ARE STOCK RETURNS PREDICTABLE?
A Century of Returns

Figure 1 shows the US monthly real S&P500 index from January 1915 to April 2004.
The real index is simply the nominal index divided by an aggregate price index and shows the changing purchasing power of holding a diversified portfolio of stocks that mimics the S&P500 index.

The stock index is non-stationary (or integrated of order one, I(1)) since its mean level is not constant and rises over time.

The monthly return on the index (excluding dividend payments) in Figure 2 appears to have a constant mean but not the conditional variance.

Figure 2: US real returns S&P500 (monthly, Feb 1915—April 2004)
The relatively large proportion of ‘outliers’ in Figure 2 (i.e. very large or very small returns) probably implies that the unconditional returns are non-normal with fat tails and the distribution may be asymmetric.

This is confirmed in the histogram of returns, where the fat tails and the negative skew are clearly visible (Figure 3).

![Figure 3](image)
Returning to Figure 2, it is evident that the volatility in the monthly returns goes through periods of calm (e.g. the 1950s and 1960s) and turbulence (e.g. 1930s, 1970s and at the turn of the 20th century).

Once returns become highly volatile, they tend to stay volatile for some time, and, when returns are relatively small, they tend to stay small for some time.

Hence, volatility is conditionally autoregressive. As volatility in econometrics is known as heteroscedasticity, the behaviour of volatility in Figure 2 is said to follow an autoregressive conditional heteroscedasticity (ARCH) process.

A special case of this class of models is the so-called GARCH(1,1) model, which when fitted to the data in Figure 2 over the sample period February 1915 to April 2004, gives the following result.

\[
R_{t+1} = 0.00315 + \epsilon_{t+1}^2 + \epsilon_{t+1}(t+1)\Omega_t\epsilon_{t+1}N(0,h_{t+1})
\]

\[2.09\]

\[
h_{t+1} = 0.00071 + 0.8791h_t + 0.0067\epsilon_{t-1}^2
\]

\[2.21\] \[33.0\] \[4.45\] (t-statistics in parentheses)
where $h_{t+1}$ is the conditional variance of returns. The mean (real) return is 0.315% per month (3.85% p.a.).

The GARCH equation for the conditional variance $h_{t+1}$ is typical of results using stock returns where volatility at time $t + 1$, $h_{t+1}$ is conditional on volatility at time $t$, $h_t$ and the squared ‘surprise’ in returns $\varepsilon_t^2$.

The relatively large coefficient on the lagged $h_t$ term of 0.8791 implies that if volatility is high (low), it stays high (low) for some time.

The unconditional volatility is $\sigma^2 = 0.00071/(1-0.8791) = 0.029339$, which implies a standard deviation of 0.07663 (7.663% per month).

The time series of $h_{t+1}$ using the above equation is shown in Figure 4, in which the persistence in volatility is evident.
GARCH-type persistence effects in conditional volatility are found in daily, weekly and monthly returns but returns at lower frequencies generally do not exhibit GARCH effects.
In other words, volatility is persistent for short-horizon returns but not for long-horizon returns.

Now let us take a look at average annual returns and volatility for stocks, bonds and bills using a long data series for the period 1900–2000 (Table 1 – Dimson, Marsh and Staunton 2002).

The arithmetic mean returns $\overline{R}$ (in real terms) for stocks in the United Kingdom, in the United States and for a world index (including the USA) are between 7.2% and 8.7% p.a. (Table 1A).

The standard deviation of these (arithmetic) returns is around 20% p.a., indicating the high risk attached to holding stocks in any particular year.

The high volatility of stock returns also means that ex-post measures of average returns are sensitive to a run of good or bad years.

If returns are $niid$ (i.e. homoscedastic and not serially correlated), then the standard error in estimating the mean return is
and therefore we can be 95% certain that the mean return (for the USA) lies approximately in the range \( \bar{R} \pm 2\sigma_R = 8.7 \pm 4 = \{4.7, 12.7\} \) this is quite a wide range of possible outcomes for the mean return.

Of course, in any one year taken at random, the actual return has a standard deviation of around 20% p.a., and Table 1 shows a fall as great as 57% (in 1974 in the UK) and a rise of 96.7% (in the UK in 1975).
Table 1  Real returns: 1900–2000

Panel A: Real Stock Returns (% p.a.)

<table>
<thead>
<tr>
<th></th>
<th>Inflation</th>
<th>Real Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>UK</td>
<td>4.3 4.1</td>
<td>7.6 20.0 2.0 5.8 -57 (in 1974) +97 (in 1975)</td>
</tr>
<tr>
<td>USA</td>
<td>3.3 3.2</td>
<td>8.7 20.2 2.0 6.7 -38 (in 1931) +57 (in 1933)</td>
</tr>
<tr>
<td>World (incl. USA)</td>
<td>n.a. n.a.</td>
<td>7.2 17.0 1.7 6.8 n.a. n.a.</td>
</tr>
</tbody>
</table>

Panel B: Real Bond Returns (% p.a.)

<table>
<thead>
<tr>
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<th>Inflation</th>
<th>Real Return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Arith. Geom.</td>
<td>Arith. Mean Standard Deviation Standard Error Geometric Mean</td>
</tr>
<tr>
<td>UK</td>
<td>4.3 4.1</td>
<td>2.3 14.5 1.4 n.a.</td>
</tr>
<tr>
<td>USA</td>
<td>3.3 3.2</td>
<td>2.1 10.0 1.0 1.6</td>
</tr>
<tr>
<td>World (incl. USA)</td>
<td>n.a. n.a.</td>
<td>1.7 10.3 1.0 1.2</td>
</tr>
</tbody>
</table>

Panel C: Real Returns on Bills (% p.a.)

<table>
<thead>
<tr>
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<th>Inflation</th>
<th>Real Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>UK</td>
<td>4.3 4.1</td>
<td>1.2 6.6 0.7</td>
</tr>
<tr>
<td>USA</td>
<td>3.3 3.2</td>
<td>1.0 4.7 0.5</td>
</tr>
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</table>
The US stock market a Sharpe ratio (i.e. the excess return-per-unit of risk) of $SR = (R - r)/\sigma$ of around 0.5.

The equity premium is the excess return of stocks over bonds or bills (Table 2).

<table>
<thead>
<tr>
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<th>Over Bills</th>
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<th>Over Bonds</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>UK</td>
<td>6.5</td>
<td>4.8</td>
<td>2.0</td>
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<tr>
<td>USA</td>
<td>7.7</td>
<td>5.8</td>
<td>2.0</td>
<td>7.0</td>
</tr>
<tr>
<td>World (incl. USA)</td>
<td>6.2</td>
<td>4.9</td>
<td>1.6</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Notes: See Table 1.

The arithmetic average equity premium over bills is higher than over bonds.

The mean equity premium is measured with substantial error, reflecting the high volatility of stock returns noted above.
The 95% confidence interval for the arithmetic mean US equity premium is 7.7% ± 4%, which again covers quite a wide range.

Risk, Return and the Sharpe Ratio

High average stock returns in the United States cannot be merely due to high productivity of the US economy or impatience (time preference) by consumers, otherwise real returns on bonds would also be high.

This risk is substantial, giving a range of 12.7 to 4.7% p.a. for $R$ in the United States (with 95% confidence). The US equity premium of 7.7% (over bills) is the reward for holding this stock market risk.

Figure 5 demonstrates the first law of finance, namely, a higher average return implies a high-level of risk (i.e. standard deviation).

The ex-post Sharpe ratio $(R - r)/\sigma$, that is, the excess return per unit of risk, in Figure 5, is around 0.5.
Figure 5  Mean and standard deviation: annual averages, US real returns (post-1947)
Importance of Equity Returns

Campbell (2001), using a variety of methods, suggests a forecast US equity real return of 6.5 to 7% (arithmetic) and 5 to 5.5% (geometric) with a forecast real interest rate of 3 to 3.5%.

This implies a forecast range for the equity premium of 3 to 4% (arithmetic) for the United States.

Historic returns imply the *ex-post average* equity return is

\[ R_t = \left( \frac{D_t}{P_{t-1}} \right) + \frac{\Delta P_t}{P_{t-1}} \]

As well as measuring the equity premium directly from historic returns data, one can try and measure long-run *expected* capital gains using ‘price ratios’.
If a variable \((X/P)t\) is stationary (mean reverting), then over the long run, 
\[
\frac{\Delta X_t}{X_{t-1}} = \frac{\Delta P_t}{P_{t-1}},
\]
and the \(X\)-variable provides an alternative estimate of the average capital gain, to use in the above equation (in place of the actual capital gain).

The most obvious candidates for \(X_t\) are dividends \(D_t\) or earnings \(E_t\). Earnings figures are invariant to share repurchases, but are more volatile than dividends, which implies their mean value is estimated with greater uncertainty.

Dividend–price and earnings–price ratios move in very long slow swings and appear to be stationary variables.

Fama and French note that dividend and earnings price ratios can forecast either future dividend growth or future expected returns.

But the dividend–price ratio has hardly any explanatory power for the future growth in dividends.

Simple Models
Think of the regression models as a ‘data description’ or the ‘stylised facts’.

Under constant expected returns, ‘predictability’ violates informational efficiency and if abnormal profits in excess of transition costs and after correcting for risk are persistent, then this also violates the Efficient Markets Hypothesis (EMH).

Tests of the EMH require an equilibrium model of asset returns. We can think of the equilibrium expected return on a risky asset as consisting of a risk-free rate $r_t$ (e.g. on Treasury Bills) and a risk premium, $r_p_t$

$$E_t R_{t+1} \equiv r_t + r_p_t$$

Equation (1) is an identity until we have an economic model of the risk premium.

Many (early) empirical tests of the EMH assume $r_p_t$ and $r_t$ are constant and consider the regression

$$R_{t+1} = k + \gamma \Omega_t + \varepsilon_{t+1}$$

(2)
where $\Omega_t = \text{information available at time } t$. Alternatively, we can use excess returns $R_{t+1} - r_t$ in (2).

A test of $\gamma' = 0$ provides evidence on the ‘informational efficiency’ element of the EMH.

These regression tests vary, depending on the information assumed:

i. data on past returns $R_{t-j} (j = 0, 1, 2, \ldots, m)$

ii. data on past forecast errors $\varepsilon_{t-j} (j = 0, 1, \ldots, m)$

iii. data on variables such as the dividend–price ratio, the earnings–price ratio, interest rates, etc.

When (i) and (ii) are examined together, this gives rise to Autoregressive Moving Average (ARMA) models, for example, the ARMA (1,1) model:
\[ R_{t+1} = k + \gamma_1 R_t + \varepsilon_{t+1} - \gamma_2 \varepsilon_t \]  \hspace{1cm} (3)

If one is only concerned with weak-form efficiency, the autocorrelation coefficients between \( R_{t+1} \) and \( R_{t-j} \) \((j = 0, 1, \ldots, m)\) can be examined to see if they are non-zero.

The EMH applies over all holding periods: a day, week, month or even over many years.

There are two approaches when testing the EMH, one is informational efficiency and the other is the ability to make abnormal profits (i.e. profits after transaction costs and correcting for \textit{ex-ante risk}).

\textbf{Smart Money and Noise Traders}

Implications for stock returns and prices of there being some nonrational or noise traders in the market.
This enables us to introduce the concepts of mean reversion and excess volatility in a fairly simple way.

We assume that the market contains a particular type of noise trader, namely, a positive feedback trader whose demand for stocks increases after there has been a price rise.

To simplify matters, we assume the rational traders or smart money believe that expected equilibrium returns are constant:

\[ E_t \left[ (P_{t+1} + D_{t+1})/P_t \right] = k^* \]  (4)

If only the smart money (fundamentals’ traders) is present in the market, prices only respond to new information or news. Price changes are random, and past returns cannot be used to predict future returns.

Now consider introducing positive feedback traders into the market.

After any good news about dividends, positive feedback traders purchase the stock, increasing its price above fundamental value.
If the rational traders recognise this mispricing, they short-sell the overvalued stock, and the price moves back towards its fundamental value.

Prices are therefore mean-reverting. All this implies:

- Prices have *overreacted* to fundamentals (i.e. news about dividends).
- Prices are more volatile than would be predicted by changes in fundamentals.

As positive feedback traders purchase the stock, then over short horizons, returns are positively serially correlated: positive returns are followed by further positive returns.

But over *long horizons*, returns are negatively serially correlated as the rational traders move prices back to their fundamental value.

Thus, in the presence of feedback traders, short-horizon returns are positively serially correlated, while long-horizon returns are negatively serially correlated.
This pattern of serial correlation over different horizons implies that buying recent ‘winners’ will tend to yield winners next period – this is a *momentum strategy*.

Over long horizons (say, 3–5 years), the negative serial correlation implies you should buy low price stocks – this is a *value-growth strategy*.

Also, the above scenario implies that returns are likely to be correlated with changes in dividends and the dividend–price ratio, so regressions of $R_{t+1}$ on $(D/P)_t$ have often been interpreted as evidence for the presence of noise traders in the market.