  | 0,47  | 0,45  |
| individual's savings \(S_{1}\)       | 0,23  | 0,23  | 0,23  | 0,23  | 0,23  | 0,23  | 0,23  | 0,23  |
| check: \(V = S + \gamma (1)\)         | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  |
| average interest rate on savings \(i\) | 0,03  | 0,03  | 0,02  | 0,17  | 0,03  | 0,03  | 0,02  | 0,17  |
| individual's consumption in 2. Period \(C_{2}\) | 0,24  | 0,23  | 0,23  | 0,26  | 0,24  | 0,24  | 0,23  | 0,24  |
| check: \(S_{2}+\prod r_{2}+\prod s_{2}+\prod r_{1}=\gamma d\) | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  |
| check: \(Y-K_{1}(1-i_{1})-Y_{M1}=\gamma d\) | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  |
| government consumption \(C_{p}=I_{KM}-I_{KG}\) | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  |
| check: \(Y+K_{1}+\gamma d +CG=\gamma d\) | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  | 0,00  |
| central bank profit \(\prod p = (1+i)^{F}\) | 0,00  | 0,07  | 0,15  | 0,00  | 0,07  | 0,07  | 0,15  | 0,00  |
| individual's saving rate \(S_{j}/Y_{Mj}\) | 0,33  | 0,33  | 0,33  | 0,33  | 0,33  | 0,33  | 0,33  | 0,33  |
| individual's consumption structure \(C_{2}/C_{1}\) | 0,51  | 0,50  | 0,51  | 0,59  | 0,51  | 0,51  | 0,50  | 0,51  |
| Utility (Flows only) \(u_{p}\)         | 0,37  | 0,37  | 0,37  | 0,38  | 0,37  | 0,37  | 0,37  | 0,38  |
| Utility (stocks only) \(u_{e}\)        | 0,55  | 0,72  | 0,35  | 0,55  | 0,55  | 0,72  | 0,35  | 0,55  |
| Utility total \(U = u_{p} + u_{e}\)    | 0,92  | 1,09  | 0,72  | 0,92  | 0,92  | 1,09  | 0,72  | 0,92  |

Samuelson, P. (1958), An exact consumption-loan model of interest with or without the social contrivance of money, Journal of Political Economy 66, pp 467 - 482