There is a growing literature that focuses on the impact of nominal and real uncertainty on a country’s macroeconomic performance. As one of the first Bloom (2009) investigated uncertainty and its role as a potential impulse driving business cycles. He finds that uncertainty seems to play a potentially important role in business cycles as high uncertainty leads firms to delay their decisions on investment and hiring and that economic slowdowns could be driven by a combination of first and second moment shocks. Using various measures of uncertainty Bloom et al. (2012) show that shocks to uncertainty lead to a temporary fall in output and investment.

Empirical research on the topic has focused on estimating macroeconomic uncertainty and its potential impact on macroeconomic performance on a national level. Thereby no attention is paid on the potentially important global aspect of the trade-off between uncertainty and macroeconomic performance. We know that macroeconomic variables are highly correlated over developed countries and there exists a rich literature on comovement of macroeconomic variables. Less attention has been paid on the correlation of uncertainty. But an increase in US inflation uncertainty may not only affect macroeconomic variables in the US but may have an impact on other countries as well. Regarding the discussion about the great recession the exposure of countries to global risk or the transmission of uncertainty is an important feature. Countries were affected by the great recession although some of those countries did not even have strong financial linkages to the US. Some of them were affected even stronger than the US where the recession started. Transmission channels of the Great recession are not fully understood by now, but it seems, there can be identified two categories of transmission mechanisms, a direct contagion effect via first moment correlation and an indirect contagion through the impact of second moments. Regarding the latter effect, Bacchetta and van Wincoop (2010) provide a solution for those phenomena developing a model that explains a simultaneous increase in risk across different countries via the transmission of financial uncertainty. Uncertainty is often related to financial variables, but there exists also a growing literature arguing that uncertainty matters for business cycles. This paper estimates a measure for global macroeconomic uncertainty and analyzes its impact on individual countries’ macroeconomic performance.

For this purpose we set up a dynamic factor model (DFM) that decomposes inflation and output growth into country-specific and global components. The conditional variances of all factors are modeled as GARCH processes and interpreted as reflecting uncertainty or risk in the underlying factor. Thus we can distinguish between country-specific and global uncertainty. The contribution of the paper is twofold. First, as its main contribution this paper provides an extension to the discussion about the importance of real and nominal uncertainty for macroeconomic performance. Specifically, we separate the effect
of country-specific risk from the effect of global risk for the conditional mean of inflation and output growth. The identification of a global dimension of risk and its impact compared to country-specific risk on the outcome of macroeconomic variables is of interest especially regarding the increasing integration of real and financial markets. To the best of our knowledge this point has not yet been analyzed in the literature. Second, this paper contributes to the discussion of the relative importance of common vs. country-specific factors in inflation and output growth. Using a DFM, Kose et al. (2003) interpret the common factor in output growth across countries as the world business cycle and find that it accounts for high shares of volatility in the economic activity of countries. Similarly Ciccarelli and Mojon (2010), Mumtaz and Surico (2012), and Neely and Rapach (2008) focus on the contribution of global inflation to fluctuations in national inflation rates. While there is quite a large literature that estimates common factors in macroeconomic aggregates no effort has been made in estimating a corresponding global risk measure. The DFM employed in this paper allows us to study the time-variation in the conditional second moments of the different factors. Thus we are able to attribute changes in the unconditional variability of output growth and inflation to be either country-specific or global. A decline in the conditional variance of a country specific component can be interpreted as evidence for increased market integration.

The prevalent approach to measuring uncertainty in macroeconomic variables is to use the time-varying conditional variance of the series. Typically univariate or bivariate GARCH models are used to model time variation in the conditional variance. In order to test the impact of these uncertainties on the conditional means two approaches have been developed. The simultaneous approach estimates GARCH-in-mean models, in which the conditional variance is included as an explanatory variable in the conditional mean equation. Under the two step procedure the GARCH model is estimated and in a second step Granger-causality tests are performed to test for potentially bi-directional causality. Grier and Perry (1998) use the two-step approach to test for the direction of causality between average inflation and its uncertainty. Applying the same approach but estimating bivariate GARCH models Fountas et al. (2006), and Fountas and Karanasos (2007) investigate possible effects between inflation and output growth and its respective uncertainties. Grier et al. (2004), and Bredin and Fountas (2009) use GARCH-in-mean models of inflation and output growth to investigate such effects.

A bivariate dynamic factor GARCH-in-mean model

Starting point of the econometric framework is a bivariate dynamic latent factor model. We denote $Y_{it}$ as output growth and $\pi_{it}$ as inflation in country $i$. The mean equation is specified as

\[
\begin{bmatrix}
Y_{it} \\
\pi_{it}
\end{bmatrix} = \begin{bmatrix}
\Gamma_y^i \\
0
\end{bmatrix} \begin{bmatrix}
R^y_t \\
R^\pi_t
\end{bmatrix} + \begin{bmatrix}
I^y_{it} \\
I^\pi_{it}
\end{bmatrix}
\]

(1)

where $R^y_t$ denotes the common or global factor in output growth and $R^\pi_t$ denotes the common inflation factor. $I^y_{it}$ and $I^\pi_{it}$ are idiosyncratic or country specific factors in output growth and inflation respectively. $\Gamma^y_i$ and $\Gamma^\pi_i$ denote the country-specific factor loadings. The common factors in inflation and output growth are modeled as independent AR processes of order $p$:

\[
R^y_t = \sum_{k=1}^{p} \rho_j R^y_{t-k} + \varepsilon^y_t
\]

(2)

\[
R^\pi_t = \sum_{k=1}^{p} \theta_j R^\pi_{t-k} + \varepsilon^\pi_t
\]

(3)
The error terms $\varepsilon_{yt}$ and $\varepsilon_{\pi t}$ are white noise processes with mean zero and variance $\sigma_{yt}$ and $\sigma_{\pi t}$, i.e.

$$
\varepsilon_{yt}^v = [\sigma_{yt}]^{1/2} v_t^v \quad \text{(4)}
$$

$$
\varepsilon_{\pi t}^\pi = [\sigma_{\pi t}]^{1/2} v_t^\pi \quad \text{(5)}
$$

where $v_t^v, v_t^\pi \sim i.i.d.(0, 1)$ and where the conditional variances $\sigma_{yt}^v$ and $\sigma_{\pi t}^\pi$ follow GARCH(1, 1) processes,

$$
\sigma_{yt}^v = V_{t-1}(\varepsilon_t^v) = \alpha_0^v + \alpha_1^v \varepsilon_{t-1}^v + \alpha_2^v \sigma_{t-1}^v \quad \text{(6)}
$$

$$
\sigma_{\pi t}^\pi = V_{t-1}(\varepsilon_t^\pi) = \alpha_0^\pi + \alpha_1^\pi \varepsilon_{t-1}^\pi + \alpha_2^\pi \sigma_{t-1}^\pi \quad \text{(7)}
$$

The idiosyncratic factors follow bivariate VAR($p$) processes

$$
x_{it} = \sum_{k=1}^{p} \Phi_{ik} x_{it-k} + \eta_{it}, \quad \eta_{it} \sim \mathcal{N}(0, H_{it}) \quad \text{(8)}
$$

where

$$
x_{it} = \begin{bmatrix} y_{it}^v \\ I_{it}^v \end{bmatrix}, \quad \Phi_{ik} = \begin{bmatrix} \phi_{i11}^v & \phi_{i12}^v \\ \phi_{i21}^v & \phi_{i22}^v \end{bmatrix}, \quad \eta_{it} = \begin{bmatrix} \eta_{it}^v \\ \eta_{it}^\pi \end{bmatrix}, \quad H_{it} = \begin{bmatrix} h_{it}^v & h_{it}^\pi \\ h_{it}^{\pi v} & h_{it}^{\pi \pi} \end{bmatrix}.
$$

We impose a constant conditional correlation (CCC) GARCH(1, 1) structure on the conditional covariance matrix $H_t$. The conditional variances and the conditional covariance are given by

$$
h_{it}^v = \beta_{01}^v + \beta_{11}^v \eta_{it-1}^v + \beta_{12}^v h_{it-1}^v \quad \text{(9)}
$$

$$
h_{it}^\pi = \beta_{02}^\pi + \beta_{11}^\pi \eta_{it-1}^\pi + \beta_{12}^\pi h_{it-1}^\pi \quad \text{(10)}
$$

$$
h_{it}^{\pi v} = \rho_1 \sqrt{h_{it}^v h_{it}^\pi} \quad \text{(11)}
$$

The model differs in two points from a standard dynamic factor model. First, by modeling output growth and inflation for each country simultaneously it constitutes a bivariate factor model. Following Mumtaz et al. (2011) we assume that the common factors are orthogonal. The country specific output growth and inflation factor are allowed to be correlated within each country. The orthogonality of the common factors allows for (time-varying) variance decomposition into common vs. idiosyncratic factors. Second, the conditional variances of all factors are time-varying and follow GARCH processes. We interpret the GARCH series as uncertainty in the underlying factor, i.e. $\sigma_{yt}$ and $\sigma_{\pi t}$ measure uncertainty in the common or global factors of output growth and inflation. Thus, we refer to them as global real and nominal uncertainty. In order to analyze the impact of global uncertainty on individual country’s macroeconomic performance we augment equation (1) to include $\sigma_{yt}$ and $\sigma_{\pi t}$ as explanatory variables, i.e.

$$
[ \Gamma_{it}^v \\ \Gamma_{it}^\pi ] = \begin{bmatrix} \Gamma_{i}^v \\ \Gamma_{i}^\pi \end{bmatrix} \begin{bmatrix} R_{it}^v \\ R_{it}^\pi \end{bmatrix} + \begin{bmatrix} \delta_{i1}^v & \delta_{i2}^v \\ \delta_{i1}^\pi & \delta_{i2}^\pi \end{bmatrix} \begin{bmatrix} \sigma_{yt}^v \\ \sigma_{\pi t}^\pi \end{bmatrix} + \begin{bmatrix} T_{it}^v \\ T_{it}^\pi \end{bmatrix} \quad \text{(12)}
$$

The coefficients $\delta_{i}$ capture the sensitivity of output growth and inflation in country $i$ to global real and nominal uncertainty. In order to keep the model tractable we focus on the impact of global uncertainty. Including country specific uncertainties as additional GARCH-in-mean variables would increase the number of parameters to be estimated significantly.
Identification

For the empirical model to be identified we impose the following restrictions. First, note that we can multiply and divide the terms \( \Gamma^r_y R^y_i \) and \( \Gamma^r_y R^w_i \) by any constant and obtain a different decomposition of \( Y_t \) and \( \pi_t \). This identification problem, known as the scale problem in factor models, states that the factor’s variance and the factor loadings are not separately identified. The scale problem is solved by imposing an unconditional variance of unity on the shocks to the two common factors. This amounts to setting \( \alpha_0^y = 1 - \alpha_1^y - \alpha_2^y \) in equation (6) and \( \alpha_0^\pi = 1 - \alpha_1^\pi - \alpha_2^\pi \) in equation (7). Second, the signs of the factor loadings and the factors are not identified since the likelihood remains the same if we multiply both \( \Gamma^r_y \) and \( R^w_i \) (or \( \Gamma^r_\pi \) and \( R^w_i \)) by \(-1\). Therefore, we impose the restrictions \( \Gamma^r_y > 0 \) and \( \Gamma^r_\pi > 0 \). Further parameter restrictions are imposed on the parameters in the \( GARCH \) equations in order to ensure that all \( GARCH \) series are non-negative and stationary. Thus, we restrict \( 0 < \alpha_1^m < 1 \), \( 0 < \alpha_2^m < 1 \), and \( 0 < \alpha_1^m + \alpha_2^m < 1 \) for \( m = \{y, \pi\} \). Similarly \( \beta_0^m > 0 \), \( 0 < \beta_1^m < 1 \), \( 0 < \beta_2^m < 1 \), and \( 0 < \beta_1^m + \beta_2^m < 1 \) for \( m = \{y, \pi\} \).

Estimation

The vast majority of the DFM literature employs a Gibbs sampling scheme in order to estimate the factors and the parameters.\(^1\) Gibbs sampling is a Bayesian estimation technique that belongs to the class of Markov Chain Monte Carlo (MCMC) methods. Instead of evaluating the full joint posterior distribution directly, the Gibbs sampler is an iterative procedure that simulates from conditional densities which have known analytical solutions. This property of sampling from conditional densities is known as conjugacy. The sequential drawing of conditional densities yields random draws of the models’ posterior density.\(^2\) However the presence of \( GARCH \) effects makes the use of Gibbs sampling impossible. The reason is that in a \( GARCH \) model the conditional variance is a function of the conditional mean. The conditional posterior density of the conditional mean contains the conditional variance which itself depends on the conditional mean. Hence the conditional posterior density does not belong to a class of known densities, i.e. there is no conjugacy.\(^3\) As a consequence the estimation procedure for the dynamic factor \( GARCH \)-in-mean model needs to evaluate the full joint posterior.\(^4\)

The estimation technique used in this paper is a Metropolis-Hastings (MH) algorithm. Similar to the Gibbs sampler the MH algorithm is a Markov chain algorithm which draws from the exact posterior density. The basic idea of the MH algorithm is to draw samples from a proposal density and then apply an acceptance rule to decide if a draw belongs to the exact posterior density. If a draw does not get accepted the previous draw is kept thereby creating dependence in the sample. MH algorithm are widely used in applied econometrics, e.g. Geweke (1995) has proposed it for the estimation of \( GARCH \) models. While the MH algorithm performs well in small or medium size models it often fails in high dimensional models. If the dimension of the posterior distribution is large, the acceptance rates of new draws from the posterior distribution is close to zero implying very low mixing of the Markov chain and thus inefficient estimation (see e.g. Au and Beck, 2001). The most commonly used variants of the MH algorithm to solve high dimensional problems are the adaptive MH and the component wise MH algorithm. The former updates the covariance matrix of the proposal distribution within the sampling process by using the empirical covariance of the chain created so far (see e.g. Haario et al., 2001). The adjustment to the MH algorithm

\(^1\)While the likelihood function of a DFM model can easily be calculated using the Kalman filter, the numerical optimization is cumbersome when the number of parameter to be estimated is large.

\(^2\)A textbook treatment of DFM estimation using the Gibbs sampler is given by Kim and Nelson (1999).

\(^3\)Gibbs sampling can work well even in DFM models with time-varying variances. For instance DFM with stochastic volatilities can be estimated using Gibbs sampling. The crucial point here is that the time-varying variance is a function of the error term and thus of the conditional mean.

\(^4\)Bauwens and Lubrano (1998) combine the Gibbs sampler with a deterministic integration rule in order to estimate \( GARCH \) models. However, this so called Griddy-Gibbs sampling algorithm is computational intensive.
algorithm used here is the component-wise approach where only parts of the Markov chain are updated in one iteration.\textsuperscript{5}

To be more specific, let $\Theta_n = (\Theta_{n1}, \ldots, \Theta_{nd})$ be a Markov chain of dimension $d$. The component wise MH algorithm divides the elements of $\Theta_n$ into $z$ components, denoted by $\Theta_{nj}$. $\Theta_{n-j}$ denotes all components except the $j$th component, i.e.

$$\Theta_{n-1} = (\Theta_{n1}, \ldots, \Theta_{nj-1}, \Theta_{nj+1}, \ldots, \Theta_{nz}).$$

Let $\theta_{n1}, \ldots, \theta_{nz}$ be the state of the components $\Theta_{n1}, \ldots, \Theta_{nz}$ at time $n$ and $\theta'_{n-j}$ be the state of $\Theta_{n-j}$ after updating the components $1, \ldots, j - 1$. The component wise MH algorithm is as follows. For each $j = 1, \ldots, z$

1. Simulate a candidate $\zeta_{nj}$ from a proposal density $\psi (\cdot | \theta_{nj}, \theta'_{n-j})$.

2. Compute the acceptance probability $\gamma$ according to

$$\gamma = \min \left\{ \frac{p (\zeta_{nj} | \theta'_{n-j}) \psi (\theta_{nj} | \zeta_{nj}, \theta'_{n-j})}{p (\theta_{nj} | \theta'_{n-j}) \psi (\zeta_{nj} | \theta_{nj}, \theta'_{n-j})}, 1 \right\}$$

where $p(\cdot)$ denotes the posterior density. Thus, $p (\theta_{nj} | \theta'_{n-j})$ is the full conditional posterior distribution for the components $\Theta_{nj}$ given the current state $\theta'_{n-j}$ of all other components.

3. Set $\Theta_{n+1j} = \zeta_{nj}$ if the candidate is accepted, otherwise set $\Theta_{n+1j} = \Theta_{nj}$.

The number of parameter in each component is not necessarily equal. While there is no fixed rule as to how many parameters should be in one component, a general guideline is that parameters which are highly correlated should be sampled together. Therefore we form the components such that the number of parameters in one component is not larger than five but still sample parameters that are likely to be correlated jointly (e.g. the AR coefficients of each factor, the GARCH parameters for a given factor etc.).

The sampling procedure requires the calculation of the posterior density given a certain draw of parameters. The posterior density is proportional to the product of the likelihood function and the prior distribution. As common in the DFM literature we use normal priors for all non-variance parameters. The priors in the GARCH equations follow inverse Gamma distributions. All priors are non-informative.

In order to calculate the likelihood function we first put the model given by equations (2)-(12) in state space form. In particular, we estimate a conditionally Gaussian linear state space system including time-varying conditional variances (see Harvey, 1989). In Appendix B we report the state space representation of the model. Given the assumption of stationarity the initialization of the filter is non-diffuse. The time-varying conditional variances complicate the otherwise standard state space framework. To deal with this we follow the approach by Harvey et al. (1992) and augment the state vector with the shocks $\varepsilon^y_t$, $\varepsilon^\pi_t$, $\eta^y_{it}$ and $\eta^\pi_{it}$. The Kalman filter then provides estimates of the conditional variance of the shocks, i.e. estimates for $\sigma^y_t$, $\sigma^\pi_t$, $h^y_{it}$, and $h^\pi_{it}$. We refer to the Appendix for more details on the approach followed. To deal with potential computational difficulties that are caused by the relatively large dimension of the observation vector we follow the univariate approach to multivariate filtering and smoothing as presented by Koopman and Durbin (2000) and Durbin and Koopman (2001, chapter 6). A major advantage of this approach is that we can avoid taking the inverse of the variance matrix of the one-step-ahead prediction errors. We refer to Koopman and Durbin (2000) for the filtering recursion and for the calculation of the likelihood.

\textsuperscript{5}These two variants may also be applied simultaneously (see e.g. Haario et al., 2005).
Data and preliminary results

The deseasonalized data are from the OECD Main Economic Indicators. As an approximation for output growth we use the annualized monthly difference of logarithmized industrial production data and for inflation we use the annualized monthly difference of the logarithmized consumer price index. The data covers the period from January 1965 to July 2012. The sample includes nine industrialized countries, Canada, France, Germany, Italy, Japan, Netherlands, Spain, United Kingdom, and the United States. Given the GARCH structure of the model the data should be of a relatively high frequency. Infact the GARCH-in-mean literature discussed in the previous section typically uses monthly data as this is the highest frequency for which output growth and inflation data are available. Data availability is also the limiting factor regarding the choice of countries to be included in the sample. However the nine countries used here represent a share of about 80 per cent of OECD countries GDP.

Following the literature on real and nominal uncertainty we use monthly data on the change of industrial production and inflation covering the period from 1970-2010. The sample includes nine industrialized countries which reflect a share of about 80 per cent of OECD output. To estimate the model we derive its state-space form and estimate it using the Kalman filter and importance sampling techniques. We use a normal distribution as importance density with the mode and variance of an numerical optimization step as mean and variance parameters.

Table 1: Garch-in-Mean effects

<table>
<thead>
<tr>
<th></th>
<th>Real Uncertainty on</th>
<th>Nominal Uncertainty on</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Output Growth $\delta_{11}$</td>
<td>Inflation $\delta_{12}$</td>
</tr>
<tr>
<td>Canada</td>
<td>-1.21**</td>
<td>0.07</td>
</tr>
<tr>
<td>France</td>
<td>0.03</td>
<td>-0.27**</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.73*</td>
<td>0.05</td>
</tr>
<tr>
<td>Italy</td>
<td>-1.61***</td>
<td>-0.19***</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.79*</td>
<td>-0.14*</td>
</tr>
<tr>
<td>Netherlands</td>
<td>1.66**</td>
<td>0.22*</td>
</tr>
<tr>
<td>Spain</td>
<td>-0.69*</td>
<td>0.18*</td>
</tr>
<tr>
<td>UK</td>
<td>0.14</td>
<td>0.13*</td>
</tr>
<tr>
<td>US</td>
<td>-0.13</td>
<td>-0.21*</td>
</tr>
</tbody>
</table>

6Data on producer prices are only available for a substantially shorter time period and are therefore neglected.

7Following Morley et al. (2011) some outliers in the inflation series are interpolated to not allow them to dominate the results. We replace them with the Median of the six adjacent observations that were not outliers themselves. The outliers are Canada 1991:1, 1994:1, Germany 1991:1-1991:12, Japan 1997:3 and UK 1990:7.

8Canada, France, Germany, Italy, Japan, Netherlands, Spain, United Kingdom, United States
Figure 1: Common risk measures

Common uncertainty IP

Common uncertainty CPI
References


URL http://ideas.repec.org/a/bla/jeurec/v10y2012i4p716-734.html